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RESEARCH ARTICLE

REPRESENTATION & NATURE OF MULTIPLE FACTORIANGULAR NUMBERS.

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Abstract

In this paper, we have defined multiple factoriangular numbers and found the values of number theoretic functions for multiple factoriangular numbers. The mathematical experimentations are used on multiple factoriangular numbers resulted to the establishment of recurrence relations for these numbers. Here we have also described the nature of multiple factoriangular numbers.

Introduction:-

Triangular number [2] is a number obtained by adding all positive integers less than or equal to a given positive integer n, i.e..

$$T_n = n(n + 1)/2$$

Factoriangular number [1] is defined as the sum of the first n natural numbers plus the factorial of n. i.e.,

$$Ft_n = n(n + 1)/2 + n!$$

ParajalRai[3] proved that there is no factoriangular number that is also factorial and also observed the patterns in factoriangular number modulo n. Romer C. Castillo [2] presented several theorems, corollaries and some conjectures for factoriangular numbers.

Here in this paper, we have defined new factoriangular numbers namely multiple factoriangular numbers and established recurrence relations for these numbers. We have also described the nature of multiple factoriangular numbers as an increasing function.

Multiple Factoriangular Numbers

A generalization of Factoriangular numbers is known as Multiple Factoriangular numbers and are defined as,

$$F_t\,(n,k) = (n!)^k + \sum n^k$$

$$\text{Where } \sum n^k = T_n(k) \text{ and } n,k \in N$$

Representation of Multiple Factoriangular Numbers

Representation of multiple factoriangular numbers by the sum of four squares and their number theoretic values are shown in Table 1.

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Table 1:-Representation of Multiple Factoriangular Numbers

S.no	$F_t(n,2)$	$\tau(F_t(n,2))$	$\Phi(F_t(n,2))$	$\sigma(F_t(n,2))$	Sum of four squares
1	2	2	1	3	$1^2+1^2+0^2+0^2$
2	9	3	6	13	$3^2+0^2+0^2+0^2$
3	50	6	20	93	$5^2+5^2+0^2+0^2$
4	606	8	200	1224	$20^2+11^2+9^2+2^2$
5	14455	12	9744	20520	$105^2+42^2+35^2+21^2$
6	518491	8	473760	565200	$483^2+375^2+324^2+199^2$
7	25401740	96	7557120	69600384	$4200^2+2378^2+1266^2+710^2$
8	1625702604	24	541602240	3795396528	$30806^2+21208^2+11610^2+9598^2$
9	131681894685	64	59104972800	247354953984	$296994^2+175235^2+112660^2+8768^2$
10	13168189440385	16	9028934690688	18060593083776	$2895929^2+1819756^2+1104112^2+501208^2$
11	1593350922240506	32	668533897241280	2807883958924800	$38517331^2+7252704^2+5497720^2+5190327^2$
12	229442532802560650	48	90484187713651200	432774774588542400	$343268700^2+269764475^2+194979255^2+28624300^2$
13	38775788043632640819	96	20452848291426265344	68935794733745372160	$4116092037^2+4075095000^2+2283956031^2+103537317^2$
14	7600054456551997441015	64	5191322176716231358080	10463232848830954536960	$60466243150^2+55092899233^2+29785417285^2+4635651749^2$
15	1710012252724199424001240	256	606410020032497902878720	4317119683080297387840000	$794738734438^2+751716887400^2+597059155914^2+396036213300^2$

Recurrence Relations for Multiple Factoriangular Numbers:-

The recurrence relation for multiple factoriangular numbers is given by

$$F_t(n+r,k+s) = (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k)-T_n(k)](n!)^s + T_{n+r}(k)\,.T_{n+r}(s) - \sum_{l\neq m}(l+r)^k(m+r)^s$$

Where, $F_t(n,k) = (n!)^k + \sum n^k$, $T_n(k)=\sum n^k$ and $l,m \in \{1,2,\ldots n\}$, $r,s \in W$.

Proof: By the definition of multiple factoriangular numbers we have,

$F_t(n,k) = (n!)^k + \sum n^k$

Where $\sum n^k = T_n(k)$

Now, $F_t(n+r,k+s) = ((n+r)!)^k + \sum(n+r)^{k+s}$

$= (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}.(n!)^{k+s} + \sum(n+r)^{k+s}$

$= (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k) - \sum n^k](n!)^s + \sum(n+r)^{k+s}$

$= (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k) - T_n(k)](n!)^s$

$+\sum(n+r)^k\sum(n+r)^s - \sum_{l\neq m}(l+r)^k(m+r)^s$

$$F_t(n+r,k+s) = (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k)-T_n(k)](n!)^s + T_{n+r}(k)\,.T_{n+r}(s) - \sum_{l\neq m}(l+r)^k(m+r)^s$$

Particular cases

Case-I: If r = 1, s = 0 then, the recurrence relation for multiple factoriangular numbers is given by,

$F_t(n+1,k) = (n+1)^k[F_t(n,k)-T_n(k)+1]+T_n(k)$

Where, $F_t(n,k) = (n!)^k + \sum n^k$ and $T_n(k)=\sum n^k$

Proof: The recurrence relation for multiple factoriangular numbers is given by

$$F_t(n+r,k+s) = (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k)-T_n(k)](n!)^s + T_{n+r}(k)\,.T_{n+r}(s) - \sum_{l\neq m}(l+r)^k(m+r)^s$$

If r = 1, s = 0 then,

$F_t(n+1,k) = (n+1)^k\,[F_t(n,k)-T_n(k)] + T_{n+1}(k)\,.T_{n+1}(0) - \sum_{l\neq m}(l+1)^k(m+1)^0$

$= (n+1)^k[F_t(n,k) - T_n(k)]+\sum(n+1)^k$

$= (n+1)^k[F_t(n,k)-T_n(k)]+ \sum n^k + (n+1)^k$

$F_t(n+1,k) = (n+1)^k[F_t(n,k)-T_n(k)+1]+T_n(k).$

Hence proved.

