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### RESEARCH ARTICLE

#### NUMERICAL SOLUTION FOR MATHEMATICAL MODEL OF EBOLA VIRUS.

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#### Abstract

Mathematical Modeling has emerged as an important tool for understanding dynamics of many infectious diseases, one of which is the Ebola virus. The main focus of the presented work was to model mathematically the transmission dynamics of Ebola virus, for this purpose, the basic Susceptible-Infected-Recovery (SIR) model of Ebola was reviewed. The basic concept was underpinning the implementation of different numerical techniques like Euler, RK-2, and RK-4 of SIR model. Most optimistic estimates for each group of individuals were obtained like the susceptible group of individuals did not change their values and remained 460, but the infected group of individuals gradually decreased their values, and only nominal increase in case of recovered group of individuals were observed due to the high mortality rate of infected group in case of Ebola.

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#### Introduction:-

Ebola virus is negative stranded RNA virus causing Ebola haemorrhagic fever. It is an infectious and fatal disease. Its signs and symptoms appear in two days and after two or three weeks fever, sore throat, and muscular pain initiates by reducing the function of the liver and kidneys following the diarrhea and rashes. Bleeding may start from the body in certain cases. Ebola virus spreads by direct contact as well as through blood transfusion. It is so dangerous that after the recovery, the breast milk and semen of the male may carry this virus for several weeks to months as per World Health Organization report (WHO, 2016).

Ebola was first identified at two different places, one is Yambuku and the other is Nazara. It was mostly confined to Central Africa, but recently was also identified in West Africa [1] A village near the Ebola River, was first affected by this disease, so the disease was named as Ebola Virus Disease (EVD). World Health Organization (WHO) reported 4656 cases of Ebola virus deaths in October 8, 2014, with most cases occurring in Liberia [9]. Natural environment of Ebola consists of 3 hosts which are from the natural host, intermediate animal host and then to humans. There are four sub types of Ebola virus: Ebola Sudan, Ebola Zaire, Ebola Ivory, and Ebola Reston [10].

Mathematical modeling is a significant and powerful tool that can be employed in analyzing the spread and control of infectious diseases such as Ebola. Mathematical models are assumed to provide understanding of methods and suggest prevention, and control strategies [11].

The basic concept in mathematical modeling is to understand the transmission dynamics of diseases. The current study mainly deals with mathematical model of Ebola Virus and implication of three numerical methods Euler,

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Runge- Kutta-2 and Runge-Kutta-4. Finally the numerical results obtained are very optimistic for each group of individuals.

**Materials and Method:-**

The motivation of this study was to solve the well-known Mathematical model called Susceptible-Infected-Recovery (SIR) model. The real physical problems in the world usually exhibited nonlinear mathematical models which includes biological issues. It was extremely challenging to obtain the exact solutions for such problems that actually represented such phenomena. It was a big task for scientific community to search for appropriate methods such as numerical or perturbation method to solve nonlinear problems [4], but the numerical methods were considered to best for such problems. Therefore numerical techniques Euler, Runge-Kutta 2 and Runge-Kutta 4 were applied to solve the proposed Susceptible-Infected-Recovery(SIR) model in this study.

**Description of Model:-**

The total population in a specific place was divided into three groups, the susceptible group S(t), the infected group I(t) and the recovered group R(t), "t" was anytime interval ([2], [3], [14]). The total population was represented by "N" and was taken to be constant for a short time interval and was given by N=S (t)+I (t)+R (t).

Population of the susceptible group reduced as the infected peoples come into contact with them by the rate of infection β. Therefore the change in population of susceptible group was equal to the negative product of -β with S (t) and I (t):

$$\frac{dS(t)}{dt} = -\beta \frac{S(t)I(t)}{N} \tag{1}$$

Population of the infected group was changed according to two different ways:

- (a) Susceptible group who joined the infected group by adding the total population of infected group with the term βS (t) I (t)
- (b) Infected group who joined the recovered group in any time interval, in such way that the total population of infected group was decreased by a term μI (t). Therefore the differential equation of infected group was written as:

$$\frac{dI(t)}{dt} = \beta \frac{S(t)I(t)}{N} - \mu I(t) \tag{2}$$

Finally, the differential equation of recovered group that based on those peoples who recovered from the Ebola virus by a rate μ.

