ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

Interaction and Quadratic effect on Cardio Vascular Disease: A Nonlinear Structural Equation Modeling

[1]S. Xavier and [2]T. Leo Alexander
[1] Assistant Professor, [2] Associate Professor, Loyola College (Autonomous) Chennai – 600 034, India

Manuscript Info

Manuscript History:

Received: 15 August 2015
Final Accepted: 22 September 2015
Published Online: October 2015

Key words:

Cardio Vascular Disease, Constrained approach, Latent Moderated Structural Equation Modeling, Nonlinear latent Structural Equation Modeling and Unconstrained approach.

[2]Corresponding Author

T. Leo Alexander

Abstract

Most of the statistical research studies concentrate on linear approaches to build the model to bring out the relationship among the variables. In the past two decades, many researchers introduced a latent nonlinear approach in Structural Equation Modeling. The Nonlinear models obtain simultaneous Interaction and Quadratic effects. In this paper, Latent Moderated Structural Equation Model, Constrained Approach and Unconstrained Approach are applied for the primary data of 405 patients who are affected by Cardio Vascular Disease. The efficacy of the model is also discussed.

1 Introduction

The estimation of nonlinear relation between variables is an important concern in different areas of the social and behavioral sciences. Several theories do not only incorporate not only linear but also nonlinear relations between variables. An interaction effect implies that a relationship between a predictor and a criterion is weakened or strengthened by a second predictor variable (Aiken and West, 1991). In social psychology, for example, an interaction effect is hypothesized in an extension of the theory of planned behavior (Ajzen, 1991). This theory suggests that the behavior is dependent on the individual's intention to perform a specific behavior and an individual's perceived ease or difficulty of performing this behavior (perceived behavioral control). In an extension of this theory, Elliott et.al. (2003) demonstrated in their study on compliance with speed-limits that prior behavior of exceeding speed limits while driving in built-up areas moderated the relationship between perceived behavioral control and subsequent behavior: increasing frequency of prior noncompliance with speed limits was associated with a decrease in the relationship between perceived behavioral control and driver's subsequent reported noncompliance with speed limits.

A quadratic effect implies that the predictor variables interact within themselves. In health psychology, for example, a quadratic effect is hypothesized in research dealing with adolescents' reputations of peer status and health behaviors. Wang et. al. (2006) investigated adolescent boys' weight-related health behaviors and cognitions expecting a curvilinear association between perceived body size and reputation-based popularity. The results showed the expected inverted U-shaped curve: Lower levels of popularity were associated with self-reported body shapes at each extreme of the silhouette scale (thin and heavy silhouettes), whereas higher levels of popularity were associated with self-reported muscular silhouettes. These findings confirm boys' body ideals toward body shapes that are neither thin nor heavy but muscular.

Most studies investigate either interaction or quadratic effects. It is also sometimes of interest to combine both types of nonlinear effects in a more complex “multiple” nonlinear model. We will cite two examples in order to explain such cases in empirical research: In educational psychology, for example, theory suggests a negative interaction between parent’s educations on child’s educational expectations (Ganzach, 1997): When the level of education of one parent is high, the educational expectations of the child will also be high, even if the level of education of the other parent is quite low. However, analyses also revealed two quadratic effects, a positively accelerated relationship between mother’s education and child’s educational expectations as well as a positively accelerated relationship between father’s education and child’s educational expectations.

In studying the relationship between teachers’ expectations and students’ perceived competence in physical education classes, Trouilloud et al. (2006) hypothesized a quadratic effect that is a negatively accelerated relationship between teachers’ early expectations and students’ later perceived competence. In the last two decades, Structural Equation Modeling (SEM) is familiar in applied behavioral and social sciences research (Schumacker and Marcoulides, 1998). They concentrated on linear and nonlinear of Structural Equation Model (SEM) in social sciences.

In this paper, we discuss collection of data as well as necessary variables in Section 3. Based on the variables, the model specification is explained and identified the significant factors in Section 4. The three SEM approaches are used for estimating parameters in nonlinear Structural Equation Modeling namely Latent Moderator Approach (LMS), Unconstrained Approach (UA) and Constrained Approach (CA) are explained and results are discussed in Sections 5, 6 and 7 respectively.

2 Preliminaries

In day today life, the cardio vascular disease is the much familiar and threatening disease to the life of the people. So we are interested to study the influencing factors to the cardio vascular disease. The survival status of the patients is also studied. In this article, we observed the two outcome variables namely ejection fraction and survival status. The Ejection Fraction and Survival status of the patients are considered as endogenous (dependent) variables. Also we observed name of the patient, Gender, Age, Body Mass Index, Place of residence (Urban / Rural), Smoking habits, Alcohol habits, Family History, Blood Glucose level, Blood Cholesterol level, Blood Pressure variables are considered as exogenous (independent) variables.

