

## RESEARCH ARTICLE

## PERFORMANCE IMPROVEMENT OF MIMO USING CS - SCHT.

Padmavathi Kora.<br>Associate Professor, Department of ECE, GRIET, India.

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#### Abstract

High speed data communications are in fact succeeded by MIMO based Alamouti technique. In order to meet the rapid developments in digital devices the mentioned technique could not meet the requirements, so there is a need to develop a new technique Conjugate Symmetric - Sequency ordered Complex Hadamard Transform (CS SCHT) to meet the enhanced speeds of the digital devices is presented in this paper. This fast and efficient algorithm to compute the CSSCHT is developed using sparse matrix factorization method, its computational load is examined and compared to that of Hadamard Transform (HT) and Sequency - ordered Complex Hadamard Transform (SCHT). The outcomes attained are compared with HT and SCHT based Alamouti OSTBC. A substantial enhancement of bit error rate (BER) is perceived from simulation results.


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## Introduction:-

For a MIMO [9] system the transceiver algorithms can be catalogued into two types, i.e., those which are required to multiply the transmission rate and those intended to increase reliability. The former is often equally stated to as spatial multiplexing and the later as transmit diversity by using OSTBC [5] technique. The Hadamard transform [6], [7] is regarded as a realistic tool for signal processing, especially in the areas of digital signal and image processing, filtering, communications and digital logic design owed to its simple implementation with the use of fast algorithms like Alamouti [12] - OSTBC diversity technique [2]. There are varieties of fast algorithms which are developed for various digital technologies, which include Haar Transform (HaT) [11], Discrete Wavelet Transform (DWT) [15], Karhunen-Loeve transform (KLT) [14], and Discrete Cosine Transform (DCT) [10] for improvement of diversity. Out of which DFT is used for signal conversion from time to frequency, DCT is used in various data and image compressions due to its superior energy compression. HADAMARD transform comprises of two levels ( $\pm 1$ ). And it can be applied only to real values. So in order to overcome this drawback we use Complex Hadamard Transform (CHT) [8] which comprises of four levels ( $\pm 1, \pm j$ ).In order to evaluate how fast of a specific row vector of a Hadamard transform would vary over a normalized time base $t \in[0,1)$ sequency ordered complex Hadamard Transform (SCHT) is used .The SCHT finds its application in spectrum estimation and image watermarking as well. The SCHT Transform can also be applied to the direct sequence (DS) CDMA systems, in which every row of SCHT matrix is employed as a complex spreading sequence which is assigned to a particular user. However SCHT coefficients are complex numbers comprising of both real and imaginary parts and they are not conjugate symmetric, hence, more memory is needed to store the coefficients for analysis and synthesis in transform implementation.

Corresponding Author:- Padmavathi Kora.
Address:- Associate Professor, Department of ECE, GRIET, India.

Hence a novel version of SCHT which surpasses the previous one called conjugate symmetric sequency-ordered complex Hadamard transform (CS-SCHT for short) [1] whose spectrum is conjugate symmetric, is projected in this paper. As CS-SCHT spectrum is conjugate symmetric, only half of spectral coefficients are required for combination and investigation. This in turn shrinks the memory requirement in processing for the applications such as real-time image watermarking and spectrum estimation.

In this paper, the Alamouti based OSTBC -MIMO with $2 \times 2$ transmit -receive antenna system is considered to improve the capability of reducing Rayleigh fading effect. A random signal is considered and modulated with BPSK and QPSK. In addition, SCHT is applied to this technique to improve the bit error rate of an MIMO system as shown in Fig 1. At the receiver end, the AWGN added signals is received by an estimator and these signals are decoded before they are demodulated. The inverse SCHT is performed at the receiver. The BER performance of BPSK and QPSK modulated signals are compared with the simulated results Alamouti scheme using HT [3] and Alamouti scheme using SCHT [4] are compared and plotted in Fig 3 and 4.


Fig.1. CS - SCHT -Alamouti $2 \times 2$ OSTBC based MIMO system model

## System Modeling:-

A MIMO channel with an input signal $s$ and added AWGN noise of $n$ is in the below equation (1):

$$
\begin{equation*}
y=Z U+a \tag{1}
\end{equation*}
$$

where $U$ represents the input signal, $Z$ represents the channel matrix, $y$ represents the signal received at the receiver along with the AWGN noise $a$.

