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RESEARCH ARTICLE

Non-Static Plane Symmetric Cosmological Model with Magnetized Anisotropic Dark Energy by Hybrid Expansion Law in $f(R,T)$ Gravity

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Abstract

In the presence of magnetized anisotropic Dark Energy (DE) fluid, some features of non-static plane symmetric universe with anisotropic Equation of State (EoS) parameter are investigated in $f(R,T)$ gravity. We choose Hybrid Expansion Law (HEL) which exhibits a transition of the universe from decelerating phase to the present accelerating phase, to obtain the determinate solution. We found that the EoS parameter (ω) for the dark energy is time-dependent and its existing range for derived models is in good agreement with data obtained from recent theoretical observations [1-4]. The physical and geometric aspects of the universe are also discussed in detail.

INTRODUCTION

Cosmological data from a wide range of sources have indicated that our Universe is undergoing an accelerating expansion [1–6]. Basically, two kinds of alternative explanations have been proposed for this unexpected observational phenomenon. One is the Dark Energy (DE) with a sufficient negative pressure, which induces a late-time accelerating cosmic expansion and other is modified gravity, which originates from the idea that the General Relativity (GR) is incorrect in the cosmic scale and therefore needs to be modify. Noteworthy amongst them are $f(R)$ theory of gravity formulated by Nojiri and Odintsov [7] and $f(R,T)$ theory of gravity proposed by Harko et al. [8]. The DE model has been characterized in a conventional manner by the EoS parameter $\omega_D = p_D/\rho_D$ which is not necessarily constant, where ρ_D is the energy density and p_D is the fluid pressure (Carroll and Hoffman [9]). The ω_D lies close to −1: it would be equal to $\omega_D = -1$ (standard ΛCDM cosmology), a little bit upper than −1 (the quintessence DE) or less than −1 (phantom DE). The possibility $\omega_D \ll -1$ is ruled out by current cosmological data. Akarsu and Kilinc [10] investigated the general form of the anisotropy parameter of the expansion for Bianchi type-III model. Sharif and Zubair [11] explored Bianchi type-I universe in the presence of magnetized anisotropic DE with variable EoS parameter. Kumar and Yadav [12] deals with a spatially homogeneous and anisotropic Bianchi type-V universe filled with DE assuming to interact minimally together with a special law of variation for the Hubble parameter, he observed that DE dominates the universe at the present epoch. Recently, Amirhashchi et al. [13] presented DE models in an anisotropic Bianchi type-VI_0 space-time by considering constant deceleration parameters (DP). Chirde and Shekh [14] investigated an anisotropic and homogenous Bianchi Type VI_0 space-time under the assumption of anisotropy of the fluid within the frame work of Lyra manifold in the presence and absence of magnetism using special form of deceleration parameter which gives an early deceleration and late time accelerating cosmological model. Saha and Yadav [15] have generated exact solutions of LRS Bianchi type-II DE model for some suitable choices of parameters which represents a transition of universe from early decelerating phase to present accelerating phase.

Modified $f(R,T)$ gravity is attracting more and more attention. In this theory the Gravitational Lagrangian is given by an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T). Adhav [16] has obtained Bianchi type-I cosmological model in $f(R,T)$ gravity. Chirde et al. [17, 18] have discussed spatially homogeneous and anisotropic Bianchi type-I and general class of Bianchi type cosmological models within the framework of theories of gravity. Recently, Rao and Neelima [19] have obtained Bianchi type-VI_0 perfect fluid model in this theory. Chandel and Ram [20] generated new classes of solutions of field equations starting from known solutions for an anisotropic Bianchi type-III cosmological model with perfect fluid in $f(R,T)$ theory of gravity. Chaubey et al. [21] has obtained a new class of Bianchi type cosmological models in $f(R,T)$ gravity. Sahoo et al. [22] investigated an axially symmetric space-time in the presence of a perfect fluid source in $f(R,T)$ gravity. While, Sharif and Zubair [23] found that the picture of equilibrium thermodynamics is not feasible in $f(R,T)$ gravity even if we specify the energy density and pressure of dark components thus the non-equilibrium treatment is used to study the laws of thermodynamics. Katore et al. [24, 25] investigated some cosmological model with DE source in $f(R,T)$ gravity. Very recently, Chirde and Shekh [26] investigated non-static plane symmetric space-time filled with DE within the frame work of same modified gravity.

