

# **RESEARCH ARTICLE**

# ON NANO GENERALIZED $\beta$ LOCALLY CLOSED SETSAND NANO GENERALIZED $\beta$ LOCALLY CLOSED CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES.

## S. B. Shalini, G. Sindhu and K. Indirani.

Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.

Manuscript Info	Abstract
Manuscript History	The aim of this paper is to introduce Nano generalized $\beta$ locally
Received: 27 May 2017 Final Accepted: 29 June 2017 Published: July 2017	closed set and Nano generalized locally closed continuous function in Nano topological spaces and also we introduce and study the relation between Nano generalized $\beta$ locally closed irresolute function and Nano generalized $\beta$ locally closed continuous function.
Key words:- N $\beta$ LC set, NG $\beta$ LC set, N $\beta$ LC	
continuous, NG $eta$ LC continuous and	

NG  $\beta$  LC irresolute functions.

Copy Right, IJAR, 2017,. All rights reserved.

### **Introduction:-**

In general, continuous function is one of the core concepts in topology. Abd El Monsef et al. [1] introduced the notion of  $\beta$ -open sets and  $\beta$ -continuous mappings in topology.Balachandran [2] introduced generalized locally closed set and GLC continuous in topology. The concept of Nano topology was introduced by LellisThivagar [4] which was defined in terms of approximations and boundary regions of a subset of a universe using an equivalence relation on it and he also defined Nano continuous functions, Nano open maps, Nano closed maps and Nano homeomorphisms and their representations in terms of Nano interior and Nano closure. In this paper we introduce Nano generalized  $\beta$  locally closed sets, Nano generalized  $\beta$  locally closed irresolute function and study some of its relation between them.

### 2. Preliminaries:-

**Definition:- 2.1[4]** Let U be the universe, R be an equivalence relation on U and  $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then  $\tau_R(X)$  satisfies the following axioms: i) U and  $\phi \in \tau_R(X)$ . ii) The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

iii) The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

Then  $\tau_R(X)$  is a topology on U called the Nano topology on U with respect to X,  $(U, \tau_R(X))$  is called the Nano topological space.

**Corresponding Author:- S. B. Shalini** Address:- Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India. **Definition:**- 2.2 [4] If  $(U, \tau_R(X))$  is a Nano topological space with respect X where  $X \subseteq U$  and if  $A \subseteq U$ , then

- 1. The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by  $N \operatorname{int}(A)$ .  $N \operatorname{int}(A)$  is the largest Nano open subset of A.
- 2. The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest Nano closed set containing A.

**Definition:** 2.3 [2] A function  $f:(X,\tau) \rightarrow (Y,\sigma)$  is called

- (i) continuous if  $f^{-1}(V)$  is open in  $(X, \tau)$  for every open set V in  $(Y, \sigma)$ .
- (ii)  $\beta$  (Semi pre) continuous if  $f^{-1}(V)$  is  $\beta$  (semi pre) open in  $(X, \tau)$  for every open set V in  $(Y, \sigma)$ .
- (iii) gcontinuous if  $f^{-1}(V)$  is g open in  $(X, \tau)$  for every open set V in  $(Y, \sigma)$ .
- (iv) g  $\beta$  continuous if  $f^{-1}(V)$  is g  $\beta$  open in  $(X, \tau)$  for every open set V in  $(Y, \sigma)$ .

**Definition:**- 2.4 [6] A subset A of a Nano topological space  $(U, \tau_R(X))$  is called Nano generalized  $\beta$  closed set (briefly Ng  $\beta$  closed) if  $N\beta cl(A) \subseteq V$  whenever  $A \subseteq V$  and V is Nano open in  $(U, \tau_R(X))$ .

**Definition:** 2.5 [7] Let  $(U, \tau_R(X))$  and  $(V, \sigma_{R'}(Y))$  be Nano topological spaces. Then a mapping  $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$  is called Ng  $\beta$  continuous on U if the inverse image of every Nano open set in V is Ng  $\beta$  open in U.

**Definition:**- 2.6 [2] A subset A of a topological space  $(X, \tau)$  is called generalized locally closed set (briefly GLC) if  $A = G \cap F$  where G is g open and F is g closed in X.