## Space - Time Block Codes:-

A typical method, to transmit data over channels affected by Rayleigh fading using multiple antennas can be achieved through Space - time block coding. The data received by STBC are divided into smaller m branches and is transmitted using m transmitting antennas. At the receiver antenna, these m signals are added with noise. ML decoding is used for detection at the receiver while the orthogonal design is used for STBC to achieve maximum order for diversity with simple decoding algorithms.

## Alamouti Scheme:-

A simple transmit diversity technique called Alamouti is applied on MIMO where there are only two transmitting antennas and any number of receiving antennas as shown in Fig 2. The transmitting antenna number does not exceed two because the full rate power cannot be realized. If there are $M_{R}$ number of receiving antennas, then a maximum of $2 M_{R}$ diversity gain can be achieved at a fixed transmission rate.


Fig 2: Schematic Diagram of Alamouti Scheme
CONJUGATE SYMMETRIC SEQUENCY - ORDERED COMPLEX HADAMARD TRANSFORM
A Hadamard matrix, $\mathrm{H}_{M}$, is demarcated as a square matrix of dimension $M \times M$ in which 1 . All records of a matrix are $\pm 1,2$. Any two divergent rows of the matrix are orthogonal, that is shown in equation (2)

$$
\begin{equation*}
H_{M}^{T} H_{M}=H_{M} H_{M}^{T}=N I_{M} \tag{2}
\end{equation*}
$$

where $H_{M}^{T}$ is the transpose of $H_{N}$ and $I_{M}$ is the identity matrix of dimension $M \times M$. In fact, a Hadamard matrix is a symmetric matrix whose row and column vectors are orthogonal to each other. The orthogonality property rests unaffected, although the row and column vectors or sign of row and column vectors are exchanged as explained by the equation (2)

In this section, we discuss about the CS-SCHT matrices. Before going directly to the generation of the CS-SCHT, we first generate CS - NCHT. By using the bit - reversal of gray codes of each row of the CS - NCHT we obtain CS - SCHT. Let $\mathcal{H}_{M}$ be any CS-NCHT matrix of dimension $M \times M$ where $M=2^{m}$ Then, it is a square matrix defined in equation (3)
$\mathcal{H}_{M}=\left[\begin{array}{cc}\mathcal{H}_{\frac{M}{2}} & \mathcal{H}_{\frac{M}{2}} \\ \mathcal{H}^{\prime}{ }_{\frac{M}{2}} S_{\frac{M}{2}} & -\mathcal{H}^{\prime}{ }_{\frac{M}{2}} S_{\frac{M}{2}}\end{array}\right]$
Where $S_{2^{m-1}}$ is represented in (4)
$S_{2^{m-1}}=\left[\begin{array}{cc}I_{2^{n-2}} & 0 \\ 0 & j I_{2^{n-2}}\end{array}\right]$
And $\mathcal{H}^{\prime}{ }_{M / 2}$ is a real Hadamard matrix whose rows are arranged in a certain manner and it is defined recursively in equation (5)
$\mathcal{H}^{\prime}{ }_{M / 2}=\left[\begin{array}{cc}\mathcal{H}^{\prime}{ }_{M / 4} & \mathcal{H}^{\prime}{ }_{M / 4} \\ \mathcal{H}^{\prime}{ }_{M / 4} I^{\prime}{ }_{M / 4} & -\mathcal{H}^{\prime}{ }_{M / 4} I^{\prime}{ }_{M / 4}\end{array}\right]$
equation (6)
(5) where $M / 4=2^{m-2}$ and $I_{2^{m-2}}^{\prime}$ is represented in

$$
I_{2^{m-2}}^{\prime}=\left[\begin{array}{cc}
I_{2^{m-3}} & 0  \tag{6}\\
0 & -I_{2^{m-3}}
\end{array}\right]
$$

is the identity matrix of $\operatorname{size} 2^{m-2} \times 2^{m-2}$ where the lower half of the elements are multiplied by ( -1 ). Consequently, $\mathcal{H}_{M / 2}$ and $\mathcal{H}_{M / 2}^{\prime}$ and can be further reduced to the dimension of $2 \times 2$, which is defined as

$$
\mathcal{H}_{2}^{\prime}=\mathcal{H}_{2}=\left[\begin{array}{cc}
1 & 1 \\
1 & -1
\end{array}\right]
$$