Case-II:If r = 0, s = 1 then, the recurrence relation for multiple factoriangular numbers is given by,
$F_t(n,k+1)=n!.F_t(n,k) + T_n(k)\ (T_n(1)-n!) -\sum_{l\neq m} l^k m$
Where, $F_t(n,k) = (n!)^k$,$T_n(k)=\sum n^k$ and l,m, ϵ {1,2,…n} , r,s ϵ W

Proof: The recurrence relation for multiple factoriangular numbers is given by
$F_t(n+r,k+s) = (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k)-T_n(k)](n!)^s$
$+T_{n+r}(k)\ .T_{n+r}(s) - \sum_{l\neq m}(l+r)^k(m+r)^s$
If r = 0, s = 1 then,
$F_t(n,k+1) = (n)!^{k+1}+ T_n(k)T_n(1) -\sum_{l\neq m} l^k m$
$= n!.[F_t(n,k)-\Sigma n^k]+ T_n(k)T_n(1) -\sum_{l\neq m} l^k m$
$= n!.\ [F_t(n,k)- T_n(k)] + T_n(k)T_n(1) -\sum_{l\neq m} l^k m$
$F_t(n,k+1) = n!.F_t(n,k) + T_n(k)\ (T_n(1)-n!) -\sum_{l\neq m} l^k m$

Hence proved.

Nature of Multiple Factoriangular Numbers
Multiple Factoriangular numbers are given by the function,
$F_t(n,k) = (n!)^k +\sum n^k$
Where $\sum n^k = T_n(k)$

There are three cases:
Case-I: n'>n, k'=k
$F_t(n',k') = (n'!)^{k'} +\sum n'^{k'}$
$> (n!)^{k'} +\sum n'^{k'}$
$= (n!)^{k'}+(1^{k'}+2^{k'}+3^{k'}+\ldots..+n^{k'}+..+n'^{k'})$
$> (n!)^{k'}+\sum n^{k'} = (n!)^k +\sum n^k$
$= F_t(n,k)$
Hence $F_t(n',k') > F_t(n,k)$

Case-II: n'=n, k'>k
$F_t(n',k') = (n'!)^{k'} +\sum n'^{k'}$
$= (n!)^k .(n!)^{k'-k} +[1^{k+(k'-k)}+2^{k+(k'-k)}+3^{k+(k'-k)}+\ldots+n^{k(k'-k)}\ldots.+n'^{k+(k'-k)}]$
$> (n!)^k+(1^k+2^k+3^k+\ldots..+n^k)$
$= F_t(n,k)$
Hence $F_t(n',k') > F_t(n,k)$

Case-III: n'>n, k'>k
$F_t(n',k') = (n'!)^{k'} +\sum n'^{k'}$
$= (n!)^{k'}(n+1)^{k'}(n+2)^{k'}\ldots..(n')^{k'}+(1^{k'}+2^{k'}+3^{k'}+\ldots..+n^{k'}+\ldots.+n'^{k'})$
$> (n!)^{k'} +\sum n^{k'}$
$= (n!)^k .(n!)^{k'-k} +[1^{k+(k'-k)}+2^{k+(k'-k)}+3^{k+(k'-k)}+\ldots...+n^{k+(k'-k)}]$
$> (n!)^k +\sum n^k$
$= F_t(n,k)$
Hence $F_t(n',k') > F_t(n,k)$

It is clear that the multiple factoriangular number i.e, the function $F_t(n,k) = (n!)^k +\sum n^k$is strictly increasing as the value of n and k increases.

Conclusion:-
The results for multiple factoriangular numbers are:
i) The recurrence relation for multiple factoriangular numbers is given by
$F_t(n+r,k+s) = (n+r)^{k+s}.(n+r-1)^{k+s}.(n+r-2)^{k+s}\ldots\ldots (n+1)^{k+s}[F_t(n,k)-T_n(k)](n!)^s$
$+T_{n+r}(k)\ .T_{n+r}(s) -\sum_{l\neq m}(l+r)^k(m+r)^s$
Where, $F_t(n,k) = (n!)^k +\sum n^k$,$T_n(k)=\sum n^k$ and l,m, ϵ {1,2,…n} , r,s ϵ W.
Case-I: If r = 1, s = 0 then, the recurrence relation for multiple factoriangular numbers is given by

$F_t(n+1,k) = (n+1)^k[F_t(n,k)-T_n(k)+1]+T_n(k)$
Case-II: If r = 0, s = 1 then, the recurrence relation for multiple factoriangular numbers is given by
$F_t(n,k+1)=n!.F_t(n,k) + T_n(k) (T_n(1)-n!) -\sum_{l\neq m} l^k m$
ii) Function $F_t (n,k) = (n!)^k +\sum n^k$ is strictly increasing function.

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