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RESEARCH ARTICLE

PERIODOGRAM ANALYSIS WITH MISSED OBSERVATION BETWEEN TWO VECTOR VALUED STOCHASTIC PROCESS.

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Abstract

The estimation of the spectral measure, covariance and spectral density functions strictly stability (r+s) vector-valued time series are considered, under the assumption that some of observations are missed. The modified periodograms with missed observation are calculated. This method is applied in the climate.

Introduction:-

Several authors discussed the properties of the smoothing periodograms using data window and considered the estimation of spectral measure of stationary process, Brillinger(1969), Dahlhaus(1985), Ghazal and Farag(2000), Teama and Bakouch(2004), Ghazal(2001,2005), Ghazal, Faraj and El-Desokey(2005), Ghazal and Elhassanein(2006), Ghazal, Mokaddis and El- Desokey(2010), Elhassanein(2013).

Let $$B(t)=\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix},\ t=0,\pm 1,\pm 2,\ ..... \qquad (1.1)$$

with $X(t)$, r vector-valued and $Y(t)$, s vector-valued a strictly stability $(r+s)$ series, where $Y(t)\approx\sum a\,(t-u)\;X(t)$. We construct the statistics $I_{WW}{}^{(T)}(\lambda)(-\infty<\lambda<\infty)$, the matrix of second order smoothing modified periodograms, $F_{BB}{}^{(T)}(\lambda)$, which is the matrix of second order spectral measures, and $f_{BB}{}^{(T)}(\lambda)$ is the matrix of second order spectral densities .

Suppose that :

$$\begin{aligned} &E\{[X(t+u)-C_x][X(t)-C_x]^T\}=C_{xx}(u)\ ,\\ &E\{[X(t+u)-C_x][Y(t)-C_y]^T\}=C_{xy}(u)\ ,\\ &E\{[Y(t+u)-C_y][Y(t)-C_y]^T\}=C_{yy}(u)\ , \end{aligned} \qquad (1.2)$$

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and we defined the second-order spectral densities by

$$f_{xx}(\lambda)=(2\pi)^{-1}\int_{-\infty}^{\infty} C_{xx}(u)\,Exp\ \ (-i\lambda u)\,du\ ,$$

$$f_{xy}(\lambda)=(2\pi)^{-1}\int_{-\infty}^{\infty} C_{xy}(u)\,Exp\ \ (-i\lambda u)\,du\ , \qquad for\ \lambda\in R \tag{1.3}$$

$$f_{yy}(\lambda)=(2\pi)^{-1}\int_{-\infty}^{\infty} C_{yy}(u)\,Exp\ \ (-i\lambda u)\,du\ ,$$

we defined the second-order spectral measures by

$$F_{xx}(\lambda)=\int_{0}^{\lambda} f_{xx}(\alpha)\ d\alpha\ , \qquad (0<\lambda<\pi)$$

$$F_{xy}(\lambda)=\int_{0}^{\lambda} f_{xy}(\alpha)\ d\alpha\ , \qquad (0<\lambda<\pi) \tag{1.4}$$

$$F_{yy}(\lambda)=\int_{0}^{\lambda} f_{yy}(u)\ d\alpha, \qquad (0<\lambda<\pi)$$

we construct estimates $C_{XX}{}^{(T)}(u)$, $C_{XY}{}^{(T)}(u)$, $C_{YY}{}^{(T)}(u)$, $f_{XX}{}^{(T)}(\lambda)$, $f_{XY}{}^{(T)}(\lambda)$, $f_{YY}{}^{(T)}(\lambda)$, $F_{XX}{}^{(T)}(\lambda)$, $F_{XY}{}^{(T)}(\lambda)$ and $F_{YY}{}^{(T)}(\lambda)$. These estimates based on the matrix of second order smoothing modified periodograms .

Assumption I.

Let $X(t)$ is a strictly stability continuous time series all of whose moments are exist. For each $j=1,\ 2,\ ...,\ k-1$ and any k-tuple $a_1,\ a_2,\ ...,\ a_k$ we have,

$$\int_{R^{K-1}} |u_j| \left| C_{a_1,....,a_k}(u_1,....,u_{k-1}) \right| d_{t_1}...d_{t_{K-1}}\ <\infty\ , \qquad k=2,\ 3,\ ...$$

where

$$C_{a_1,....,a_k}(u_1,...,u_{k-1})=cum\{X_{a_1}(t+u_1),\ X_{a_1}(t+u_2),\ ...,X_{a_k}(t)\ \},$$
$$(a_1,\ ...,a_k=1,\ 2,\ ...,r,\ \ u_1,\ ...,u_{k-1},\ \ t\in R,\ \ k=2\ ,\ 3,\ ....)$$

Assumption II.

Let $\Psi(\alpha), \alpha\in R$ is a weight function which is bounded and has bounded first derivative such that

$$\int_{-\pi}^{\pi}\Psi(\alpha)d\alpha=1.$$

Given $B_T>0$ we then set

$$\Psi^{(T)}(\alpha)=B_T^{-1}\Psi^{(T)}(B_T^{-1}\alpha).$$

Assumption III.