The general model is represented by the following equations consisting of measurement and structural models:

$$Y = \nu + \Lambda\eta + \varepsilon \tag{2.1}$$

and

$$\eta = \alpha + B\eta + \xi, \tag{2.2}$$

where Y is the vector of p observed variables in a considered study (p >1), ν the p × 1 vector of observed variable mean intercepts, Λ is the $p \times q$ matrix of factor loadings, η is the of q x 1 latent factors assumed in it (q > 0), ε the vector of p pertinent residuals (error terms), α is the $q \times 1$ vector of latent variable intercepts, B is a q x q matrix of latent regression coefficients and ξ is the $q \times 1$ vector of corresponding latent disturbance terms.

Based on the general equation (2.1) and (2.2), we obtain the following the structural equation model for the three factors namely blood factor (ξ_1), life style factor (ξ_2) and physical factor (ξ_3) with manifest endogenous variables ejection fraction (Y_1) and survival status (Y_2) are given in the following structural equation models:

$$Y_1 = \alpha_1 + \beta_{11}\xi_1 + \beta_{12}\xi_2 + \beta_{13}\xi_3 + \zeta_1 \tag{2.3}$$

$$Y_2 = \alpha_2 + \beta_{21}\xi_1 + \beta_{22}\xi_2 + \beta_{23}\xi_3 + \zeta_2 \tag{2.4}$$

The general matrix expression is given in the following equation:

$$Y_1 = \alpha_1 + \Gamma_1\xi_1 + \zeta_1 \tag{2.5}$$

$$Y_2 = \alpha_2 + \Gamma_2\xi_2 + \zeta_2 \tag{2.6}$$

where $\Gamma_1 = (\beta_{11} \quad \beta_{12} \quad \beta_{13})$, $\Gamma_2 = (\beta_{21} \quad \beta_{22} \quad \beta_{23})$, $\xi_1 = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$ and $\xi_2 = \begin{pmatrix} \xi_1{}^* \\ \xi_2{}^* \\ \xi_3{}^* \end{pmatrix}$.

In the above equations (2.5) and (2.6), Y_1 and Y_2 are the two manifest endogenous variables, α_1 and α_2 are the latent intercepts, Γ_1 and Γ_2 are the coefficient vectors for the linear effects of n latent predictors, $\xi_1$and ξ_2 are the latent factors and finally ζ_1 and ζ_2 are the latent disturbance. The above model in equations (2.5) and (2.6), are constructed in Amos (version 16.0) and finalized the significant factors. In this models equations (2.5) and (2.6), only two factors namely blood factor (ξ_1) and life style factor (ξ_2) are significant on the manifest endogenous variables ejection fraction (Y_1) and survival status (Y_2). The physical factor (ξ_3) in not significant on both the variables Y_1 and Y_2.

3 Data characteristics

This study is conducted in Cardio Vascular Disease patient in Chennai City. A total of 405 samples are collected with help of Dr. Immanuel who is running private hospital in Chennai. There are 13 variables information of patient were observed in clinical laboratory namely, Name of the patient, Gender, Age, Body Mass Index (BMI), Place of residence (Urban / Rural), Smoking habits, Alcohol habits, Family History, Blood Glucose level (BGL), Blood Cholesterol level (BCL), Blood Pressure (BP), Ejection Fraction (EF) of the patients and their Survival status . The variables Age, BMI, BGL, BCL, BP and EF are continuous variables and Gender, Place of residence, smoking habits, alcohol habits and family history are categorical variables. The data were collected between the years 2010 and 2013.