#### 3. Nano generalized $\beta$ locally closed sets:-

**Definition:- 3.1** A subset A of a Nano topological space  $(U, \tau_R(X))$  is called Nano generalized  $\beta$  locally closed set (briefly NG  $\beta$  LC) if  $A = G \cap F$  where G is Ng  $\beta$  open and F is Ng  $\beta$  closed in  $(U, \tau_R(X))$ .

**Definition:** 3.2 A subset A of a Nano topological space  $(U, \tau_R(X))$  is called  $(NG \beta LC)^*$  set if  $A = G \cap F$  where G is Ng  $\beta$  open and F is Nano closed in  $(U, \tau_R(X))$ .

**Definition:** 3.3 A subset A of a Nano topological space  $(U, \tau_R(X))$  is called  $(NG \beta LC)^{**}$  set if  $A = G \cap F$  where G is Nano open and F is Ng  $\beta$  closed in  $(U, \tau_R(X))$ .

**Definition:** 3.4 A subset A of a Nano topological space  $(U, \tau_R(X))$  is called strongly Nano generalized  $\beta$  locally closed set (briefly strongly NG  $\beta$  LC) if  $A = G \cap F$  where G is strongly Ng  $\beta$  open and F is strongly Ng  $\beta$  closed in  $(U, \tau_R(X))$ .

**Theorem:-** 3.5 Every strongly NG  $\beta$  LC set is NG  $\beta$  LC set. **Proof:-** Every strongly Ng  $\beta$  closed set is Ng  $\beta$  closed set proof follows.

Remark: 3.6:- The converse of the above theorem need not be true which is seen in the below example.

**Example:- 3.7** Let  $U = \{a, b, c, d, e\}$  with  $U/R = \{\{a, b\}, \{c, e\}, \{d\}\}$  and  $X = \{a, d\}$ . Then the Nano topology is defined as  $\tau_R(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$ . Here the set  $\{a, b, c, d\}$  is NG  $\beta$  LC set but not strongly NG  $\beta$  LC set in U.

**Remark: 3.8:** Let  $(U, \tau_R(X))$  and  $(V, \sigma_{R'}(Y))$  be Nano topological spaces. Then

- (i) Every NLC set is NG  $\beta$  LC set.
- (ii) Every NGLC set NG  $\beta$  LC set.
- (iii) Every (NGLC)<sup>\*</sup> set is NG  $\beta$  LC set.
- (iv) Every (NGLC)<sup>\*\*</sup> set is NG  $\beta$  LC set.
- (v) Every NLC set is  $(NG \beta LC)^*$  set.
- (vi) Every NLC set is  $(NG \beta LC)^{**}$  set.

Similarly we prove for strongly NG  $\beta$  LC set.

**Theorem:-** 3.9 If a subset A of a Nano topological space  $(U, \tau_R(X))$  is  $(NG \beta LC)^*$  set then there exist a  $Ng\beta$  open set G such that  $A = G \cap \beta cl(A)$ .

**Proof:** Assume that *A* is  $(NG \ \beta \ LC)^*$  set. Then there exist a  $Ng\beta$  open set *G* and Nano closed set *F* such that  $A = G \cap F$ . Since  $A \subseteq F$  and *F* is Nano closed,  $A \subseteq N\beta cl(A) \subseteq F$ . Then  $A \subseteq G$  and  $A \subseteq N\beta cl(A)$  and hence  $A \subseteq G \cap \beta cl(A)$ . To prove the reverse inclusion, if  $x \in G \cap \beta cl(A)$  then  $x \in G$  and  $x \in \beta cl(A) \subseteq F$ . So  $x \in G \cap F = A$ . Hence  $G \cap \beta cl(A) \subseteq A$ . Therefore  $A = G \cap \beta cl(A)$ .

Theorem: - 3.10 For a subset of the following conditions.

- i)  $A \in NG\beta LC^*(U, X)$ .
- ii)  $A = G \cap N\beta cl(A)$  for some  $Ng\beta$  open set G.
- iii)  $N\beta cl(A) A$  is  $Ng\beta$  closed.
- iv)  $A \cup (U N\beta cl(A))$  is  $Ng\beta$  closed.

are related as follows: i) and ii) are equivalent, iii) and iv) are equivalent and iv) implies ii).