Let $h_a^{(T)}(t)$ is bounded has bounded variation and vanishes for $0<t<T-1$, is called data window and satisfies

$$\frac{1}{T}\int_0^T h_a^{(T)}\, d_t \underset{T\to\infty}{\longrightarrow} \int_0^1 h_a(u)\,du, \qquad a=\overline{1,r}$$

$$G^{(T)}_{a_1,\ldots,a_k}(\lambda)=\int_0^T\left[\prod_{j=1}^{k} h_{a_j}^{(T)}(t)\right]\exp\{-i\lambda t\}dt,$$

for $-\infty<\lambda<\infty$, $a_1,\ldots,a_k=1,\ldots, r$

Let $H_a(t), a=1, 2, \ldots, r\,(t\in R)$ be a process independent of $B(t)$ such that for every t ,

$$P[H_a(t)=1]=p_a \;, \qquad P[H_a(t)=0]=q_a \;. \tag{1.5}$$

Note that $$E\{H_a(t)\}=P \;. \tag{1.6}$$

The success of recording an observation not depends on the fail of another and so they are independent . We may then define the modified series as

$$W(t)=H(t)B(t) \;, \tag{1.7}$$

where

$$W_a(t)=H_a(t)B_a(t), \tag{1.8}$$

and

$$H_a(t)=\begin{cases}1 \;, & \text{if } X_a(t), Y_a(t) \text{ are observed}\\ 0 \;, & otherwise\end{cases} \;, \tag{1.9}$$

We construct the expanded finite Fourier transform with data window with missed observations as :

$$d_a^{(T)}(\lambda)=\left[2\pi\int_0^T\left(h_a^{(T)}(t)\right)^2\right]^{-1/2}\int_{-\infty}^{\infty} h_a^{(T)}(t)\, W_a(t)\exp\{-i\lambda t\}dt, \qquad for\ \lambda\in R, \tag{1.10}$$

The paper is organized as follows : In Section(1) Introduction, Section (2) we will considered the smoothing modified periodograms, We will study the statistical properties of the spectral measure and spectral density in Section (3), application on our theoretical study in the climate in Section (4).

2. The Smoothing Modified Periodograms.

In this section, the modified periodogram will be constructed. Using expanded finite Fourier transform (1.10) we construct the modified periodogram as:

$$I_{ab}^{(T)}(\lambda)=\{2\pi G_{ab}^{(T)}(0)\}^{-1}\partial_a^{(T)}(\lambda)\overline{\partial_b^{(T)}(\lambda)} \;, \tag{2.1}$$

where

$$\partial_{X_a}^{(T)}(\lambda)=\int_0^T h_a(t)W_a(t)\,\exp(-i\lambda t)d\,t\,. \tag{2.2}$$

The bar denotes the complex conjugate . The moments of modified periodogram will be given in the following theorems.

Theorem 2.1

Let $W_a(t)=H_a(t)B_a(t)$, $a=1, 2, \ldots, \min(r,s)$ are missed observations on the strictly stability continuous series which satisfies Assumption I with mean zero, $h_a(u), -\infty<u<\infty$ satisfies Assumption (III) for $a=1, \ldots, \min(r,s),$ and let

$$I_{WW}{}^{(T)}(\lambda) = \left[I_{ab}{}^{(T)}(\lambda)\right] = \left[\left\{2\pi G_{ab}{}^{(T)}(0)\right\}^{-1}\partial_a{}^{(T)}(\lambda)\overline{\partial_b{}^{(T)}(\lambda)}\right],$$

then

$$E\left[I_{ab}{}^{(T)}(\lambda)\right] = \begin{bmatrix} P_{a_1a_2} f_{a_1a_2}(\lambda) & P_{a_1b_2} f_{a_1b_2}(\lambda)A(\lambda)^T \\ P_{b_1a_2} A(\lambda) f_{b_1a_2}(\lambda) & P_{b_1b_2} A(\lambda) f_{b_1b_2}(\lambda)A(\lambda)^T \end{bmatrix} + \begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix}, \quad (2.3)$$

Where $O(T^{-1})$ is uniform in λ and $A(\lambda) = f_{yx}(\lambda) f_{xx}(\lambda)^{-1}$.

and

$$Cov\left[I_{a_1b_1}{}^{(T)}(\lambda), I_{a_2b_2}{}^{(T)}(\mu)\right] = \left\{G_{a_1b_1}{}^{(T)}(0)G_{a_2b_2}{}^{(T)}(0)\right\}^{-1} \times$$
$$\times\left[(P^4 G_{a_1a_2}(\lambda-\mu)\overline{G_{b_1b_2}(\lambda-\mu)}VZ + P^4 G_{a_1b_2}(\lambda+\mu)\overline{G_{b_1a_2}(\lambda+\mu)}VZ\right] +$$
$$+ T^{-2} M^{(T)}_{a_1b_1a_2b_2}{}^{(T)}(\lambda,\mu) + O(T^{-1}), \quad (2.4)$$

where

$$V = \begin{bmatrix} f_{a_1a_2}(\lambda) & f_{a_1b_2}(\lambda)A(\lambda)^T \\ A(\lambda) f_{b_1a_2}(\lambda) & A(\lambda) f_{b_1b_2}(\lambda)A(\lambda)^T \end{bmatrix}, Z = \begin{bmatrix} f_{a_1a_2}(-\lambda) & f_{a_1b_2}(-\lambda)A(\lambda)^T \\ A(\lambda) f_{b_1a_2}(-\lambda) & A(\lambda) f_{b_1b_2}(-\lambda)A(\lambda)^T \end{bmatrix},$$

$$O = \begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix}$$

Lemma 2.1.
Let $h_a{}^{(T)}(t),\ t \in R,\ a = 1, ..., \min(r,s)$ is bounded by a constant L and satisfying

$$\left|h_a{}^{(T)}(t+u) - h_a(t)\right| \le C|u| \ ,$$

then

$$\left|\int_0^T h_{a_1}{}^{(T)}(t) h_{a_2}{}^{(T)}(t)\exp(-i\lambda t)\,dt\right| \le \frac{1}{|\lambda/2|} + LC \ , \quad (2.5)$$

for some constants L, C and $\lambda,\ \lambda \in R,\ \lambda \ne 0,\ a_1, a_2 = 1,..., \min(r,s)$.