$$\frac{dR(t)}{dt} = \mu I(t) \tag{3}$$

**Euler Method:-**

Values of parameters used were β = 0.000318 and μ = 0.0175 and the initial conditions were S (0) =460, I (0) =12, R (0) =0

The Euler method

$$\begin{cases} w_0 = \alpha \\ w_{i+1} = w_i + hf(t_i, w_i) \end{cases} , i = 0, 1, 2, \dots, N - 1$$

The time interval was [0 90] and N=10 so

$$h = \frac{b-a}{N}$$

$$h = \frac{90-0}{10} = 9$$

$$t_0 = 0, t_1 = 9, t_2 = 18, t_3 = 27, \dots, t_{10} = 90$$

$$w_0 = \begin{pmatrix} S(0) \\ I(0) \\ R(0) \end{pmatrix} = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} = \begin{pmatrix} w_{1,0} \\ w_{2,0} \\ w_{3,0} \end{pmatrix}$$

Iteration-1 for i= 0

$$w_1 = w_0 + hf(t_0, w_0)$$

$$w_1 = \begin{pmatrix} w_{1,0} \\ w_{2,0} \\ w_{3,0} \end{pmatrix} + hf(0, w_0)$$

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + 9 \begin{pmatrix} -0.00038 \\ -0.20962 \\ 0.21 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 459.99658 \\ 10.11342 \\ 1.89 \end{pmatrix}$$

The successive iterations of Euler method were given in Table-1.

**Table 1:-** Solution of SIR Model by Euler method up to 10 iterations.

| i  | t <sub>i</sub> | S(t <sub>i</sub> ) | w <sub>1,i</sub> | I(t <sub>i</sub> ) | w <sub>2,i</sub> | R(t <sub>i</sub> ) | w <sub>3,i</sub> |
|----|----------------|--------------------|------------------|--------------------|------------------|--------------------|------------------|
| 0  | 0              | 460                | 460              | 12                 | 12               | 0                  | 0                |
| 1  | 9              | 460                | 459.9965         | 10.2513            | 10.1134          | 1.7486             | 1.8900           |
| 2  | 8              | 460                | 459.9937         | 8.75747            | 8.52339          | 3.2425             | 3.4828           |
| 3  | 27             | 460                | 459.9912         | 7.48130            | 7.1833           | 4.5187             | 4.8252           |
| 4  | 36             | 460                | 459.9892         | 6.39110            | 6.0540           | 5.6089             | 5.9566           |
| 5  | 45             | 460                | 459.9874         | 5.45977            | 5.1022           | 6.5402             | 6.9102           |
| 6  | 54             | 460                | 459.9860         | 4.66415            | 4.3000           | 7.3358             | 7.7138           |
| 7  | 63             | 460                | 459.9847         | 3.98448            | 3.6239           | 8.0155             | 8.3910           |
| 8  | 72             | 460                | 459.9838         | 3.40385            | 3.0541           | 8.5961             | 8.9618           |
| 9  | 81             | 460                | 459.9829         | 2.90783            | 2.5740           | 9.0921             | 9.4429           |
| 10 | 90             | 460                | 459.9821         | 2.48409            | 2.1693           | 9.5159             | 9.8483           |

**Runge-Kutta-2 Method**

Solution of the SIR model by RK-2 Heun’s method is presented as.

$$w_0 = \alpha$$

$$\left\{ w_{i+1} = w_i + \frac{h}{4} \left[ f(t_i, w_i) + 3f \left( t_i + \frac{2h}{3}, w_i + \frac{2h}{3} f(t_i, w_i) \right) \right] \right\}, \quad i = 0, 1, 2, \dots, N - 1$$

The time interval was [0 90] and N=10

so

$$h = \frac{b-a}{N}$$

$$h = \frac{90-0}{10} = 9$$

$$t_0 = 0, t_1 = 9, t_2 = 18, t_3 = 27, \dots, t_{10} = 90$$

$$w_0 = \begin{pmatrix} S(0) \\ I(0) \\ R(0) \end{pmatrix} = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} = \begin{pmatrix} w_{1,0} \\ w_{2,0} \\ w_{3,0} \end{pmatrix}$$

Iteration-1 for i= 0

$$w_1 = w_0 + \frac{h}{4} \left[ f(t_0, w_0) + 3f \left( t_0 + \frac{2h}{3}, w_0 + \frac{2h}{3} f(t_0, w_0) \right) \right]$$