2. Brief review of $f(R,T)$ Gravity

The $f(R,T)$ Gravity is the generalization of (GR). In this theory, the field equations are derived from a variation, Hilbert-Einstein type principle which is given as

$$S = \frac{1}{16\pi}\int \sqrt{-g}\; f(R,T)\, d^4x + \int \sqrt{-g}\; L_m d^4 x\,, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T) of the matter T_{ij} $(T = g^{ij}T_{ij})$. L_m is the matter Lagrangian density.

The stress energy tensor of matter is defined as

$$T_{ij} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}\;. \tag{2}$$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives, equation (2) leads to

$$T_{ij} = g_{ij}L_m - \frac{\delta(L_m)}{\delta g^{ij}}\;. \tag{3}$$

Varying the action S with respect to the metric tensor components g_{ij}, the gravitational field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}\,, \tag{4}$$

with $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ and ∇_i is the covariant derivative.

The contraction of equation (4) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = \left(8\pi - f_T(R,T)\right)T - f_T(R,T)\Theta \text{ with } \Theta = g^{ij}\Theta_{ij}\,. \tag{5}$$

Equation (5) gives a relation between Ricci scalar and the trace of energy momentum tensor.

Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij}\,, \tag{6}$$

where $u^i = (0,0,0,1)$ denotes the four velocity vector in co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ and p is energy density and pressure of the fluid respectively.

The variation of stress energy of perfect fluid has the following expression

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \tag{7}$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. [8] gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)\, f_3(T) \end{cases}. \tag{8}$$

In this paper, we have focused to the first class $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of tress energy tensor of the form $f(T) = \mu T$ where μ is constant. For this choice the gravitational field equations of $f(R,T)$ gravity becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij}, \tag{9}$$

where the prime denotes differentiation with respect to the argument. If the matter source is a perfect fluid then the field equations (in view of Eq. (7)) becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}. \tag{10}$$

3. Metric, Field Equations and Kinematical parameters

We consider a Riemannian space-time described by the line element

$$ds^2 = e^{2h}\left(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2\right), \tag{11}$$

where r, θ, z are the usual cylindrical polar coordinates and *h* & *s* are functions of *t* alone. It is well known that this line element is plane symmetric.

The Energy momentum tensor for magnetized anisotropic DE is given by

$$\begin{aligned} T_i^{\,j} &= diag[\rho + \rho_B, -px + \rho_B, -p_y + \rho_B, -p_z - \rho_B] \\ &= diag[\rho + \rho_B, -(\omega+\delta)\rho + \rho_B, -(\omega+\delta)\rho + \rho_B, -\omega\rho - \rho_B], \end{aligned} \tag{12}$$

where ρ is the energy density of the fluid, p_x, p_y, p_z and $\omega_x, \omega_y, \omega_z$ are the directional pressures and EoS parameters along the *x*, *y*, *z*- axes respectively, δ be the deviation from the free EoS parameter (hence the deviation free pressure) on *x*-axis and *y*-axis and ρ_B stands for energy density of magnetic field.

In the presence of magnetized DE source given in equation (12), the field equations (10) corresponding to the metric (11) lead to the following set of linearly independent differential equations

$$e^{-2h}\left(2\ddot{h} + \dot{h}^2 + \frac{2\dot{h}\dot{s}}{s} + \frac{\ddot{s}}{s}\right) = \rho\,[(8\pi+2\mu)(\omega+\delta) - \mu(1-3\omega-2\delta)] - (8\pi+2\mu)\,\rho_B - 2\mu\,p, \tag{13}$$

$$e^{-2h}\left(2\ddot{h} + \dot{h}^2\right) = \rho\,[(8\pi+2\mu)\omega - \mu(1-3\omega-2\delta)] + 8\pi\,\rho_B - 2\mu\,p, \tag{14}$$

$$e^{-2h}\left(\frac{2\dot{h}\dot{s}}{s} + 3\dot{h}^2\right) = (-\rho)[(8\pi+2\mu) + \mu(1-3\omega-2\delta)] - (8\pi+2\mu)\,\rho_B - 2\mu\,p. \tag{15}$$