In this way, an $M \times M$ CS-NCHT matrix can be stated using the low order CS-NCHT matrices of order $(M / 2) \times$ $(M / 2)$, and the smallest dimension of the CS-NCHT matrix will be the size of $4 \times 4$ as shown in equation (7)
$\mathcal{H}_{M}=\left[\begin{array}{cc}\mathcal{H}_{2} & \mathcal{H}_{2} \\ \mathcal{H}_{2}{ }_{2} S_{2} & -\mathcal{H}^{\prime}{ }_{2} S_{2}\end{array}\right]=\left[\begin{array}{cccc}1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \\ 1 & -j & -1 & j\end{array}\right]$
As a result, any CS-NCHT matrix of dimension $M \times M$ can be illustrated by WHT matrix and direct block matrix operator as represented in equation (8)
$\mathcal{H}_{M}=W_{M} \cdot A_{(m-1),(m-1)}^{\prime} \ldots . A_{2,2}^{\prime}, A_{1,1}^{\prime}$
(8) where $M=2^{m}, W_{M}$ is the $M \times M$ matrix,
$A_{1,1}=\left[I_{2}^{m-1}, S_{2}^{m-1}\right]^{T}$
$A_{2,2}^{\prime}=\left[I_{2^{(m-2)}}, S_{2^{(m-2)},} I_{2^{(m-2)}}, I_{2^{(m-2)}}^{\prime}\right]^{T}$
$A_{m-1, m-1}^{\prime}=\left[I_{2}, S_{2}, I_{2}, I_{2}^{\prime}, \ldots . . I_{2}, I_{2}^{\prime}\right]^{T}$
where (. $)^{T}$ in equation (9) represents the transpose, and. denotes the direct block matrix operator which was previously defined.
Let us, for example, consider $M=8$. Then $m=\log _{2} 8=3$ and (7) becomes (10)
$\mathcal{H}_{8}=w_{8} \cdot A_{(2,2)}^{\prime} \cdot A_{1,1}^{\prime}$
Substituting the corresponding values defined in (10) into (7),
we have equation (11)
$\mathcal{H}_{8}=\left[\begin{array}{cccc}W_{2} & W_{2} & W_{2} & W_{2} \\ W_{2} & -W_{2} & W_{2} & -W_{2} \\ W_{2} & W_{2} & -W_{2} & -W_{2} \\ W_{2} & -W_{2} & -W_{2} & W_{2}\end{array}\right] \cdot\left[\begin{array}{c}I_{2} \\ S_{2} \\ I_{2} \\ I_{2}^{\prime}\end{array}\right] \cdot\left[\begin{array}{l}I_{4} \\ S_{4}\end{array}\right]$
where $W_{2}=\left[\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right]$ is the $2 \times 2$ matrix. Therefore $H_{8}$ is obtained as shown in equation (12)

$$
\mathcal{H}_{8}=\left[\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1  \tag{12}\\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & 1 & j & j & -1 & -1 & -j & -j \\
1 & -1 & j & -j & 1 & -1 & j & -j \\
1 & -1 & -j & j & 1 & -1 & -j & j \\
1 & 1 & -j & -j & -1 & -1 & j & j
\end{array}\right]
$$

As such any CS - NCHT matrix of dimension $M \times M$ where $M=2^{m}$ can be achieved. Having defined the CSNCHT, a bit reversal conversion of CS-NCHT matrix produces CS-SCHT matrix and vice versa. Let $\mathcal{H}_{M}$ be any CSSCHT matrix of size $M \times M$. Then it is defined in equation (13)

$$
\begin{equation*}
H_{M}(r, c)=H_{M}(b(r), c) \tag{13}
\end{equation*}
$$

Where r and c are the row and column indexes of a matrix such that $0 \leq r, c \leq M-1$ and $\mathrm{b}(\mathrm{r})$ is the decimal number obtained by the bit-reversed operation of the decimal. As an example, $\mathcal{H}_{8}$ is obtained as shown in (14)