#### Proof:-

i)  $\Rightarrow$  ii) Let  $A \in NG\beta LC^*(U, X)$ . Then there exist a Ng  $\beta$  open set G and a Nano closed set F such that  $A = G \cap F$ . Since  $A \subseteq G$  and  $A \subseteq N\beta cl(A)$  we have  $A = G \cap N\beta cl(A)$ . Conversely since  $N\beta cl(A) \subseteq F$ ,  $G \cap N\beta cl(A) \subseteq G \cap F = A$  which implies that  $A = G \cap N\beta cl(A)$ . ii)  $\Rightarrow$  i) Since G is Ng  $\beta$  open and  $N\beta cl(A)$  is Nano closed  $G \cap N\beta cl(A) \in NG\beta LC^*(U, X)$ . iii)  $\Rightarrow$  iv) Let  $P = N\beta cl(A) - A$ . Then  $A \cup (U - N\beta cl(A))$  is Ng  $\beta$ open. Since  $U - P = A \cup (U - N\beta cl(A))$  holds and hence U - F is Ng  $\beta$  open. iv)  $\Rightarrow$  iii) Let  $Q = A \cup (U - N\beta cl(A))$ . Then U - Q is Ng  $\beta$  closed and  $U - Q = N\beta cl(A) - A$  holds.  $G = A \cup (U - N\beta cl(A)).$ iv)⇒ii) Let Then G  $Ng\beta$ is open. Now  $G \cap N\beta cl(A) = [A \cup (U - N\beta cl(A))] \cap N\beta cl(A) = N\beta cl(A) \cap A \cup [N\beta cl(A) \cap (U - N\beta cl(A))]$  $= A \cup \phi = A$ . Therefore  $A = G \cap N\beta cl(A)$ .

**Definition:-** 3.11 A Nano topological space  $(U, \tau_R(X))$  is said to be a Ng  $\beta$  submaximal if every Nano dense subset of  $(U, \tau_R(X))$  is Ng  $\beta$  open in  $(U, \tau_R(X))$ .

**Theorem:-** 3.12 Every Nano submaximal space is Ng  $\beta$  submaximal but bot conversely.

**Proof:** Let U be a Nano submaximal and A be a Nano dense subset of U. Then A is Nano open. But every Nano open set is Ng  $\beta$  open and so A is Ng  $\beta$  open. Therefore U is Ng  $\beta$  submaximal.

**Proposition:** 3.13 The following results are true for any two subsets A and B of a Nano topological space  $(U, \tau_R(X))$ .

- i) If  $A, B \in NG\beta LC^*(U, X)$  then  $A \cap B \in NG\beta LC^*(U, X)$ .
- ii) If  $A \in NG\beta LC(U, X)$  and B is Ng  $\beta$  open then  $A \cap B \in NG\beta LC(U, X)$ .
- iii) If  $A \in NG\beta LC^*(U, X)$  and B is Ng  $\beta$  open then  $A \cap B \in NG\beta LC^*(U, X)$ .
- iv) If  $A \in NG\beta LC^{**}(U, X)$  and B is strongly Ng  $\beta$  open then  $A \cap B \in NG\beta LC(U, X)$ .
- v) If  $A \in NG\beta LC^{**}(U, X)$  and B is Nano open then  $A \cap B \in NG\beta LC^{**}(U, X)$ .

**Theorem:** 3.14 A Nano topological space  $(U, \tau_R(X))$  is Ng  $\beta$  submaximal iff  $NG\beta LC^*(U, X) = P(X)$ .

**Proof:** (Necessary) Let  $A \in P(X)$ . Let  $G = A \cup (U - N\beta cl(A))$ . Then  $N\beta cl(G) = U$ . Since U is Ng  $\beta$  submaximal, G is Ng  $\beta$  open. By theorem 3.11,  $A \in NG\beta LC^*(U, X)$  and so  $NG\beta LC^*(U, X) = P(X)$ . (Sufficient) Let A be a Nano dense subset of U. Then  $A \cup (U - N\beta cl(A)) = A \cup \phi = A$ . Since  $A \in NG\beta LC^*(U, X)$ , by theorem 3.11 A is Ng  $\beta$  open in U. Hence U is Ng  $\beta$  submaximal.