Corollary 2.1.
Under the conditions of theorem (2.1) we have

$$E\left[I_{ab}{}^{(T)}(\lambda)\right] \to P^2 \begin{bmatrix} f_{a_1a_2}(\lambda) & f_{a_1b_2}(\lambda)A(\lambda)^T \\ A(\lambda) f_{b_1a_2}(\lambda) & A(\lambda) f_{b_1b_2}(\lambda)A(\lambda)^T \end{bmatrix} \text{as } T \to \infty,$$
$$a, b = 1, ..., \min(r,s),\ \lambda \in R.$$

Proof
The prove comes directly from (2.**3**) by taking the limits for both sides and then using the given conditions .

Corollary 2.2.

Under the conditions of theorem (2.1) then for all $\lambda, \mu \in R$,

$$\lim_{T\to\infty} Cov\left[I_{a_1b_1}{}^{(T)}(\lambda), I_{a_2b_2}{}^{(T)}(\mu)\right] =$$

$$= \begin{cases} P^4\delta(\lambda-\mu)VZ + P^4\delta(\lambda+\mu)VZ, & if\ \lambda \pm \mu = 0 \\ 0, & if\ \lambda \pm \mu \neq 0 \end{cases}$$

Where $\delta(\lambda-\mu)$ is the Kroncker delta function which is given by :

$$\delta(\lambda) = \begin{cases} 1, & \lambda = 0 \\ 0, & ow \end{cases}$$

Proof:

When $\lambda \pm \mu = 0$, and by using the Assumption III then we get from (2.**4**)

$$Cov\left[I_{a_1b_1}{}^{(T)}(\lambda), I_{a_2b_2}{}^{(T)}(\mu)\right] = P^4\delta(\lambda-\mu)VZ + P^4\delta(\lambda+\mu)VZ + O(T^{-1})$$

In the limit , then

$$Cov\left[I_{a_1b_1}{}^{(T)}(\lambda), I_{a_2b_2}{}^{(T)}(\mu)\right] = P^4\delta(\lambda-\mu)VZ + P^4\delta(\lambda+\mu)VZ\ .$$

Now, when $\lambda \pm \mu \neq 0$, $\lambda,\ \mu \in R$, then take the modulus for both sides of (2.4) and then using lemma (2.1) and the boundedness of $f_{ab}(\lambda)\ a,\ b = 1, 2, ..., \min(r,s),\ \lambda \in R,$ we obtain

$$\left|Cov\left[I_{a_1b_1}{}^{(T)}(\lambda), I_{a_2b_2}{}^{(T)}(\mu)\right]\right| \leq \left\{G_{a_1b_1}{}^{(T)}(0)G_{a_2b_2}{}^{(T)}(0)\right\}^{-1} \times$$

$$\times \left\{\left[\frac{2L_1\nu_1}{|\sin(\lambda+\mu)/2|}\right]^2 K^2 + \left[\frac{2L_2\nu_2}{|\sin(\lambda-\mu)/2|}\right]^2 K^2\right\} +$$

$$+ T^{-2}\left|T^{-2}M^{(T)}_{a_1b_1a_2b_2}(\lambda,\mu)\right| + (\boldsymbol{T^{-1}}),$$

where , for some constant K, we have

$$\left|T^{-2}M^{(T)}_{a_1b_1a_2b_2}(\lambda,\mu)\right| \leq K\left\{\left[\frac{2L_1\nu_1}{|\sin(\lambda+\mu)/2|}\right] + \left[\frac{2L_2\nu_2}{|\sin(\lambda-\mu)/2|}\right] \times\right.$$

$$\left.\times \left[\frac{2L_3\nu_3}{|\sin(\lambda+\mu)/2|}\right] + \left[\frac{2L_4\nu_4}{|\sin(\lambda-\mu)/2|}\right]\right\},$$

using lemma (2.1) we get $Cov\left[I_{a_1b_1}{}^{(T)}(\lambda), I_{a_2b_2}{}^{(T)}(\mu)\right] \to 0$ as $T \to \infty$ .hence, the corollary is obtained. In the case of $\lambda = \pm\mu$ corollary (2.2) indicates corollary (2.3) as the following .

Corollary 2.3.

Under the conditions of theorem(2.1) and corollary (2.2) then,

$$\lim_{T\to\infty} D[I_{ab}{}^{(T)}(\lambda)] = \begin{cases} P^4\delta(\lambda-\mu)VZ\ , & if\ \lambda = \mu = \omega \neq 0 \\ P^4\delta(\lambda-\mu)VZ + P^4\delta(\lambda+\mu)VZ, & if\ \lambda = \mu = \omega = 0 \end{cases}$$

Proof

By substituting about $\lambda = \mu = \omega,\ \omega \in R$, $a_1 = a_2 = a$, $b_1 = b_2 = b$, $a, b = 1, .., \min(r,s)$ into corollary(2.2) we get

$$\lim_{T\to\infty} D[I_{ab}{}^{(T)}(\lambda)] = P^4\delta(\omega-\omega)VZ + P^4\delta(\omega+\omega)VZ\,,$$

when $\omega \neq 0$, by noting that $f_{ab}(\omega) = f_{ba}(-\omega)$ into $V,\ Z$, $a, b = 1, ..., \min(r,s)$, $\omega \in R$ then

$$\lim_{T\to\infty} D[I_{ab}{}^{(T)}(\lambda)] = P^4\delta(\omega-\omega)VZ\,.$$

When $\omega = 0$, then we obtain

$$\lim_{T\to\infty} D[I_{ab}{}^{(T)}(\lambda)] = P^4VZ + P^4VZ\,.$$

Hence the proof is complete .