$$
\mathcal{H}_{8}=\left[\begin{array}{l}
\mathcal{H}_{8}(0, c)  \tag{14}\\
\mathcal{H}_{8}(1, c) \\
\mathcal{H}_{8}(2, c) \\
\mathcal{H}_{8}(3, c) \\
\mathcal{H}_{8}(4, c) \\
\mathcal{H}_{8}(5, c) \\
\mathcal{H}_{8}(6, c) \\
\mathcal{H}_{8}(7, c)
\end{array}\right]=\mathcal{H}_{8}=\left[\begin{array}{l}
\mathcal{H}_{8}(0, c) \\
\mathcal{H}_{8}(4, c) \\
\mathcal{H}_{8}(2, c) \\
\mathcal{H}_{8}(6, c) \\
\mathcal{H}_{8}(1, c) \\
\mathcal{H}_{8}(5, c) \\
\mathcal{H}_{8}(3, c) \\
\mathcal{H}_{8}(7, c)
\end{array}\right]
$$

in which the row vectors of the matrix are arranged in increasing order of zero crossings in the unit circle of a complex plane. Besides, the $r^{t h}$ row of the matrix is the conjugate of the $(M-r)^{t h}$ row vector where $r=$ $1,2,3 \ldots .(M / 2)-1$ and,$M=2^{m}$ hence, the spectrum obtained by using this matrix is shown to be conjugatesymmetric. The $0^{t h}$ and $(M / 2)^{t h}$ row vectors correspond to the DC and Nyquist frequency components in the DFT matrix, respectively. As such any CS-SCHT matrix of dimension $M \times M$ can be generated. This completes the construction of the CS-SCHT. Having developed the CS-SCHT matrix, the CS-SCHT of an M-point complex signal vector $y_{M}=[y(0), y(1), \ldots . . y(m-1)]^{T}$ is defined using equation (15)

$$
\begin{equation*}
Y_{M}=\frac{I}{M} H_{M} y_{M} \tag{15}
\end{equation*}
$$

where $[Y(0), Y(1), \ldots Y(m-1)]^{T}$ is the transformed complex column vector $($.$) , denotes the complex conjugate and$ $H_{M}$ is the CS-SCHT matrix as defined in (13). The data sequence can be uniquely recovered from the inverse transform, that is, represented as shown in equation (16)

$$
\begin{equation*}
y_{M}=H_{M}^{T} Y_{M} ` \tag{16}
\end{equation*}
$$

since $H_{M}^{T} H_{M}{ }^{`}=H_{M}^{*} H_{M}=M I_{M}$ (unitary property) where (.) represents the complex conjugate transpose.
These values are applied to $2 \times 2$ Alamouti scheme. This scheme generates two symbols $u_{1}$ and $u_{2}$ for transmission for first period and symbols $-u_{2}^{*}$ and $u_{1}^{*}$ for the second period to transmit two symbols all together as represented in equation (17).

$$
U=\left[\begin{array}{cc}
u_{1} & u_{2}  \tag{17}\\
u_{2}^{*} & -u_{1}^{*}
\end{array}\right], Z=\left[\begin{array}{l}
z_{1} \\
z_{2}
\end{array}\right]
$$

where $u_{1}=d_{1}+d_{2}$ and $u_{2}=d_{1}-d_{2}$
The output signal at an antenna is given as represented in equation (18)

$$
\left[\begin{array}{l}
y_{1}  \tag{18}\\
y_{2}
\end{array}\right]=\left[\begin{array}{cc}
u_{1} & u_{2} \\
u_{2}^{*} & -u_{1}^{*}
\end{array}\right]\left[\begin{array}{l}
z_{1} \\
z_{2}
\end{array}\right]+\left[\begin{array}{l}
a_{1} \\
a_{2}
\end{array}\right]=\left[\begin{array}{lr}
\left(d_{1}+d_{2}\right) & \left(d_{1}-d_{2}\right) \\
\left(d_{1}-d_{2}\right)^{*} & -\left(d_{1}+d_{2}\right)^{*}
\end{array}\right]\left[\begin{array}{l}
z_{1} \\
z_{2}
\end{array}\right]+\left[\begin{array}{l}
a_{1} \\
a_{2}
\end{array}\right]
$$

These will be recurrent for the next time slots also which are given as shown in following equation (19)
$\left[\begin{array}{l}y_{3} \\ y_{4}\end{array}\right]=\left[\begin{array}{cc}u_{3} & u_{4} \\ u_{4}^{*} & -u_{3}^{*}\end{array}\right]\left[\begin{array}{l}Z_{3} \\ z_{4}\end{array}\right]+\left[\begin{array}{l}a_{3} \\ a_{4}\end{array}\right]$

$$
=\left[\begin{array}{cc}
\left(d_{3}+d_{4}\right) & \left(d_{3}-d_{4}\right)  \tag{19}\\
\left(d_{3}-d_{4}\right)^{*} & -\left(d_{3}+d_{4}\right)^{*}
\end{array}\right]\left[\begin{array}{l}
z_{3} \\
z_{4}
\end{array}\right]+\left[\begin{array}{l}
a_{3} \\
a_{4}
\end{array}\right]
$$