### 4. Ng $\beta$ lc continuous map and ng $\beta$ lc irresolute map:-

**Definition:** 4.1 A function  $f:(U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$  is called NG  $\beta$  LC continuous (NG  $\beta$  LC\* continuous and NG  $\beta$  LC\*\* continuous) if  $f^{-1}(G) \in NG\beta LC(U, X)$  ( $f^{-1}(G) \in NG\beta LC*(U, X)$  and  $f^{-1}(G) \in NG\beta LC**(U, X)$ ) for every Nano open set G of V.

**Theorem:** 4.2 Afunction  $f: U \to V$  be a function then we have the following.

- i) If f is NLC continuous then f is NG  $\beta$  LC continuous (NG  $\beta$  LC\* continuous and NG  $\beta$  LC\*\* continuous).
- ii) If f is NG  $\beta$  LC\* continuous or NG  $\beta$  LC\*\* continuous then f is NG  $\beta$  LC continuous.

**Proof:** i) Suppose that f is NLC continuous. Let G be a Nano open set in U. Then  $f^{-1}(G)$  is NLC set in U. Since every NLC set is NG  $\beta$  LC set (NG  $\beta$  LC\* set and NG  $\beta$  LC\*\* set ), it follows that f is NG  $\beta$  LC continuous (NG  $\beta$  LC\* continuous and NG  $\beta$  LC\*\* continuous).

ii)Let  $f: U \to V$  be NG  $\beta$  LC\* continuous or NG  $\beta$  LC\*\* continuous. Since every NG  $\beta$  LC\* set or NG  $\beta$  LC\*\* set is NG  $\beta$  LC set. Therefore f is NG  $\beta$  LC continuous.

**Theorem:** 4.3 If  $f: U \to V$  is NG  $\beta$  LC continuous and  $g: V \to W$  is Nano continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC continuous.

**Proof:-** Let F be Nano closed in W. Then  $g^{-1}(F)$  is Nano closed V, since g is Nano continuous and then  $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$  is NG  $\beta$  LC set in U, as f is NG  $\beta$  LC continuous. Thus  $g \circ f$  is NG  $\beta$  LC continuous.

**Theorem:** 4.4 If  $f: U \to V$  is NG  $\beta$  LC\* continuous and  $g: V \to W$  is Nano continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC\* continuous.

**Theorem:** 4.5 If  $f: U \to V$  is NG  $\beta$  LC\*\* continuous nd  $g: V \to W$  is Nano continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC\*\* continuous.

Proof of the theorem 4.4 and 4.5 are similar to the theorem 4.3.

**Definition:** 4.6 A function  $f: (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$  is called NG  $\beta$  LC irresolute (NG  $\beta$  LC\* irresolute and NG  $\beta$  LC\*\* irresolute) if  $f^{-1}(G) \in NG\beta LC(U, X)$  ( $f^{-1}(G) \in NG\beta LC*(U, X)$  and  $f^{-1}(G) \in NG\beta LC**(U, X)$ ) for every  $G \in NG\beta LC(V, Y)$  ( $G \in NG\beta LC*(V, Y)$ ) and  $G \in NG\beta LC**(V, Y)$ ).

**Theorem:** 4.7 A function  $f: U \to V$  be a function then we have the following.

- i) If f is NLC irresolute then f is NG  $\beta$  LC irresolute (NG  $\beta$  LC\* irresolute and NG  $\beta$  LC\*\* irresolute).
- ii) If f is NG  $\beta$  LC irresolute (NG  $\beta$  LC\* irresolute or NG  $\beta$  LC\*\* irresolute) then f is NG  $\beta$  LC continuous (NG  $\beta$  LC\* continuous and NG  $\beta$  LC\*\* continuous).

**Proof:** i) Suppose that f is NLC continuous. Let G be a NLC set in U. Then  $f^{-1}(G)$  is NLC set in U. Since every NLC set is NG  $\beta$  LC set (NG  $\beta$  LC\* set and NG  $\beta$  LC\*\* set ), it follows that f is NG  $\beta$  LC irresolute (NG  $\beta$  LC\* irresolute and NG  $\beta$  LC\*\* irresolute).

ii)Let G be a Nano open set in U and G is NG  $\beta$  LC set in V. Then  $f^{-1}(G) \in NG\beta LC(U, X)$  as f is NG  $\beta$  LC irresolute (NG  $\beta$  LC\* irresolute or NG  $\beta$  LC\*\* irresolute). Hence f is NG  $\beta$  LC continuous (NG  $\beta$  LC\* continuous).