3. Asymptotic moments of spectral measure and spectral density function

In this section we will study the statistical properties of $F_{xx}{}^{(T)}(\lambda)$ and $f_{xx}{}^{(T)}(\lambda)$ by deriving mean and covariance. Let $f_{xx}{}^{(T)}(\lambda)$ be defined as (1.3) and $F_{xx}{}^{(T)}(\lambda)$ be defined as (1.4), then from Theorem (2.1) we get,

Theorem 3.1

Let $W(t)$ satisfies Assumption I then**:**

$$E\left\{F_{ab}{}^{(T)}(\lambda)\right\} = P^2\begin{bmatrix} F_{a_1a_2}(\lambda) & F_{a_1b_2}(\lambda)A(\lambda)^T \\ A(\lambda)F_{b_1a_2}(\lambda) & A(\lambda)F_{b_1b_2}(\lambda)A(\lambda)^T \end{bmatrix} + \begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix}, \qquad (3.1)$$

where $O(T^{-1})$ is uniform in λ .

$$\operatorname{cov}\left\{F_{a_1b_1}{}^{(T)}(\lambda_1), F_{a_2b_2}{}^{(T)}(\lambda_2)\right\} = \left[\left\{G_{a_1b_1}{}^{(T)}(0)G_{a_2b_2}{}^{(T)}(0)\right\}^{-1}\right] \times$$
$$\times P^4 G_{a_1a_2b_1b_2}(0)\left[\int_{-\infty}^{\lambda_1} VZd\alpha_1 + \int_{-\infty}^{\lambda_1} VZd\alpha_1\right] + O(T^{-1}) \qquad (3.2)$$

Corollary 3.1.

Let $W_a(t) = H_a(t)B_a(t)$, $a = 1, 2, ..., \min(r,s)$ are missed observations on the strictly stability continuous series which satisfies Assumption(I) with mean zero, $h_a(t), -\infty < t < \infty$, be data window satisfies Assumption (III) for $a = 1, ..., \min(r,s)$, and let

$$I_{ww}{}^{(T)}(\lambda) = \left[I_{ab}{}^{(T)}(\lambda)\right] = \left[\left\{2\pi G_{ab}{}^{(T)}(0)\right\}^{-1} d_a{}^{(T)}(\lambda)\overline{d_b{}^{(T)}(\lambda)}\right],$$

then

$$E\left[F_{ab}{}^{(T)}(\lambda)\right] \underset{T\to\infty}{\longrightarrow} P^2\begin{bmatrix} F_{a_1a_2}(\lambda) & F_{a_1b_2}(\lambda)A(\lambda)^T \\ A(\lambda)F_{b_1a_2}(\lambda) & A(\lambda)F_{b_1b_2}(\lambda)A(\lambda)^T \end{bmatrix}, \qquad (3.3)$$

for all $a, b = 1, ..., \min(r,s)$.

proof

Formula (3.3) comes directly by taking the limits for both sides of (3.1) and the proof is complete.

Corollary 3.2.

Under the conditions of theorem (3.1) if the spectral density function $f_{ab}(x)$ is bounded by a constant k, $a, b = 1, ..., \min(r,s)$ and continuous at point $x = \lambda,\ \lambda \in R,$ then

$$\lim_{T\to\infty} Cov\left[F_{a_1b_1}{}^{(T)}(\lambda), F_{a_2b_2}{}^{(T)}(\mu)\right] = 0 ,$$

for $a_j, b_j = 1, ..., \min(r,s)$, $j = 1, ..., k,\ k = 1, 2, ...$

Proof

Taking the modulus on both sides of (3.2) , we get

$$\left|Cov\left[F_{a_1b_1}{}^{(T)}(\lambda), F_{a_2b_2}{}^{(T)}(\mu)\right]\right| \le (2\pi)\left|p^4\right| G_{a_1a_2b_1b_2}(0)\times$$

$$\times\left\{G_{a_1b_1}{}^{(T)}(0) G_{a_2b_2}{}^{(T)}(0)\right\}^{-1} \times \left|P^4\right| G_{a_1a_2b_1b_2}(0)\times \left[\int_{-\infty}^{\lambda_1} |V||Z|\, d\alpha_1 + \int_{-\infty}^{\lambda_1} |V||Z|\, d\alpha_1\right] + O(T^{-1})$$

.

Using Assumption (III) and the boundedness of $f_{ab}(\lambda)$, $a, b = 1, ..., \min(r,s),\ \lambda \in R$ we get

$$Cov\left[F_{a_1b_1}{}^{(T)}(\lambda), F_{a_2b_2}{}^{(T)}(\mu)\right] = O(T^{-1}) \underset{T\to\infty}{\longrightarrow} = 0 \quad .$$

Then the corollary is obtained .

Lemma 3.1.

Let $h_a^{(T)}(t), -\infty < t < \infty,$ be data window satisfies Assumption III for $a = 1, ..., \min(r,s)$ then $h_a^{(T)}(t)$ satisfies the following properties

1. $\displaystyle\int_{t_2=0}^{T} \Psi_{a_2b_2}{}^{(T)}(\lambda_2 - \alpha_2) G_{a_1a_2}(\lambda - \mu)\, dt_2 = 2\pi \Psi_{a_2b_2}{}^{(T)}(\lambda_2 - \mu) + O(1)$,
2. $\displaystyle\int_{t_2=0}^{T} \Psi_{a_2b_2}{}^{(T)}(\lambda_2 - \alpha_2) G_{a_1a_2}(\lambda + \mu)\overline{G_{b_1b_2}(\lambda + \mu)}\, dt_2 =$

$$= 2\pi G_{a_1a_2b_1b_2}{}^{(T)}(0) \Psi_{a_2b_2}{}^{(T)}(\lambda_2 - \alpha_1) + O(T^{-1}) .$$