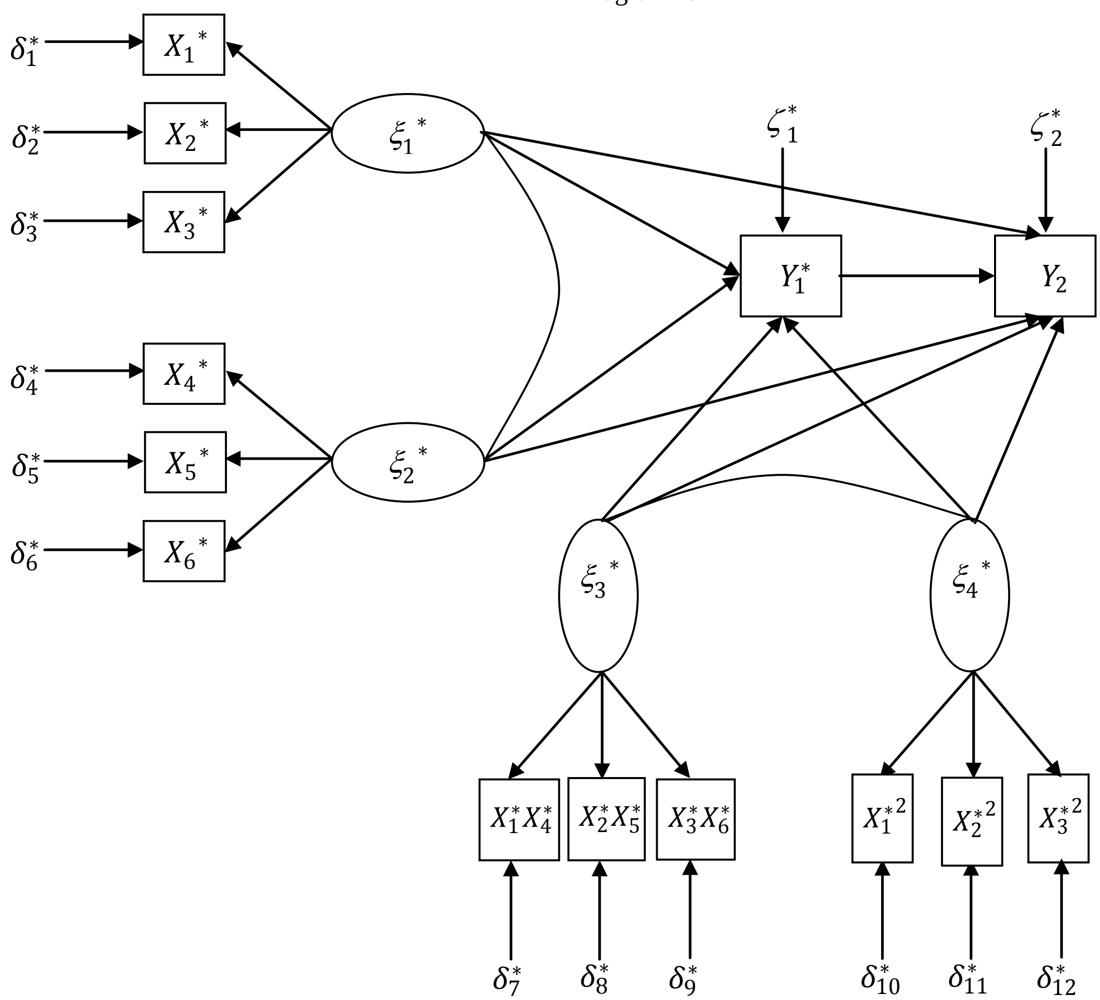
4 Model Specification

We have classified into three factors namely Blood factor, Life Style factor and Physical factor based on the nature of the independent variables. The Blood factor is measured by three variables namely Blood Glucose, Blood Cholesterol and Blood Pressure. The Life Style factor is measured by four variables namely Place of residence (Urban / Rural), Smoking habits, Alcohol habits and Family History of the patients. The Physical factor is measured by three variables namely Age of the patients, Body Mass Index and Gender of the patients. The three factors are considered as major factors which influence the Cardio Vascular Disease and Survival status. Out of three factors, the physical factor is not significant on the manifest endogenous variables ejection fraction and survival status. Finally the two factors namely blood factor and life style factor are used to build the model in order to explain the relationship among the variables are shown in Diagram 4.1.

We have constructed two major factors namely blood factor and life style factor and they are denoted as ξ_1^* and ξ_2^* respectively. The blood factor (ξ_1^*) is measured by three variables namely blood glucose (X_1^*), blood cholesterol (X_2^*) and blood pressure (X_3^*). The life style factor (ξ_2^*) is measured by three variables namely Smoking (X_4^*), Alcohol (X_5^*) and Family History (X_6^*). The Ejection Fraction and Survival status are the two manifest endogenous variables which are influenced by two linear factors namely blood factor (ξ_1^*) and life style factor (ξ_2^*). Normally the linear effects are studied on endogenous variables. In this Paper, we are interested to study linear and nonlinear effects on manifest endogenous variable Ejection Fraction (Y_1^*) and Survival status (Y_2).

We have constructed two types of measurements models namely latent measurement model and nonlinear latent measurement model. The latent measurement model is explained by two linear factors namely ξ_1^* and ξ_2^*. The nonlinear latent measurement model is explained by one interaction term (ξ_3^*) and one quadratic term (ξ_4^*). It is shown in the following Diagram 4.1. Also the linear effect, interaction effect and quadratic effect of three methods namely Linear Structural Equation Method, Unconstrained Method and Constrained Method are studied on Y_1^* and Y_2. Finally the efficacy of the methods is studied. In this paper, we study how the survival status is influenced by linear and non linear terms and the influence of Ejection Fraction on Survival status.

Diagram 4.1



There are two measurement models used in this section namely linear latent measurement model and nonlinear latent measurement model and they are shown in Diagram 4.1. The nonlinear structural equation model includes two latent exogenous variables ξ_1^* and ξ_2^*, a latent interaction term $\xi_1^*\xi_2^* = \xi_3^*$, latent quadratic term $\xi_1^{*2} = \xi_4^*$, two manifest endogenous variable Y_1^* and Y_2 and a disturbance term ζ_1^* and ζ_2^*. The parameters β_{11}^* and β_{12}^* are linear effect of ξ_1^* and ξ_2^* , the notation ω_{13}^* is the interaction effect of ξ_3^* and ω_{14}^* is the quadratic effect of ξ_4^* for the manifest endogenous variable Y_1^*. The parameter β_{21} is the effect of Y_1^* on Y_2 and the notation ω_{23} is the interaction effect of ξ_3^* and ω_{24} is the quadratic effect of ξ_4^* for the manifest endogenous variable Y_2.