The overhead dot represents the differentiation with respect to time *t*. We have the following equation from the Bianchi identity,

$$\dot{\rho} + (1+\omega)\rho\left(\frac{\dot{s}}{s} + 3\dot{h}\right) + \rho\left(2\delta\dot{h}\right) + 2\rho_B\left(\frac{\dot{s}}{s} + \dot{h}\right) + \dot{\rho}_B = 0. \tag{16}$$

Now we define some parameters for the universe which are important in cosmological observations. The spatial volume (V) and the generalized Hubble's parameter (H) for the space-time (11) are defined by

$$V = a^3 = rse^{4h}, \tag{17}$$

$$H = \frac{\dot{a}}{a}, \tag{18}$$

where a be the average scale factor of the Universe.

The physical quantities of observational interest in cosmology such as the expansion scalar (θ), deceleration parameter (q) the anisotropy parameter (A_m), and the shear scalar (σ^2) defined as follows

$$\theta = 3H, \tag{19}$$

$$q = -1 - \frac{\dot{H}}{H^2}, \tag{20}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \text{ where } \Delta H_i = H_i - H \ (i = 1,2,3). \tag{21}$$

$$\sigma^2 = \frac{3}{2}A_m H^2. \tag{22}$$

4. Solution of the field equations:

The set of field equations (13) - (15) are a coupled system of non-linear differential equations and we seek physical solution to the field equations for applications in cosmology and astrophysics. There are only three independent equations with six unknowns $h, s, \rho, \omega, \delta$ and ρ_B. In order to obtain an explicit solution of the system we need two additional constraints.

i) According to the work of Thorne [27], velocity red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30 % range approximately [28, 29] and red shift studies place the limit $(\sigma / H) \le 0.3$ where σ and H are shear scalar and Hubble parameter respectively. Collins et al. [30] have mentioned that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition (σ / H) is constant, it gives

$$e^h = \beta s^n, \tag{23}$$

where β is constant and $n > 0$.

ii) The EoS parameter (ω) is proportional to skewness parameter (δ) [24 -26] (Mathematical condition) such that

$$\omega + \delta = 0. \tag{24}$$

Following a very recent work of Akarsu et al. [31] and Azizur Rahman and Ansari [32], we consider an *ansatz*

$$a = a(t) = a_0\left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)}, \tag{25}$$

where γ and ξ are non-negative constants, a_0 and t_0 represent the present value of scale factor and age of the universe respectively. We call the relation (25) as Hybrid Expansion Law (HEL) which is a combination of a power-law and an exponential function. It is observed that $\gamma = 0$ yields the exponential law cosmology while $\xi = 0$ gives power law cosmology. Further, the scale factor given by equation (25) yields a time-dependent deceleration parameter which exhibits a transition of the universe from the early decelerating phase to the present accelerating phase.

The Bianchi identity from the equation (16) can be expressed as

$$\dot{\rho} + (1+\omega)\rho\left[\frac{\dot{s}}{s} + 3\dot{h}\right] + \rho \quad (2\delta \dot{h}) = 0, \tag{26}$$

and

$$2\rho_B\left(\frac{\dot{s}}{s} + \dot{h}\right) + \dot{\rho}_B = 0,$$

i.e

$$\rho_B = \frac{c e^{-2h}}{s^2}. \tag{27}$$

Using equations (17) and (25) we obtained the metric potentials as

$$s = \left(r\beta^4\right)^{\frac{-1}{4n+1}} \left[a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \right]^{\frac{3}{4n+1}}, \tag{28}$$

$$e^{2h} = \left(r^{-2n}\beta^2\right)^{\frac{1}{4n+1}} \left[a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \right]^{\frac{6n}{4n+1}}. \tag{29}$$

Using equations (28) and (29), spatially homogeneous non-static plane symmetric universe within the framework of $f(R,T)$ gravity becomes

$$ds^2 = \left(r^{-2n}\beta^2\right)^{\frac{1}{4n+1}} \left[a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \right]^{\frac{6n}{4n+1}} \left(dt^2 - dr^2 - r^2 d\theta^2 - \left(r\beta^4\right)^{\frac{-2}{4n+1}} \left[a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \right]^{\frac{6}{4n+1}} dz^2 \right). \tag{30}$$