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + \frac{9}{4} \left[ f \left( 0, \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} \right) + 3f \left( 0 + \frac{2(9)}{3}, \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + \frac{2(9)}{3} f \left( 0, \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} \right) \right) \right]$$

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + 2.25 \left[ \begin{pmatrix} -0.00038 \\ -0.20962 \\ 0.21 \end{pmatrix} + 3f \left( 6, \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + 6 \begin{pmatrix} -0.00038 \\ -0.20962 \\ 0.21 \end{pmatrix} \right) \right]$$

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + 2.25 \left[ \begin{pmatrix} -0.00038 \\ -0.20962 \\ 0.21 \end{pmatrix} + 3f \left( 6, \begin{pmatrix} 459.99772 \\ 10.74228 \\ 1.26 \end{pmatrix} \right) \right]$$

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + 2.25 \left[ \begin{pmatrix} -0.00038 \\ -0.20962 \\ 0.21 \end{pmatrix} + \begin{pmatrix} -0.00101 \\ -0.56295 \\ 0.56397 \end{pmatrix} \right]$$

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + \begin{pmatrix} -0.00313 \\ -1.73828 \\ 1.74143 \end{pmatrix}$$

$$w_1 = \begin{pmatrix} 459.99687 \\ 10.26172 \\ 1.74143 \end{pmatrix} = \begin{pmatrix} w_{1,1} \\ w_{2,1} \\ w_{3,1} \end{pmatrix}$$

The successive iterations of RK-2 Heun’s method are given in Table-2.

**Table 2:-**Solution of SIR Model by RK-2 method up to 10 iterations.

| i  | t <sub>i</sub> | S(t <sub>i</sub> ) | w <sub>1,i</sub> | I(t <sub>i</sub> ) | w <sub>2,i</sub> | R(t <sub>i</sub> ) | w <sub>3,i</sub> |
|----|----------------|--------------------|------------------|--------------------|------------------|--------------------|------------------|
| 0  | 0              | 460                | 460              | 12                 | 12               | 0                  | 0                |
| 1  | 9              | 460                | 459.99687        | 10.25132           | 10.26172         | 1.74868            | 1.74143          |
| 2  | 18             | 460                | 459.99419        | 8.75747            | 8.77523          | 3.24253            | 3.23059          |
| 3  | 27             | 460                | 459.99189        | 7.48130            | 7.50112          | 4.51870            | 4.50470          |
| 4  | 36             | 460                | 459.98993        | 6.39110            | 6.41453          | 5.60890            | 5.59325          |
| 5  | 45             | 460                | 459.98871        | 5.45977            | 5.46631          | 6.54023            | 6.54718          |
| 6  | 54             | 460                | 459.98731        | 4.66415            | 4.67447          | 7.33585            | 7.34044          |
| 7  | 63             | 460                | 459.98601        | 3.98448            | 3.99733          | 8.01552            | 8.01879          |
| 8  | 72             | 460                | 459.98497        | 3.40385            | 3.41827          | 8.59615            | 8.59886          |
| 9  | 81             | 460                | 459.98405        | 2.90783            | 2.92311          | 9.09217            | 9.09492          |
| 10 | 90             | 460                | 459.98331        | 2.48409            | 2.49968          | 9.51591            | 9.51591          |

**Runge-Kutta-4 Method**

The approximate solution by RK-4 method was

$$\begin{cases} w_0 = \alpha \\ w_{i+1} = w_i + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \end{cases}, \quad i = 0, 1, 2, \dots, N - 1$$

Where

$$k_1 = hf(t_i, w_i)$$

$$k_2 = hf \left( t_i + \frac{h}{2}, w_i + \frac{k_1}{2} \right)$$

$$k_3 = hf \left( t_i + \frac{h}{2}, w_i + \frac{k_2}{2} \right)$$

$$k_4 = hf(t_i + h, w_i + k_3)$$

Iteration-1 for i= 0

$$k_1 = 9f(t_0, w_0)$$

$$k_1 = 9f \left( 0, \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} -0.00342 \\ -1.88658 \\ 1.89 \end{pmatrix}$$