The structural equation of the nonlinear model with an intercept term α_1^* is given in the following structural equation model:

$$Y_2 = \alpha_1^* + \beta_{21}Y_1^* + \omega_{23}\xi_3^* + \omega_{24}\xi_4^* + \zeta_2^* \ ,$$

where

$$Y_1^* = \alpha^* + \beta_{11}^*\xi_1^* + \beta_{12}^*\xi_2^* + \omega_{13}^*\ \xi_3^* + \omega_{14}^*\ \xi_4^* + \zeta_1^*.$$

The general matrix expression is given in the following equation:

$$Y_2 = \alpha_1^* + \Gamma^*\psi + \xi^{*'}\Omega^*\xi^* + \zeta_2^* \ ,$$

where $\Gamma = (0 \quad \beta_{21})$, $\psi = \begin{pmatrix} 0 \\ Y_1^* \end{pmatrix}, \xi^* = (\xi_1^* \quad \xi_2^*)$ and $\Omega^* = \begin{pmatrix} 0 & 0 \\ \omega_{13} & \omega_{14} \end{pmatrix}$.

In the above Equation, Y_2 is the manifest endogenous variable, α_1^* is the latent intercept, Γ^* is the coefficient vector for the linear effects, Ω^* is the coefficient matrix of the non-linear effects and finally ζ_2^* is the latent disturbance. In this paper, the linear measurement model and nonlinear measurement model are explained in Sections 4.1 and 4.2 respectively in order to estimate the parameters.

4.1 Linear Latent Measurement Model

The latent exogenous variable ξ_1^* is measured by three indicators X_1^*, X_2^* and X_3^* and ξ_2^* is measured by three indicators X_4^*, X_5^* and X_6^* and it is shown in the following measurement model equation:

$$\begin{bmatrix} X_1^* \\ X_2^* \\ X_3^* \\ X_4^* \\ X_5^* \\ X_6^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{12}^* & 0 \\ \lambda_{13}^* & 0 \\ 0 & 1 \\ 0 & \lambda_{25}^* \\ 0 & \lambda_{26}^* \end{bmatrix} \begin{bmatrix} \xi_1^* \\ \xi_2^* \end{bmatrix} + \begin{bmatrix} \delta_1^* \\ \delta_2^* \\ \delta_3^* \\ \delta_4^* \\ \delta_5^* \\ \delta_6^* \end{bmatrix}$$

The parameters λ_{12}^* and λ_{13}^* are the factor loadings of the measurement model ξ_1^* and the parameters λ_{25}^* and λ_{26}^* are the factor loadings of the measurement model ξ_2^*. The factor loadings of X_2^*, X_3^*, X_5^* and X_6^* are λ_{12}^* $\lambda_{13}^*, \lambda_{25}^*$ and λ_{26}^* respectively which are freely estimated parameters. The parameters $\lambda_{11}^*$and λ_{24}^* are the factor loading of the indicators X_1^* and X_4^* respectively which are fixed parameters and they are denoted as $\lambda_{11}^* = 1$ and $\lambda_{24}^* = 1$. The errors of the indicator variables $X_1^*, X_2^*, X_3^*, X_4^*, X_5^*$ and are X_6^* $\delta_1^*, \delta_2^*, \delta_3^*, \delta_4^*, \delta_5^*$ and δ_6^* respectively. The endogenous manifest variable is denoted as $Y_1^*\ and\ Y_2$.

4.2 Nonlinear latent Measurement Model

The latent exogenous variable ξ_3^* is measured by three interaction indicators $X_1^*X_4^*$, $X_2^*X_5^*$ and $X_3^*X_6^*$. The latent exogenous variable ξ_4^* is also measured by three quadratic indicators ${X_1^*}^2, {X_2^*}^2$ and ${X_3^*}^2$. The two nonlinear exogenous terms are described in the following equation.

$$\begin{bmatrix} X_1^*X_4^* \\ X_2^*X_5^* \\ X_3^*X_6^* \\ {X_1^*}^2 \\ {X_2^*}^2 \\ {X_3^*}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{38}^* & 0 \\ \lambda_{39}^* & 0 \\ 0 & 1 \\ 0 & \lambda_{42}^* \\ 0 & \lambda_{43}^* \end{bmatrix} \begin{bmatrix} \xi_3^* \\ \xi_4^* \end{bmatrix} + \begin{bmatrix} \delta_7^* \\ \delta_8^* \\ \delta_9^* \\ \delta_{10}^* \\ \delta_{11}^* \\ \delta_{12}^* \end{bmatrix}$$

The parameters λ_{38}^* and λ_{39}^* are the factor loadings of the nonlinear measurement term ξ_3^* . The parameter λ_{42}^* and λ_{43}^* are the factor loadings of the nonlinear measurement term ξ_4^* . The factor loadings λ_{38}^* and λ_{39}^* are of the interaction indictors $X_2^*X_5^*$ and $X_3^*X_6^*$ respectively. The parameter λ_{42}^* and λ_{43}^* are the factor loading of the quadratic indicators ${X_2^*}^2$ and ${X_3^*}^2$ respectively. The parameters λ_{38} , λ_{39} λ_{42} and λ_{43} are freely estimating parameters. The parameters λ_{37}^* and λ_{41}^* are the factor loadings of the indicators $X_1^*X_4^*$ and ${X_1^*}^2$ respectively are fixed parameters and they are denoted as $\lambda_{37}^* = 1$ and $\lambda_{41}^* = 1$. The errors of the indicator variables $X_1^*X_4^*$, $X_2^*X_5^*, X_3^*X_6^*$, ${X_1^*}^2$, ${X_2^*}^2$ and ${X_3^*}^2$ are δ_7^*, δ_8^*, δ_9^*, $\delta_{10}^*, \delta_{11}^*$ and δ_{12}^* respectively.