The energy density for magnetic field as

$$\rho_B = \frac{c\left(r^{2(n+1)}\beta^6\right)^{\frac{1}{4n+1}}}{\left[a_0 \left(\frac{t}{t_0}\right)^{\gamma} e^{\xi\left(\frac{t}{t_0}-1\right)} \right]^{\frac{6(n+1)}{4n+1}}}. \tag{31}$$

Above equation (31) gives the expression of energy density for magnetic field. Its graphical performance is shown in the figure (1). From figure (1) it is observed that the energy density for magnetic field is not constant, positive decreasing function of time, at an initial stage of the universe it is very high but with the expansion it is decreases and approaches to zero at infinite expansion.

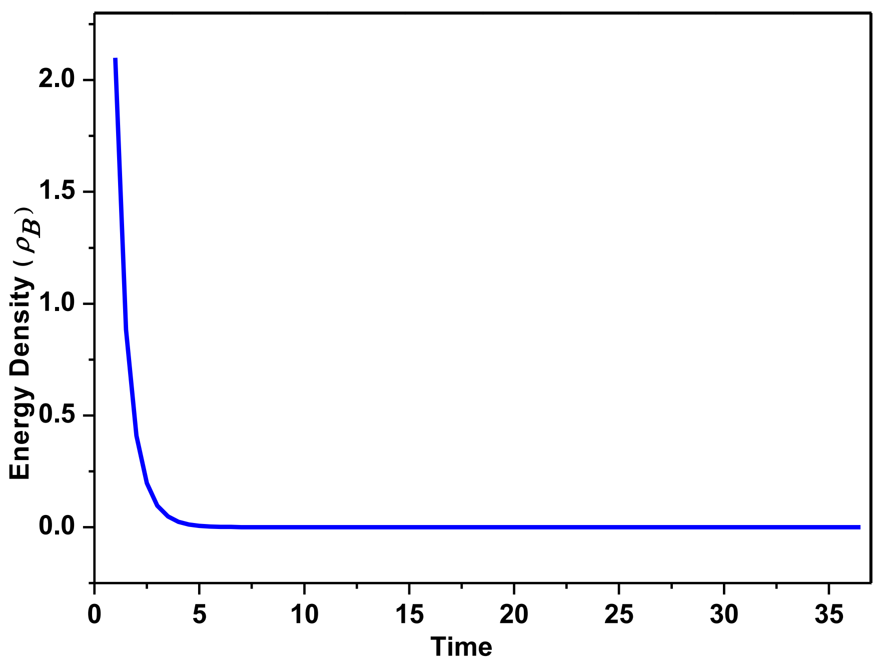

Figure (1): The energy density for magnetic field versus time t.

The generalized Hubble parameter and the spatial volume becomes

$$H = \frac{3}{t_0}\left(\gamma \frac{t_0}{t} + \xi\right), \tag{32}$$

$$V = \left[a_0 \left(\frac{t}{t_0} \right)^{\gamma} e^{\xi \left(\frac{t}{t_0} - 1 \right)} \right]^3, \tag{33}$$

The expansion scalar and shear scalar are given by

$$\theta = \frac{9}{t_0} \left(\gamma \frac{t_0}{t} + \xi \right), \tag{34}$$

$$\sigma^2 = \frac{9 \left(12 n^2 + 4n + 3 \right) \left(\gamma t_0 + \xi t \right)^2}{2 \left(4n + 1 \right)^2 t^2 {t_0}^2}, \tag{35}$$

In this derived universe it is observed that, the Hubble parameter, spatial volume, expansion scalar, shear scalar all are time dependent. Initially when the universe starts to expand all are infinitely large and at an infinite expansion it approaches to null.

The mean anisotropy parameter turns out to be

$$A_m = \frac{12 n^2 + 4n + 3}{3 \left(4n + 1 \right)^2}. \tag{36}$$

The mean anisotropy parameter given in equation (40) is constant throughout the expansion of the universe. Hence, the universe does not approach to isotropy.