By using the above signals, the output at each antenna can be obtained as represented in equation (20)

$$
\left[\begin{array}{c}
y_{1}^{1}  \tag{20}\\
y_{2}^{1} \\
y_{1}^{2 *} \\
y_{2}^{2 *}
\end{array}\right]=\left[\begin{array}{cc}
z_{11} & z_{12} \\
z_{21} & z_{22} \\
z_{12}^{*} & -z_{11}^{*} \\
z_{22}^{*} & -z_{21}^{*}
\end{array}\right]\left[\begin{array}{c}
d_{1}+d_{2} \\
\left(d_{1}-d_{2}\right.
\end{array}\right]+\left[\begin{array}{c}
a_{1}^{1} \\
a_{2}^{1} \\
a_{1}^{2 *} \\
a_{2}^{*}
\end{array}\right]
$$

The original signals are reconstructed from the received signals by using the inverse SCHT as shown below in (21)

$$
\begin{equation*}
\tilde{u}=H_{M} \vec{u} \tag{21}
\end{equation*}
$$

By doing the inverse SCHT, the original signal is regenerated at the receiver. Now BER is to be calculated.

## III. ERROR PERFORMANCE SIMULATION RESULTS

In this section bit error rate of CS - SCHT Alamouti based MIMO is calculated at respective SNR values by using following equation (22)

$$
\begin{equation*}
B E R=\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_{b}}{N_{0}}}\right) \tag{22}
\end{equation*}
$$

By considering the following Fig 3, it is observed that the Alamouti - CS - SCHT achieved a drastic diminution of BER compared to the Alamouti - HT and Alamouti - SCHT using BPSK modulation. These bit error rates can be further reduced when the modulation technique is changed to QPSK as shown in the Fig 4.

For BPSK modulation Alamouti - CS - SCHT achieves a BER of $0.8 \times 10^{-4}$ at 15 dB , Alamouti - SCHT has $1.5 \times 10^{-3}$ and $5.0 \times 10^{-3}$ for Alamouti - HT scheme. This is can be further reduced by Alamouti - CS - SCHT to $0.6 \times 10^{-4}$ for QPSK modulation.

The computational complexity of MIMO - OSTBC based Alamouti system using SCHT and CS - SCHT is compared in TABLE I.


Fig. 3:- Comparison of BER of CS-SCHT - Alamouti, Alamouti - SCHT and Alamouti - HT schemes for BPSK modulation


Fig. 4:- Comparison of BER of CS-SCHT - Alamouti, Alamouti - SCHT and Alamouti - HT schemes for QPSK modulation

Table 1:- Comparison of Computational Complexity of CS - SCHT and SCHT

| m | CS-SCHT | SCHT |
| :--- | :--- | :--- |
| 3 | 3 | 4 |
| 4 | 7 | 12 |
| 5 | 15 | 32 |
| 6 | 31 | 80 |
| 7 | 63 | 192 |
| 8 | 127 | 448 |
| 9 | 255 | 1024 |
| 10 | 511 | 2304 |

## Conclusion:-

Since the CS-SCHT spectrum of a real-valued data sequence is conjugate-symmetric and then transforms itself is sequency - ordered and pertinent for complex-valued functions, which makes it distinctive among the remaining transforms. Only half of the spectrum is needed for investigation and synthesis by applying conjugate - symmetry property. This attains saving memory in the processing of the transform in the applications when compared to HT and SCHT. The construction of the CS-SCHT is based on the CS-NCHT. This has been shown that the above algorithms can be derived from the WHT and direct block matrix operation. The fast CS-SCHT algorithm is derived using the sparse matrix factorization approach and its computational complexity is very much reduced when compared to that of SCHT and HT transforms It has been investigated that less number of complex multiplications by the trivial twiddle factors, i.e., are required to compute an m-point CS-SCHT when compared to that of SCHT using fast algorithm. The CS-SCHT proves to be a better choice to replace the DFT (at the expense of some reduction in accuracy of estimation) in signal analysis and synthesis as compared to the SCHT with simple implementation and less computational complexity $M$.

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