The three approaches namely Latent Moderator Approach (LMS), Unconstrained Approach (UA) and Constrained Approach (CA) are used to estimate the parameters of the indicator variables which are shown in Diagram 4.1. Subsequently, their results are discussed in the following Sections 5, 6 and 7 respectively.

5 Latent Moderated Structural Equation Model (LMS)

Latent Moderated Structural Equation method has been developed by Klenin and Moosbrugger (2000) for estimation of multiple latent interaction and quadratic effects for non normal latent variables. The manifest non-linear indicators are not needed in latent moderated structural equation method for the estimation of the non-linear effects. Instead, the latent criterion variable is non-normally distributed when non-linear effects are in the data; the distribution of the latent criterion is utilized to estimate the non-linear effects.

The latent exogenous predictor and the latent exogenous moderator variables are assumed to be bivariate normally distributed. If follows that for each value of the moderator variable the conditional distribution of the predictor variable and the conditional distribution of the latent criterion variable is normal. Therefore, the non-normal density function of the joint indicator vector is approximated by a finite mixture distribution of multivariate normally distributed components. In order to estimate the model parameters, model implied mean vectors and covariance matrices of the mixture components are utilized.

The latent moderated structural equation only assumes that the latent predictor variables ξ_1 and ξ_2 and all error variables δ_1, δ_2, δ_3, δ_4, δ_5, δ_6 and ξ are to be normally distributed. The Maximum Likelihood estimates are computed with the Expectation – Maximization algorithm (Dempster et al. 1977). The parameters are estimated by using latent moderated approach in the statistical software Mplus and the syntax is taken from Moosbrugger, H., et al. (2010),Testing of nonlinear effects in Structural Equation Modeling: A comparison of alternative estimation approaches, Rotterdam, NL, Sense publications. The fit indices and coefficients of LMS approach are obtained by using Mplus 5.0 package and given in the following Table 5.1 and Table 5.2 respectively.

Table 5.1

Methods	χ^2/df	RMSEA	CFI	SRMR	Result
Latent Moderated approach	3.528	0.041	0.906	0.042	Model is significant

In latent approach, the χ^2/df value 3.528 is less than the recommended value 5, the RMSEA value 0.041 is also less than 0.05, the CFI value 0.906 is higher than the guideline value 0.9 and SRMR is less than 0.05. All fit indices of latent moderated approach satisfied the required conditions for accepting the given model in Diagram 4.1. In overall, the fit indices of LMS Method confirm that the derived model has a high explanatory power in terms of describing the interrelationship among the latent exogenous and latent endogenous constructs. The coefficient values of latent exogenous variables and its standard error and probability value are given in the following Table 5.2.

Table 5.2

Method	Paths	Coefficient value	Standard error	p-value	Result
Latent Moderated Approach	$\xi_1^* \to Y_1^*$	-0.421	0.018	0.000	Significant
	$\xi_2^* \to Y_1^*$	-0.307	0.027	0.000	Significant
	$\xi_3^* \to Y_1^*$	-0.506	0.015	0.000	Significant
	$\xi_4^* \to Y_1^*$	-0.327	0.024	0.000	Significant
	$\xi_1^* \to Y_2$	-0.562	0.026	0.000	Significant
	$\xi_2^* \to Y_2$	-0.427	0.020	0.000	Significant
	$\xi_3^* \to Y_2$	-0.610	0.019	0.000	Significant
	$\xi_4^* \to Y_2$	-0.405	0.015	0.000	Significant
	$Y_1^* \to Y_2$	0.597	0.004	0.000	Significant

From the Table 5.2, the coefficients, standard errors and significant values are given for the Latent Moderated Approach Method. In Latent Moderated Approach, the linear latent exogenous variables ξ_1^* and ξ_2^*, the nonlinear latent exogenous variable ξ_3^* and latent quadratic exogenous variable ξ_4^* are significant at 1 % level. There are four latent exogenous variable are explaining the manifest endogenous variables Y_1^* *and* Y_2 out of which the most contributing variable is ξ_3^* the next contributing variable is ξ_1^* and least contributing variables are ξ_4^* *and* ξ_2^*.