The deceleration parameter,

$$q = -1 + \frac{\gamma {t_0}^2}{3 \left(\gamma t_0 + \xi t \right)^2}, \tag{37}$$

In our derived Universe, the deceleration parameter comes out to be time dependent. The variation of deceleration parameter is depicted in figure (2). From figure (2), it is clear that at an initial stage of the universe the value of deceleration parameter is positive and becomes negative at late time. Hence, the deceleration parameter gives a transition from a decelerating expansion phase to the present accelerating phase of the universe. Also, we observed that it decreases rapidly and approaches −1 asymptotically which shows de-Sitter like expansion at late time.

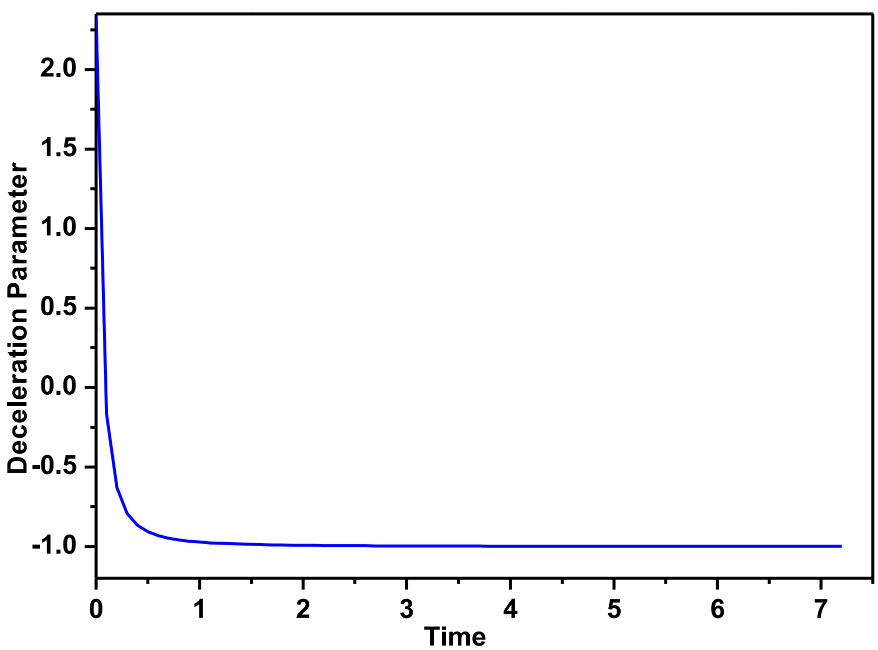

Figure (2): the deceleration parameter versus time t.

Energy density for DE is

$$\rho = \frac{1}{(8\pi + 2\mu)}\left[\frac{9\left(1-2n^2\right)(\gamma t_0 + \xi t)^2 - 3\left(8n^2 + 6n + 1\right)\gamma t_0^{\,2}}{(4n+1)^2\, t^2\, t_0^{\,2}\, \left(r^{-2n}\beta^2 a^{6n}\right)^{\frac{1}{4n+1}}}\right]. \tag{38}$$

Figure (3) (blue line) depicts that the variation of energy density versus time *t*, it is observed that the energy density distribution is positive decreasing functions of time *t*. At an initial stage from where the model starts to expand the energy density having positive value and then it shows decreasing behavior whereas at infinite expansion of the universe it approaches to zero i.e. $\rho \to 0$ thus the model is asymptotically empty.

The EoS parameter is

$$\omega = \left[\frac{-9(n+1)(\gamma t_0 + \xi t)^2 + 3(4n+1)\gamma t_0^{\,2} - 2c\,(4n+1)^2\, t^2\, t_0^{\,2}\, \left(r^{2(n+1)}\beta^6 a^{-6n}\right)^{\frac{1}{4n+1}}}{9\left(1-2n^2\right)(\gamma t_0 + \xi t)^2 - 3\left(8n^2 + 6n + 1\right)\gamma t_0^{\,2}}\right]. \tag{39}$$

From equation (39), we observed that the EoS parameter of DE (ω) is time dependent. The graphical behavior of EoS parameter verses time *t* is shown in figure (3) (red line). At the initial stage when the universe started to expand for slight interval of time, the EoS of the universe having value $\omega > 0$ i.e. the model behave as like matter dominated once at early stages while at late times it becomes $\omega < 0$. With the expansion the EoS parameter extending from Phantom $\omega < -1$ region to quintessence $\omega > -1$ region; this is a situation in the universe where the quintessence field dominated, while for late interval of time it shows dusty universe.