$$k_2 = 9f \left( t_0 + \frac{9}{2}, w_0 + \frac{k_1}{2} \right)$$

$$k_2 = 9f\left(4.5, \begin{pmatrix} 459.99829 \\ 11.05671 \\ 0.945 \end{pmatrix}\right) = \begin{pmatrix} -0.00315 \\ -1.73835 \\ 1.74143 \end{pmatrix}$$

$$k_3 = 9f\left(t_0 + \frac{9}{2}, w_0 + \frac{k_2}{2}\right)$$

$$k_3 = 9f\left(4.5, \begin{pmatrix} 459.99842 \\ 11.13082 \\ 0.87072 \end{pmatrix}\right) = \begin{pmatrix} -0.00315 \\ -1.74996 \\ 1.75310 \end{pmatrix}$$

$$k_4 = 9f(t_0 + 9, w_0 + k_3)$$

$$k_4 = 9f\left(9, \begin{pmatrix} 459.99685 \\ 10.25004 \\ 1.75310 \end{pmatrix}\right) = \begin{pmatrix} -0.00288 \\ -1.61145 \\ 1.61438 \end{pmatrix}$$

The obtained results were

$$w_1 = \begin{pmatrix} 460 \\ 12 \\ 0 \end{pmatrix} + \frac{1}{6} \left[ \begin{pmatrix} -0.00342 \\ -1.88658 \\ 1.89 \end{pmatrix} + 2 \begin{pmatrix} -0.00315 \\ -1.73835 \\ 1.74143 \end{pmatrix} + 2 \begin{pmatrix} -0.00315 \\ -1.74996 \\ 1.75310 \end{pmatrix} + \begin{pmatrix} -0.00288 \\ -1.61145 \\ 1.61438 \end{pmatrix} \right]$$

$$w_1 = \begin{pmatrix} 459.99685 \\ 10.25422 \\ 1.74891 \end{pmatrix} = \begin{pmatrix} w_{1,1} \\ w_{2,1} \\ w_{3,1} \end{pmatrix}$$

Similarly, the successive iterations of the RK-4 method were presented in Table-3.

**Table 3:-**Solution of SIR Model by RK-4 method up to 10 iterations.

| i  | t <sub>i</sub> | S(t <sub>i</sub> ) | w <sub>1,i</sub> | I(t <sub>i</sub> ) | w <sub>2,i</sub> | R(t <sub>i</sub> ) | w <sub>3,i</sub> |
|----|----------------|--------------------|------------------|--------------------|------------------|--------------------|------------------|
| 0  | 0              | 460                | 460              | 12                 | 12               | 0                  | 0                |
| 1  | 9              | 460                | 459.99685        | 10.25132           | 10.25422         | 1.74868            | 1.74891          |
| 2  | 18             | 460                | 459.99416        | 8.75747            | 8.76241          | 3.24253            | 3.24338          |
| 3  | 27             | 460                | 459.99185        | 7.48130            | 7.48765          | 4.51870            | 4.52043          |
| 4  | 36             | 460                | 459.98987        | 6.39110            | 6.39833          | 5.60890            | 5.61169          |
| 5  | 45             | 460                | 459.98817        | 5.45977            | 5.59877          | 6.54023            | 6.41293          |
| 6  | 54             | 460                | 459.98671        | 4.66415            | 4.78427          | 7.33585            | 7.22891          |
| 7  | 63             | 460                | 459.98546        | 3.98448            | 4.09809          | 8.01552            | 7.91631          |
| 8  | 72             | 460                | 459.98441        | 3.40385            | 3.51744          | 8.59615            | 8.49799          |
| 9  | 81             | 460                | 459.98351        | 2.90783            | 3.00576          | 9.09217            | 9.01060          |
| 10 | 90             | 460                | 459.98271        | 2.48409            | 2.56848          | 9.51591            | 9.44866          |

**Results and Discussion:-**

Results obtained in case of all three numerical techniques were found to be exactly the same as were found by homtopyperturbation method in [4]. Solution of the model is presented graphically in Fig-1. Moreover we solve the model analytically by linearization as follows:

The system of equations 1-3 was nonlinear which was linearized as:

Let

$$f_1 = \frac{dS(t)}{dt} = -\beta \frac{S(t)I(t)}{N}$$

$$f_2 = \frac{dI(t)}{dt} = \beta \frac{S(t)I(t)}{N} - \mu I(t)$$

$$f_3 = \frac{dR(t)}{dt} = \mu I(t)$$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial R} \end{pmatrix}_{(0,0,0)}$$

$$J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & \mu & 0 \end{pmatrix}$$

For Eigenvalues  $|\lambda I - J| = 0$

$$\begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & \mu & 0 \end{vmatrix} = 0$$

$$\lambda(\lambda(\lambda + \mu) + 0) = 0$$

$$\lambda = 0, 0, -\mu$$

Eigen vectors were found as

$$(\lambda I - J)x = 0$$

For  $\lambda = 0, 0, -\mu$ , Eigenvectors obtained were

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

And the solution of SIR Model was

$$X = C_1 e^{\lambda_1 t} V_1 + C_2 e^{\lambda_2 t} V_2 + C_3 e^{\lambda_3 t} V_3$$

$$\begin{pmatrix} S(t) \\ I(t) \\ R(t) \end{pmatrix} = C_1 e^{(0)t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + C_2 e^{(0)t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_3 e^{-\mu t} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$S(t) = C_1 e^0(0) + C_2 e^0(1) + C_3 e^{-\mu t}(0)$$

$$S(t) = C_2$$

$$I(t) = C_1 e^0(0) + C_2 e^0(0) + C_3 e^{-\mu t}(-1)$$

$$I(t) = -C_3 e^{-\mu t}$$

$$R(t) = C_1 e^0(1) + C_2 e^0(0) + C_3 e^{-\mu t}(1)$$

$$R(t) = C_1 + C_3 e^{-\mu t}$$

By the use of initial conditions  $S(0) = 460, I(0) = 12, R(0) = 0$  at  $t = 0$  obtained values were

$$C_1 = 12, C_2 = 460 \text{ and } C_3 = -12$$

Final analytical solution for nonlinear SIR model was:

$$S(t) = 460 \tag{4}$$

$$I(t) = 12e^{-\mu t} = 2.4841 \tag{5}$$

$$R(t) = 12 - 12e^{-\mu t} = 12(1 - e^{-\mu t}) = 9.5159 \tag{6}$$

These results are presented in Fig-1 for justification purpose. Each group of individuals i.e., Susceptible, Infected, and recovered were taken along y-axis versus time along x-axis.

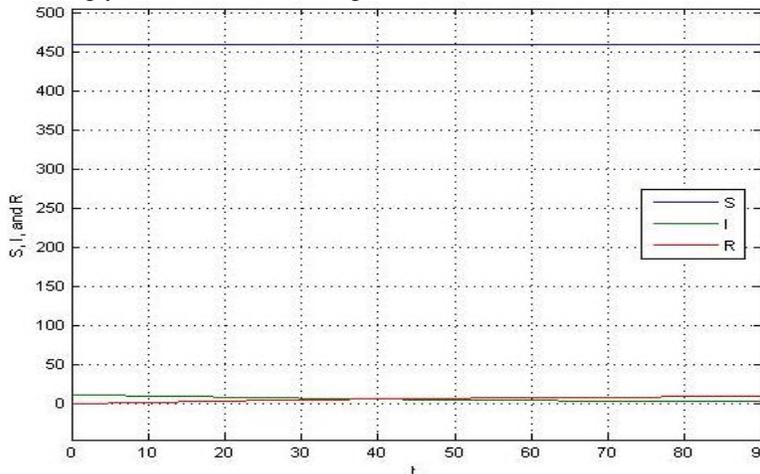


Figure1:-Solution of SIR Model (Time along X-axis & S, I,R along Y-axis).

Different numerical methods like Euler, Runge- Kutta 2, RK-2 and RK-4 were used to solve the model numerically, the results were presented in Tables 1, 2 and 3 were exactly the same as in Equations 4, 5 and 6 obtained by analytical method .

### Conclusion and Future Work:-

Solution of model by three numerical methods Euler, Runge-Kutta-2 (Heuns) and Runge-Kutta-4 were presented in Tables 1, 2 and 3. Conclusion was drawn that most optimistic estimates for each group of individuals were obtained like the susceptible group did not change and remained 460, but the infected group gradually decreased its values, due to the high mortality rate of infected group, and the recovered group slightly increased its values. This study can be extended further for parameter estimation and sensitivity analysis of the model.

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