Theorem 3.2

Let $W_a(t) = B_a(t) H_a(t),\ t = 0, \pm 1, ...$, $a = 1, ..., \min(r+s)$ are missed observations on the strictly stability continuous $B_a(t), a = 1, ..., \min(r+s)$, $t \in R$ which satisfies Assumption I with mean zero, $h_a(t), a = 1, ..., \min(r+s)$,$t \in R$ be data window satisfies Assumption III, and let

$$f_{ab}{}^{(T)}(\lambda) = \int_{t=0}^{T} \Psi^{(T)}(\lambda - \alpha) \mathrm{I}_{ab}{}^{(T)}(\alpha)\, dt , \qquad (3.4)$$

Where $\Psi^{(T)}(\lambda - \alpha)$ is weight function which is defined in assumption II. Then

$$E\left\{f_{ab}{}^{(T)}(\lambda)\right\} = P^2 \int_{t=0}^{T} \Psi_{ab}{}^{(T)}(\lambda - \alpha) \begin{bmatrix} f_{a_1a_2}(\alpha) & f_{a_1b_2}(\alpha) A(\alpha)^T \\ A(\alpha) f_{b_1a_2}(\alpha) & A(\alpha) f_{b_1b_2}(\alpha) A(\alpha)^T \end{bmatrix} dt +$$

$$+\begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix}, \tag{3.5}$$

and

$$\operatorname{cov}\left\{ f_{a_1b_1}{}^{(T)}(\lambda), f_{a_2b_2}{}^{(T)}(\lambda) \right\} = 2\pi P^4 \left\{ G_{a_1b_1}{}^{(T)}(0) G_{a_2b_2}{}^{(T)}(0) \right\}^{-1} \times$$

$$\times G_{a_1a_2b_1b_2}(0) \left[\int_{t_1=0}^{T} \Psi_{a_1b_1}{}^{(T)}(\alpha) \Psi_{a_2b_2}{}^{(T)}(\lambda_2 - \lambda_1 + \alpha) VZ\, dt_1 + \right.$$

$$\left. + \int_{t_1=0}^{T} \Psi_{a_1b_1}{}^{(T)}(\alpha) \Psi_{a_2b_2}{}^{(T)}\left(\frac{\lambda_2 + \lambda_1}{B_T} - \alpha\right) VZ\, dt_1 \right] + O(T^{-2}), \tag{3.6}$$

when $B_T = 1$

$$\operatorname{cov}\left\{ f_{a_1b_1}{}^{(T)}(\lambda), f_{a_2b_2}{}^{(T)}(\lambda) \right\} =$$

$$= \left\{ \int_{t_1=0}^{T} \Psi_{a_1b_1}{}^{(T)}(\alpha) \Psi_{a_2b_2}{}^{(T)}(\alpha)\, dt_1 \right\} \left[\delta(\lambda_1 - \lambda_2) VZ + \delta(\lambda_1 + \lambda_2) VZ \right] +$$

$$\left. + \int_{t_1=0}^{T} \Psi_{a_1b_1}{}^{(T)}(\alpha) \Psi_{a_2b_2}{}^{(T)}\left(\frac{\lambda_2 + \lambda_1}{B_T} - \alpha\right) VZ\, dt_1 \right] + O(B_T^{-1} T^{-2}), \tag{3.7}$$

when $B_T^{-1} \to 0 \quad B_T T \to \infty$ as $T \to \infty$.

Corollary 3.3

Under the conditions of Theorem (3.2) if $\lambda \neq 0,\ \lambda \in R$ and $B_T \to 0$ as $T \to \infty$, then

$$E\left\{ f_{ab}{}^{(T)}(\lambda) \right\} \underset{T\to\infty}{\to} P^2 \begin{bmatrix} f_{a_1a_2}(\alpha) & f_{a_1b_2}(\alpha) A(\alpha)^T \\ A(\alpha) f_{b_1a_2}(\alpha) & A(\alpha) f_{b_1b_2}(\alpha) A(\alpha)^T \end{bmatrix}.$$

Proof

proof comes directly by taking the limits for both sides of formula (3.5) as $T \to \infty$

Corollary 4.2.

Under the conditions of theorem (3.2) if the spectral density function $f_{ab}(x)$ is bounded by a constant M, $a, b = 1, ..., \min(r+s)$ and continuous at a point $x = \lambda,\ \lambda \in R$ and $B_T \to 0$, $B_T T \to \infty$ as $T \to \infty$, then

$$\operatorname{cov}\left\{ f_{a_1b_1}{}^{(T)}(\lambda), f_{a_2b_2}{}^{(T)}(\lambda) \right\} \xrightarrow[T\to\infty]{} 0$$

for all $a_j, b_j = 1, ..., \min(r+s),\ \lambda_j \in R,\ j = 1, ..., k,\ k = 1, 2, ...$

Proof

Taking the modulus for both sides of equation (3.6), then using Assumption III and the boundedness of $f_{ab}(\lambda)$ by constant M, we get

$$\operatorname{cov}\left\{ f_{a_1b_1}{}^{(T)}(\lambda), f_{a_2b_2}{}^{(T)}(\lambda) \right\} = O(B_T^{-1}) = O(B_T^{-1} T^{-1}) \xrightarrow[T\to\infty]{} 0.$$

Then the corollary is obtained .