6 Unconstrained Method

Marsh et al. (2004) revised the constrained approach in that they did not impose any complicated nonlinear constraints to define relations between product indicators and the latent nonlinear terms and denoted this new approach as Unconstrained approach. In the unconstrained approach extended to quadratic nonlinear terms (Kelava et al., 2008) factor loadings as well as error variables and covariances are estimated directly without using any constraints. Additionally, parameters based on assumptions of normality are also not constrained, so that this approach does not need any constraints on the latent variances, too. The covariances between the linear and nonlinear latent variables may be freely estimated if it is assumed that the variables are non normally distributed. The only constraints used in this approach are the constraints on the latent means which are shown in the following:

$$Mean\,Vector = \begin{pmatrix} 0 \\ 0 \\ \phi_{21} \\ \phi_{11} \end{pmatrix} \quad and \quad Covariacne\,Matrix = \begin{pmatrix} \phi_{11} & & & \\ \phi_{21} & \phi_{22} & & \\ 0 & 0 & \phi_{33} & \\ 0 & 0 & \phi_{43} & \phi_{44} \end{pmatrix}.$$

An advantage of the unconstrained approach for applied researchers clearly is that the syntax is much easier to set up than for the extended constrained approach as no complicated, nonlinear constraints are required. The fit indices and coefficients of unconstrained approach are obtained by using Mplus 5.0 package and given in the following Table 6.1 and Table 6.2 respectively.

Table 6.1

Methods	χ^2/df	RMSEA	CFI	SRMR	Result
Unconstrained approach	1.392	0.038	0.962	0.039	Model is significant

In unconstrained approach, the χ^2/df value 1.392 is less than the recommended value 2, the RMSEA value 0.038 is less than 0.05, the value of CFI 0.962 is more than the guideline value 0.9 and the value of SRMR 0.039 is also less than 0.05. The fit indices values of unconstrained approach are also satisfied the recommended values and the model is accepted. Based on the fit indices it is inferred that the given model is accepted which means the endogenous variables Y_1^* and Y_2 are highly explained by the latent linear term, interaction term and quadratic term. The coefficient values of latent exogenous variables and its standard error and probability value are given in the following Table 6.2.

Table 6.2

Method	Paths	Coefficient value	Standard error	p-value	Result
Unconstrained Approach	$\xi_1^* \to Y_1^*$	-0.438	0.011	0.000	Significant
	$\xi_2^* \to Y_1^*$	-0.299	0.013	0.000	Significant
	$\xi_3^* \to Y_1^*$	-0.548	0.009	0.000	Significant
	$\xi_4^* \to Y_1^*$	-0.337	0.017	0.000	Significant
	$\xi_1^* \to Y_2$	-0.609	0.013	0.000	Significant
	$\xi_2^* \to Y_2$	-0.475	0.019	0.000	Significant
	$\xi_3^* \to Y_2$	-0.692	0.008	0.000	Significant
	$\xi_4^* \to Y_2$	-0.486	0.010	0.000	Significant
	$Y_1^* \to Y_2$	-0.684	0.003	0.000	Significant

From the Table 6.2, the coefficients, standard errors and significant values are given for the Unconstrained Approach Method. In Unconstrained Approach, the linear latent exogenous variables ξ_1^* and ξ_2^*, the nonlinear latent exogenous variable ξ_3^* and latent quadratic exogenous variable ξ_4^* are significant at 1 % level. Out of four latent factors, the leading contributor to Y_1^* and Y_2 are ξ_3^* the second leading term is ξ_1^*, the third leading latent term is ξ_4^* and ξ_2^* is the least contributor on manifest endogenous variables Y_1^* and Y_2.

7 Constrained Method

Joreskog and Yang (1996) developed constrained approach and Marsh et al. (2004). The unconstrained approach is developed by Kenny and Judd (1984). In that study, the latent nonlinear terms ξ_3^* and ξ_4^* are specified for the both models constrained and unconstrained approaches. In order to estimate the parameters, the maximum likelihood method is used and it is to be assumed that all latent variables are normally distributed. Kenny and Judd (1984) first described, how to analyze a latent nonlinear model with an interaction or a quadratic effect and also they included all possible manifest variables considered as indicators in latent interaction term ξ_3^* . Later Marsh et al. (2004) suggested to use the matched pair strategy in order to avoid overlapping information in the quadratic and interaction term. As per Kenny and Judd (1984), it is required nine product indicators for latent interaction term, but only three product indicators are obtained as per Marsh et al. (2000) namely $X_1^*X_4^*$, $X_2^*X_5^*$ and $X_3^*X_6^*$.

If X_2^* is an indicator of ξ_1^* and X_5^* is an indicator of ξ_2^* (with $X_2^* = \lambda_{12}^*\xi_1^* + \delta_2^*$ and $X_5^* = \lambda_{52}^*\xi_2^* + \delta_5^*$), then the indicator $X_2^*X_5^*$ of the interaction term ξ_3^* would be specified as follows:

$$X_2^*X_5^* = (\lambda_{12}^*\xi_1^* + \delta_2^*)(\lambda_{25}^*\xi_2^* + \delta_5^*)$$
$$= \lambda_{38}^*\xi_3^* + \delta_8^* .$$