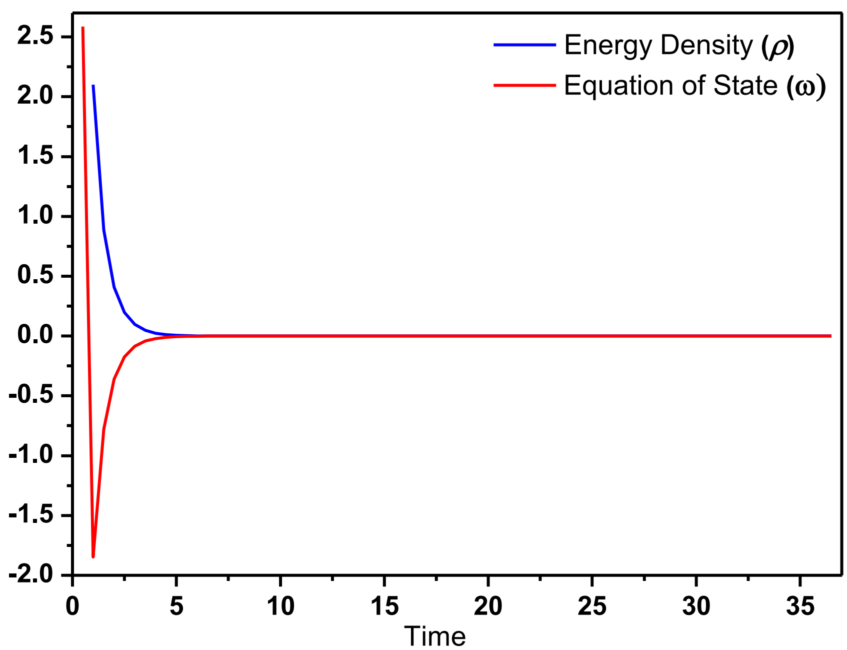

Figure (3): Energy density and Equation of state parameter of the DE verses time t.

The skewness parameter is

$$\delta = \left[\frac{9(n+1)(\gamma t_0 + \xi t)^2 - 3(4n+1)\gamma t_0^{\,2} + 2c\,(4n+1)^2\, t^2\, t_0^{\,2}\, \left(r^{2(n+1)}\beta^6 a^{-6n}\right)^{\frac{1}{4n+1}}}{9\left(1-2n^2\right)(\gamma t_0 + \xi t)^2 - 3\left(8n^2 + 6n + 1\right)\gamma t_0^{\,2}}\right]. \tag{40}$$

7. Conclusions

In this paper, we have studied non static plane symmetric anisotropic universe filled with magnetized dark energy. We choose a kinematical *ansatz* called hybrid expansion law which yields power-law and exponential law cosmologies in special cases to deduce the exact solutions of the field equations. Also we have discussed some geometrical and physical properties of the model with following observations.

- It is observed that the present model exhibits point type singularity and it evolves with a zero volume at time $t = 0$.
- We see that the deceleration parameter decreases rapidly and approaches to -1 asymptotically which shows de-Sitter like expansion at late time. For this model, the deceleration parameter gives a transition from a decelerating expansion phase to the present accelerating phase of the universe.
- Note that $(\sigma/\theta) \neq 0$ as well as the mean anisotropy parameter $A_m \neq 0$ which implies that our model is anisotropic at all-time.
- It is seen that the DE density of fluid decrease as the universe expands. Thus our model approaches towards a flat universe at late time. Thus our model is in good agreement with the recent observation.
- The EoS parameter of the derived universe shows different stages like matter dominated once, Phantom and quintessence phase and dust universe.
- Also, it is interesting to note that here in the absence of magnetic field all the results (Physical and geometrical parameters) are resembles with the work of [25, 26].

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