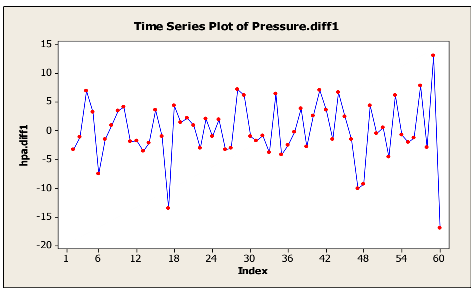
Application on the Theoretical Study:
We will apply our theoretical case study in climate as following :

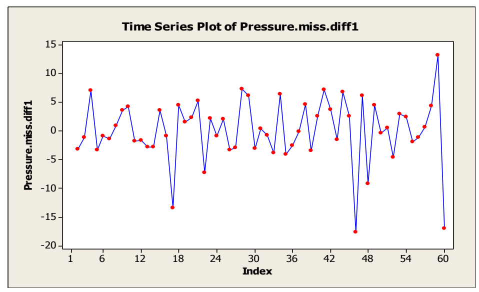
4.1. Studying the Atmospheric Pressure and Maximum temperature.
The data in this research represents the daily maximum temperatures and atmospheric pressure in Tripoli for the period from 1/1/ 2016 to 29/2/2016.

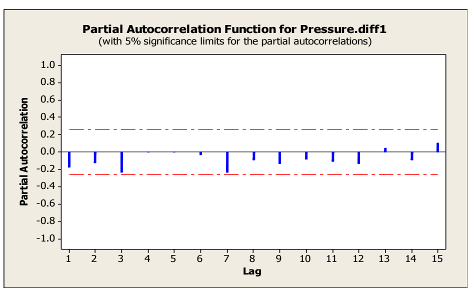
4.1.1. Studying the Atmospheric pressure.
In this study we will comparison between our results, model of strictly stability time series (the Atmospheric pressure) with some missing observations and the classical results, where all observations are available.

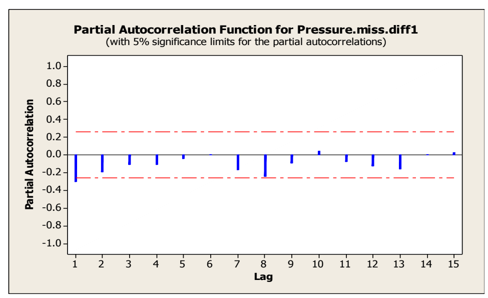
Let $X_a(t)$ is the data of the daily of the atmospheric pressure where all observations are available (classical case) suppose that there is some missing observations in a random way (our study), table 4.1.1 shows the comparison between our results, spectral analysis of strictly stability time series with some missing observations and the classic results, where all observations are available.

Table 4.1.1:- comparison of the results with and without missed observations of the Atmospheric pressure
.

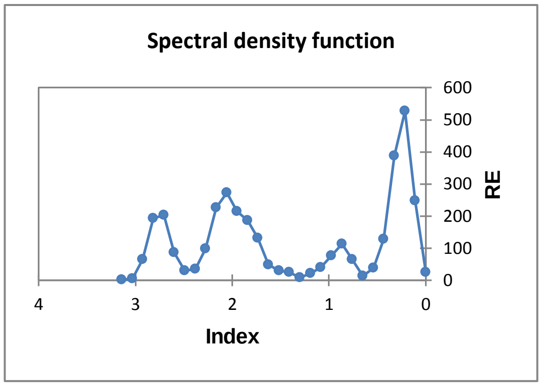
<table>
<tr><th>without missed observations</th><th>with missed observations</th></tr>
<tr><td>

The daily Atmospheric pressure</td><td>

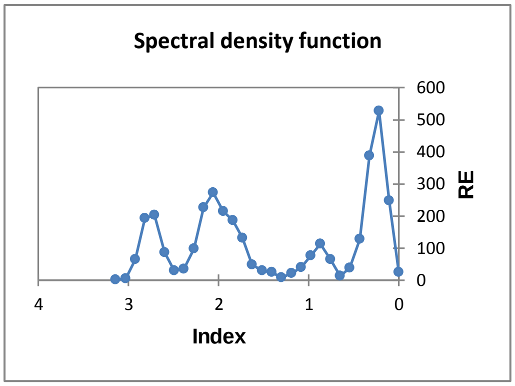
The daily Atmospheric pressure</td></tr>
<tr><td>

PACF for the daily Atmospheric pressure</td><td>

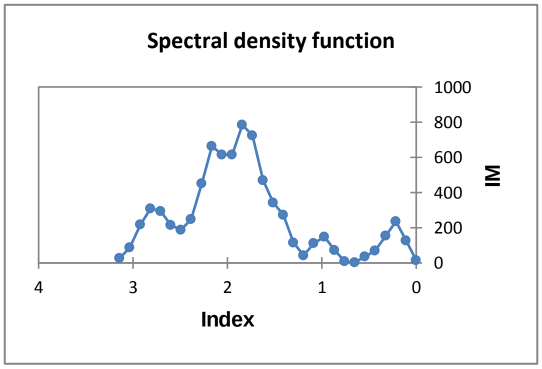
PACF for the daily Atmospheric pressure</td></tr>
</table>



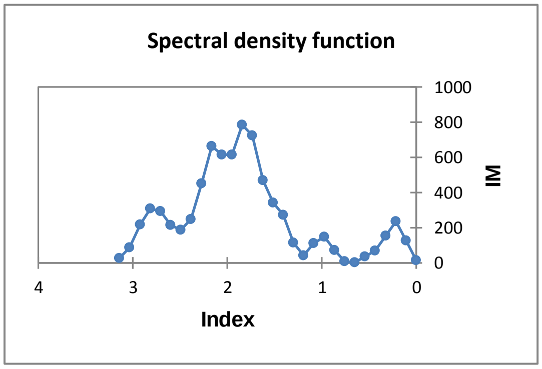
Real part of Atmospheric Pressure without missed observations