As the parameters in this measurement equation cannot be estimated directly, several constraints are needed. The variance decomposition of the interaction indicator $X_2^*X_5^*$ required for model specification and is given as follows:

$$Var(X_2^*X_5^*) = Var(\lambda_{38}^*\xi_3^* + \delta_8^*)$$
$$= {\lambda_{38}^*}^2 Var(\xi_3^*) + Var(\delta_8^*)$$

and includes the following constraints

$${\lambda_{38}^*}^2 = {\lambda_{12}^*}^2 {\lambda_{25}^*}^2$$

$Var(\xi_3^*) = Var(\xi_1^*) + Var(\xi_2^*) + Cov(\xi_1^*, \xi_2^*)^2$
$Var(\delta_8^*) = {\lambda_{12}^*}^2 Var(\xi_1^*) Var(\delta_5^*) + {\lambda_{25}^*}^2 Var(\xi_2^*) Var(\delta_2^*) + Var(\delta_2^*) Var(\delta_5^*)$.
All indicators of the linear terms ξ_1^* and ξ_2^* should be used in the formation of the indicators of each latent non linear term, but each of the multiple indicators should be used only once for each nonlinear term.
The above measurement model for the predictor variables of the nonlinear model is therefore as follows:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_1X_4 \\ X_2X_5 \\ X_3X_6 \\ X_1^2 \\ X_2^2 \\ X_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \lambda_{12} & 0 & 0 & 0 \\ \lambda_{13} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \lambda_{25} & 0 & 0 \\ 0 & \lambda_{26} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \lambda_{38} & 0 \\ 0 & 0 & \lambda_{39} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda_{42} \\ 0 & 0 & 0 & \lambda_{43} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_1\xi_2 \\ \xi_1^2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \\ \delta_{10} \\ \delta_{11} \\ \delta_{12} \end{bmatrix}.$$

Under the supposition that ξ_1, ξ_2, ζ and all error terms δ are multivariate normal, uncorrelated and have zero means. Joreskog and Yang (1996) proposed a model with a latent mean structure. Using the constrained approach extended for multiple nonlinear term by Kelava et.al.,(2008), the mean vector and covariance matrix of ξ_1, ξ_2, $\xi_1\xi_2$ and ξ_1^2 are, respectively.

$$Mean\ vector = \begin{bmatrix} 0 \\ 0 \\ \phi_{21} \\ \phi_{11} \end{bmatrix} \quad and \quad Covariance\ Matrix = \begin{bmatrix} \phi_{11} & & & \\ \phi_{21} & \phi_{22} & & \\ 0 & 0 & \phi_{11}\phi_{22} + \phi_{21}^2 & \\ 0 & 0 & 2\phi_{11}\phi_{21} & 2\phi_{11}^2 \end{bmatrix}.$$

Furthermore, even if ξ_1 and ξ_2 are centered so as to have zero means, $E(\xi_1\xi_2) = Cov(\xi_1, \xi_2) = \phi_{21}$, will typically not be zero. Extending Joreskog and Yang (1996) latent interaction model to a model with quadratic terms (Kelava et al., 2008). In order to estimate the parameters in extended constrained approach, the syntax (Kelava et al., 2008) $E(\xi_1^2) = Var(\xi_1) = \phi_{11}$ and $E(\xi_2^2) = Var(\xi_2) = \phi_{22}$ will also not be zero. Hence the mean structure is always necessary and should always be specified. It is used in LISREL package and it is taken from Moosbrugger, H., et.al. (2010),Testing of nonlinear effects in Structural Equation Modeling: A comparison of alternative estimation approaches, Rotterdam, NL, Sense publications.

The fit indices and coefficients of constrained approach are obtained by using Mplus 5.0 package and given in the following Table 7.1 and Table 7.2 respectively.

Table 7.1

Methods	χ^2/df	RMSEA	CFI	SRMR	Result
Constrained approach	1.046	0.003	0.995	0.028	Model is significant

From Table 7.1, in constrained approach, the value of χ^2/df 1.046 is less than the guideline value 2, the RMSEA value 0.003 is less than 0.05, the CFI value 0.995 is more than 0.9 and it is close to 1 and SMR value 0.028 is less than 0.05. All fit indices highly satisfied the guideline values. Based on the fit indices it is inferred that the given model is accepted which means the endogenous variables Y_1^* and Y_2 are highly explained by the latent linear term, interaction term and quadratic term.

The fit indices of the constrained approach highly satisfy the guideline values than the latent moderated approach and unconstrained method. The performance of the constrained approach is better than the latent moderated approach and unconstrained method. The coefficient values of latent exogenous variables and its standard error and probability values are given in the following Table 7.2.

Table 7.2

Method	Paths	Coefficient value	Standard error	p-value	Result
Constrained Approach	$\xi_1^* \to Y_1^*$	-0.457	0.003	0.000	Significant
	$\xi_2^* \to Y_1^*$	-0.308	0.014	0.000	Significant
	$\xi_3^* \to Y_1^*$	-0.619	0.008	0.000	Significant
	$\xi_4^* \to Y_1^*$	-0.364	0.006	0.000	Significant
	$\xi_1^* \to Y_2$	-0.689	0.018	0.000	Significant
	$\xi_2^* \to Y_2$	-0.507	0.012	0.000	Significant
	$\xi_3^* \to Y_2$	-0.724	0.006	0.000	Significant
	$\xi_4^* \to Y_2$	-0.531	0.002	0.000	Significant
	$Y_1^* \to Y_2$	0.703	0.001	0.000	Significant

From the Table 7.2, the coefficients, standard errors and significant values are given for the Constrained Approach Method. In Constrained Approach, the linear latent exogenous variables ξ_1^* and ξ_2^*, the nonlinear latent exogenous variable ξ_3^* and latent quadratic exogenous variable ξ_4^* are significant at 1 % level of significant. Out of four latent factors, the leading contributor to Y_1^* *and* Y_2 is ξ_3^* the second leading term is ξ_1^* , the third leading latent term is ξ_4^* and least contributor on manifest endogenous variables is ξ_2^*.