**Theorem:** 4.8 If  $f: U \to V$  is NG  $\beta$  LC irresolute and  $g: V \to W$  is NG  $\beta$  LC continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC continuous.

**Proof:** Let F be Nanoclosedin W. Then  $g^{-1}(F)$  is NG  $\beta$  LC set in V, since g is NG  $\beta$  LC continuous. Again since f is NG  $\beta$  LC irresolute,  $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$  is NG  $\beta$  LC set in U. Thus  $g \circ f$  is Ng  $\beta$  LC continuous.

**Theorem:** 4.9 If  $f: U \to V$  is NG  $\beta LC^*$  irresolute and  $g: V \to W$  is NG  $\beta LC^*$  continuous, then  $g \circ f: U \to W$  is NG  $\beta LC^*$  continuous.

**Theorem:** 4.10 If  $f: U \to V$  is NG  $\beta$  LC\*\* irresolute and  $g: V \to W$  is NG  $\beta$  LC\*\* continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC\*\* continuous.

Proof of the theorem 4.9 and 4.10 are similar to the theorem 4.8. Similarly we can prove the following theorems.

**Theorem:** 4.11 If  $f: U \to V$  is NG  $\beta$  LC irresolute and  $g: V \to W$  is NG  $\beta$  LC irresolute, then  $g \circ f: U \to W$  is NG  $\beta$  LC irresolute.

**Theorem:** 4.12 If  $f: U \to V$  is NG  $\beta$  LC\* irresolute and  $g: V \to W$  is NG  $\beta$  LC\* irresolute, then  $g \circ f: U \to W$  is NG  $\beta$  LC\* irresolute.

**Theorem:** 4.13 If  $f: U \to V$  is NG  $\beta LC^{**}$  irresolute and  $g: V \to W$  is NG  $\beta LC^{**}$  irresolute, then  $g \circ f: U \to W$  is NG  $\beta LC^{**}$  irresolute.

**Theorem:** 4.14 If  $f: U \to V$  is NG  $\beta$  LC irresolute and  $g: V \to W$  is NLC continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC continuous.

**Theorem:** 4.15 If  $f: U \to V$  is NG  $\beta$  LC irresolute and  $g: V \to W$  is Ng  $\beta$  continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC continuous

**Theorem:** 4.16 If  $f: U \to V$  is NG  $\beta$  LC\* irresolute and  $g: V \to W$  is NG  $\beta$  LC\* continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC\* continuous.

**Theorem:** 4.17 If  $f: U \to V$  is NG  $\beta$  LC\*\* irresolute and  $g: V \to W$  is NG  $\beta$  LC\*\* continuous, then  $g \circ f: U \to W$  is NG  $\beta$  LC\*\* continuous.

#### **References:-**

- Abd EL-Monsef M. E., EL-Deep S. N. and Mahmoud R. A., β-Open Sets and β continuous mappings, Bull. Fac. Sci., Assiut Univ., 12 (1983), 77-90.
- Balachandran, K., Sundaram, P. and Maki, H., Generalized locally closed sets and GLC-continuous functions, Indian J. Pure. Appl. Math., 27(1996), 235-244.
- 3. Gnanmbal. Y., On Nano  $\beta$  open sets, Int. Jr. of Engineering, 1(2) (2015), 1-6.
- 4. LellisThivagar .M., and Carmel Richard, On Nano forms of weakly open sets, International
- 5. Journal of Mathematics and Statistics Invention, 1(1) (2013), 31 37.
- LellisThivagar .M., and Carmel Richard, On Nano Continuity, Mathematical Theory and Modeling, 3(7)(2013), 32-37.
- 7. Shalini. S.B., and Indirani. K., Characterisation of nano generalized  $\beta$  closed sets in nano topological spaces, International Journal Sciences and Applied Research, 4(1)(2017), 7-11.
- 8. Shalini. S.B., and Indirani. K., On Nano generalized  $\beta$  continuous functions and Nano generalized  $\beta$  irresolute functions in Nano topological spaces, IOSR Journal of Mathematics (IOSR-JM), 13(1)(2017), 79-86.