Real part of Atmospheric Pressure with missed observations



Imaginary part of Atmospheric Pressure without missed observations



Imaginary part of Atmospheric Pressure with missed observations

<table>
<tr>
<td>ARIMA Model: : Parameters without missed observations
ARIMA(1,1,1)
Final Estimates of Parameters
Type Coef SE Coef T P
AR 1 0.6033 0.1374 4.39 0.000
MA 1 0.9460 0.0813 11.63 0.000
Constant 0.04098 0.05103 0.80 0.425

Differencing: 1 regular difference
Number of observations: Original series 60, after differencing 59
Residuals: SS = 1339.81 (backforecasts excluded)
MS = 23.93 DF = 56

Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Square 5.4 11.1 22.7 28.7
DF 9 21 33 45
P-Value 0.797 0.960 0.910 0.972</td>
<td>ARIMA Model: Parameters with missed observations
ARIMA(1,1,1)
Final Estimates of Parameters

Type Coef SE Coef T P
AR 1 0.4868 0.1403 3.47 0.001
MA 1 0.9454 0.0743 12.73 0.000
Constant 0.04450 0.05131 0.87 0.389

Differencing: 1 regular difference
Number of observations: Original series 60, after differencing 59
Residuals: SS = 1399.75 (backforecasts excluded)
MS = 25.00 DF = 56

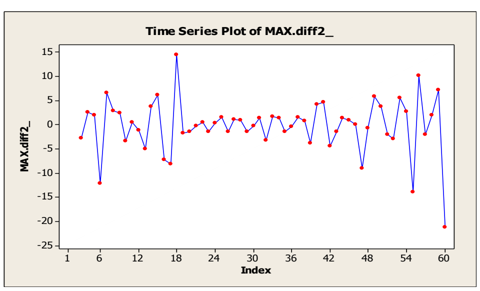
Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Square 4.3 8.8 18.3 34.4
DF 9 21 33 45
P-Value 0.891 0.991 0.982 0.876</td>
</tr>
</table>

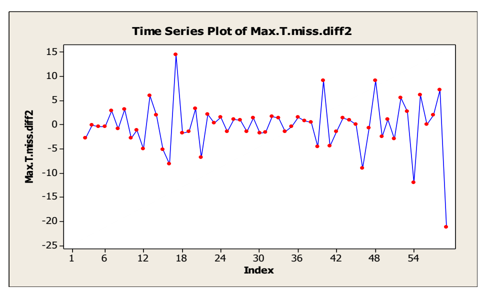
4.1.2. Studying the Maximum temperature:-

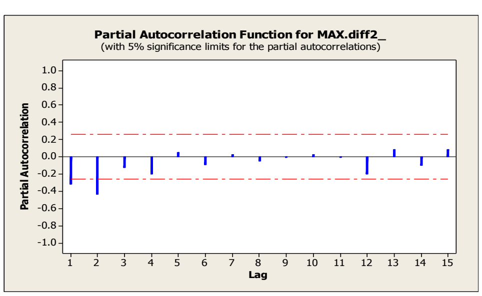
In this study we will comparison between our results, model of strictly stability time series (the Maximum temperature) with some missing observations and the classical results, where all observations are available.

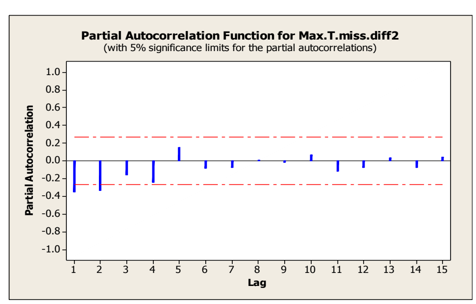
Let $Y_a(t)$ is the data of the daily of the Maximum temperature where all observations are available (classical case) suppose that there is some missing observations in a random way (our study), table 4.1.2 shows the comparison between our results, spectral analysis of strictly stability time series with some missing observations and the classic results, where all observations are available.

Table (4.1.2) comparison of the results with and without missed observations of the Maximum temperature.

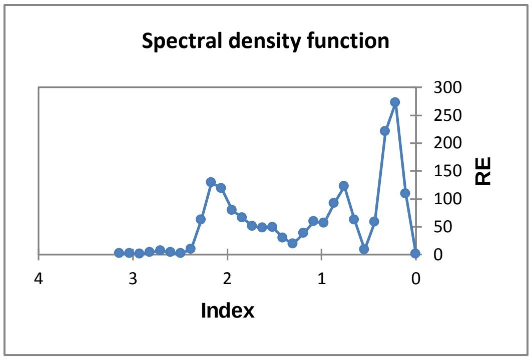
<table>
<tr><td>without missed observations</td><td>with missed observations</td></tr>
<tr><td>

The daily Maximum temperature</td><td>

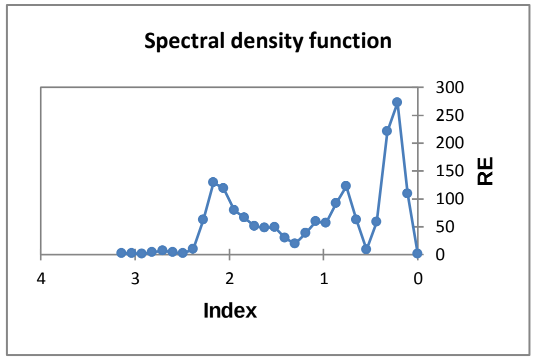
The daily Maximum temperature</td></tr>
<tr><td>