The coefficient of each latent exogenous variables value are significantly contributing to the manifest endogenous variables Y_1^* *and* Y_2. Their coefficient values are also clearly explaining the relationship among the variables. In Constrained approach method, the coefficient value of the linear latent exogenous variables ξ_1^* and ξ_2^*, the latent interaction exogenous variable ξ_3^* and latent quadratic exogenous variable ξ_4^* are considerably higher than the latent moderated method and unconstrained method. Hence it is concluded that the constrained method is the best method to explain the relationship between latent exogenous variables and manifest endogenous variables.

8 Conclusion

When considering the three methods namely latent moderated structural model, unconstrained approach, constrained approach all the three methods explain the relationship among the variables very well. Comparing unconstrained approach with latent moderated approach, the unconstrained method is better explaining endogenous variables Y_1^* *and* Y_2 by the latent exogenous variables. The constrained method is compared with latent moderated method and unconstrained method. The fit indices of constrained method highly satisfy the guideline values than the other two methods. It is inferred that the constrained approach is the best method to explain the relationship between the latent exogenous variables ξ_1^* , ξ_2^* (blood factor, life style factor), ξ_3^* (interaction term), ξ_4^* (quadratic term) and the manifest endogenous variable Y_1^* (Ejection Fraction) and Y_2 (Survival status).

References

Aiken, L.S., et.al (1991). Multiple regression: Testing and interpreting interactions. Newbury Park, CA: Sage Publications.

Ajzen, I. (1991). The theory of planned behavior. *Organizational Behavior and Human Decision Processes*, *50*, 179–211.

Algina, J. and Moulder, B.C. (2001). A Note on Estimating the Joreskog-Yong Model for Latent Variables Interaction using LISREL 8.3, Structural Equation Modeling, 8(1) 40-52.

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society, Series B*, *39*, 1–38.

Elliott, M. A., Armitage, C. J., & Baughan, C. J. (2003). Drivers' compliance with speed limits: An application of the theory of planned behavior. *Journal of Applied Psychology*, *88*, 964–972.

Ganzach, Y. (1997). Misleading interaction and curvilinear terms. *Psychological Methods*, *3*, 235–247.

Jaccard, J., & Wan, C. K. (1995). Measurement error in the analysis of interaction effects between continuous predictors using multiple regression: Multiple indicator and structural equation approaches. *Psychological Bulletin*, *117*, 348–357.

Jöreskog, K. G., & Yang, F. (1996). Nonlinear structural equation models: The Kenny-Judd model with interaction effects. In G. Marcoulides & R. Schumacker (Eds.), *Advanced structural equation modeling* (pp. 57–87). Mahwah, NJ: Lawrence Erlbaum Associates.

Kelava, A., Moosbrugger, H., Dimitruk, P., & Schermelleh-Engel, K. (2008). Multicollinearity and missing constraints: A comparison of three approaches for the analysis of latent nonlinear effects. *Methodology, 4*, 51–66.

Kenny, D. A., & Judd, C. M. (1984). Estimating the nonlinear and interactive effects of latent variables. *Psychological Bulletin*, *96*, 201–210.

Klein, A. G., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. *Psychometrika*, *65*, 457–474.

Marsh, H. W., Wen, Z., & Hau, K. T. (2004). Structural equation models of latent interactions: Evaluation of alternative estimation strategies and indicator construction. *Psychological Methods*, *9*, 275–300.

Marsh, H. W., Wen, Z. L., & Hau, K. T. (2006). Structural equation models of latent interaction and quadratic effects. In G. R. Hancock & R. O. Mueller (Eds.), *A second course in structural equation modeling*. Greenwich, CT: Information Age Publishing.

Moosbrugger, H., et. al.(2010),Testing of nonlinear effects in Structural Equation Modeling: A comparison of alternative estimation approaches, Rotterdam, NL, Sense publications.

Schermelleh-Engel, K., Klein, A., & Moosbrugger, H. (1998). Estimating nonlinear effects using a latent moderated structural equations approach. In R. E. Schumacker & G. A. Marcoulides (Eds.), *Interaction and nonlinear effects in structural equation modeling* (pp. 203–238). Mahwah, NJ:Lawrence Erlbaum Associates.

Trouilloud, D., Sarrazin, P., Bressoux, P., & Bois, J. (2006). Relation between teachers' early expectations and students' later perceived competence in physical education classes: Autonomy supportive climate as a moderator. *Journal of Educational Psychology*, *98*, 75–86.

Wang, S. S., Houshyar, S., & Prinstein, M. J. (2006). Adolescent girls' and boys' weight-related health behaviors and cognitions: Associations with reputation- and preference-based peer status. *Health Psychology*, *25*, 658–663.