PACF for the daily Maximum temperature</td><td>

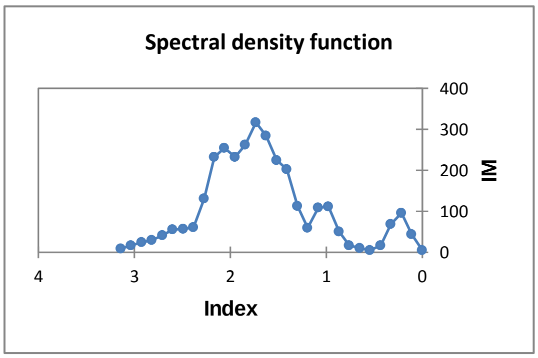
PACF for the daily Maximum temperature</td></tr>
</table>



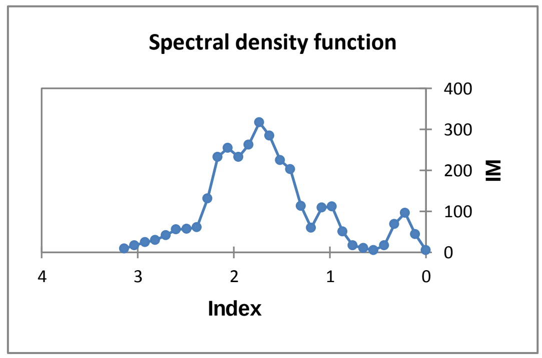
Real part of Maximum temperature without missed observations



Real part of Maximum temperature with missed observations



Imaginary part of Maximum temperature without missed observations



Imaginary part of Maximum temperature with missed observations

RIMA Model: Maximum temperature without missed observations

ARIMA(2,2,1)

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	-0.2172	0.1418	-1.53	0.031
AR 2	-0.3010	0.1541	-1.95	0.056
MA 1	1.0024	0.0920	10.90	0.000
Constant	-0.00053	0.01891	-0.03	0.978

Differencing: 2 regular differences

Number of observations: Original series 60, after differencing 58

Residuals: SS = 725.838 (backforecasts excluded)

MS = 13.441 DF = 54

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	5.5	9.4	16.7	27.2
DF	8	20	32	44
P-Value	0.705	0.978	0.988	0.978

RIMA Model: Maximum temperature with missed observations

ARIMA(2,2,1)

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	-0.2436	0.1383	-1.76	0.034
AR 2	-0.3080	0.1561	-1.97	0.044
MA 1	0.9563	0.1034	9.25	0.000
Constant	0.00223	0.03716	0.06	0.952

Differencing: 2 regular differences

Number of observations: Original series 60, after differencing 58

Residuals: SS = 680.271 (backforecasts excluded)

MS = 12.598 DF = 54

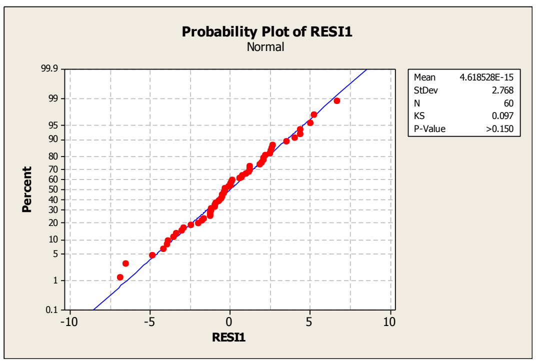
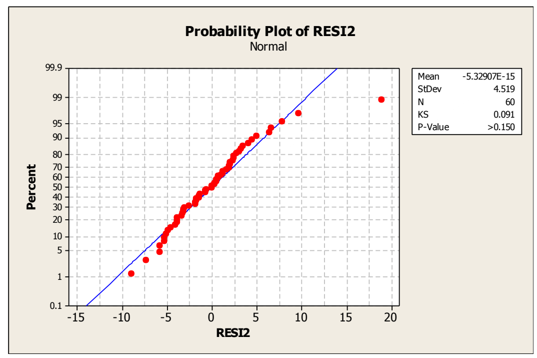
Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	10.1	16.2	26.2	47.0
DF	8	20	32	44
P-Value	0.259	0.702	0.753	0.351

4.1.3. Studying the Regression Between the Atmospheric pressure and the Maximum temperature

In this study we will comparison between our results, regression model between the Atmospheric pressure and the Maximum temperature with some missing observations and the classical results, where all observations are available, the comparison between two cases is shown in table (4.1.3) .

Table 4.1.3:- Comparison of the results with and without missed observations of the regression analysis

Without missed observations	**With missed observations**
The regression equation is Pressure = 3.27 + 1.36 MaxTemp Predictor Coef SE Coef T P Constant 3.266 1.779 -1.84 0.041 MaxTemp 1.35816 0.08596 15.80 0.000 S = 2.79138 R-Sq = 81.1% R-Sq(adj) = 80.8% Analysis of Variance Source DF SS MS F P Regression 1 1945.1 1945.1 249.64 0.000 Residual Error 58 451.9 7.8 Total 59 2397.0 Durbin-Watson statistic = 1.55496	The regression equation is Pressure.miss = 4.80 + 0.933 Max.T.miss Predictor Coef SE Coef T P Constant 4.805 3.087 1.56 0.025 Max.T.miss 0.9325 0.1491 6.26 0.000 S = 4.55758 R-Sq = 80.8% R-Sq(adj) = 79.9% Analysis of Variance Source DF SS MS F P Regression 1 812.85 812.85 39.13 0.000 Residual Error 58 1204.75 20.77 Total 59 2017.60 Durbin-Watson statistic = 1.5732
 Normal-plot of standardized Residuals Without missed observations	 Normal-plot of standardized Residuals With missed observations

Materials and Methods:-

We used SPSS and MINITAB, XL.STAT the software programming to solve our numerical example .

Results and Discussion:-

1. The study of the time series with missed observations and with the modified periodogram had the same results as the study of the classical time series .
2. The study of regression model between classical time series X(t) ,Y(t) had the same results as case of missed observations .

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