ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com
Journal DOI: 10.21474/IJAR01

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

MATHEMATICAL MODEL ON ANALYSIS OF AWARENESS IN CONTROLLING DENGUE DISEASE.

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Manuscript Info

Manuscript History:

Received: 19 May 2016
Final Accepted: 22 June 2016
Published Online: July 2016

Key words:
Epidemic model,
Equilibrium point,
Awareness,
Basic reproduction number,
Stability,
Sensitivity analysis

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Abstract

Dengue is a viral disease transmitted to humans by the bite of infected Aedes mosquitoes. The disease has become a major public health burden in recent years. There is no vaccine and no effective treatment for dengue disease. So, spreading awareness about the disease transmission in people can play a crucial role in controlling the transmission of the disease. Present paper considers an SIR epidemic model to investigate the impact of awareness in controlling the disease. It is assumed that due to awareness, some susceptible hosts do not come in contact with mosquitoes, they use mosquito repellent and take all possible precautions, some infected hosts get isolated so that they do not transmit the disease; and some infected hosts approach the doctor soon and recover fast from the disease. Basic reproduction number which determines whether the disease dies out or takes hold is calculated using next generation matrix method. Local and global stability of disease free equilibrium point of the model are analyzed. Sensitivity analysis of the model is performed to investigate the relative importance of the awareness parameters to the disease transmission.

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Introduction:-

Dengue is one of the most rapidly spreading diseases in the world. An estimated 2.5 billion people live in the areas where dengue is epidemic. The disease is recognized in over 100 countries and an estimated 50 – 100 million dengue infections occur annually [9]. Dengue is caused by any one of four serotypes of viruses, DEN 1 - DEN 4. A person infected by one of the serotypes of the dengue viruses will never be infected by the same serotype, but he/she loses immunity to other serotypes of the viruses and then becomes more susceptible in developing dengue hemorrhagic fever [5].

Till date, there is no specific drug that can treat dengue effectively. Different strategies of control of disease have been proved to be inefficient. Many mathematical models of dengue disease have been proposed to analyze and control the disease [3, 4, 6, 10- 13, 15, 16, 18, 20]. Awareness program is an influential source of knowledge transfer and dissemination process. It plays an important role in gathering and reproducing information. It is considered as the most effective epidemic management program which can reduce the burden of the disease [6]. Thus, spread of awareness can help in controlling the transmission of the disease. In the present paper, we have considered that

- Some susceptible hosts do not come in contact with infected mosquitoes as they take all possible precautions.
- Some infected hosts get the supportive treatment in time and recover fast.
- Some infected hosts who are aware of the disease transmission do not transmit the disease due to awareness towards the disease transmission.

In the absence of awareness parameters, present model reduces to the model proposed by Esteva and Vargas [3].

Formulation of the model:-

For the formulation of the model, total host (human) population is divided into three mutually disjoint classes, Susceptible, Infected and Recovered. Total vector (mosquito) population is divided into two classes only, Susceptible and Infected as the infection of the vectors end with their death.

It is assumed that the fraction p_1 of total susceptible host population use mosquito repellent so that they do not come in contact with the infected mosquitoes. The fraction p_2 of total infected hosts approach the doctor soon for the supportive treatment and recover fast by the recovery rate improving factor $\alpha > 1$. The fraction p_3 of total infected hosts do not transmit the disease as they are isolated. So, only the fraction $(1 - p_1)$ of susceptible hosts and the fraction $(1 - p_3)$ of infected hosts interact with the mosquitoes. Let S_h, I_h and R_h denote the number of susceptibles, infectives and immunes (recovered from disease) in the host population; and S_v and I_v denote the number of susceptibles and infectives in vector population.

Table 1:- Parameters, their meanings and dimensions

Parameters	Biological Meaning	Dimensions
μ_h	Birth/Death rate in the host population	Time^{-1}
γ_h	Recovery rate in host population	Time^{-1}
β_h	Transmission probability from vector to host	Dimensionless
β_v	Transmission probability from host to vector	Dimensionless
μ_v	Birth/Death rate in the vector population	Time^{-1}
b	Biting rate of vector	Time^{-1}
A	Recruitment rate of vector	Mosquitoes× Time^{-1}

The system of differential equations that describe the dynamics of the interaction between hosts and vectors is given by

$$\begin{aligned}
\frac{dS_h}{dt} &= \mu_h N_h - (1-p_1)\frac{b\beta_h}{N_h} S_h I_v - \mu_h S_h \\
\frac{dI_h}{dt} &= (1-p_1)\frac{b\beta_h}{N_h} S_h I_v - [\alpha\, p_2\, \gamma_h + (1-p_2)\gamma_h] I_h - \mu_h\, I_h \\
\frac{dR_h}{dt} &= [\alpha p_2 \gamma_h + (1-p_2)\gamma_h] I_h - \mu_h R_h \\
\frac{dS_v}{dt} &= A - (1-p_3)\frac{b\beta_v}{N_h} S_v I_h - \mu_v\, S_v \\
\frac{dI_v}{dt} &= (1-p_3)\frac{b\beta_v}{N_h} S_v I_h - \mu_v\, I_v
\end{aligned} \tag{1}$$

Both host and vector populations are considered constant. So,

$$R_h = N_h - S_h - I_h, \qquad S_v = N_v - I_v$$

Again,

$$\frac{dN_v}{dt} = 0 \implies N_v = \frac{A}{\mu_v}$$

Hence, the system (1) has the same qualitative dynamics as

$$\begin{aligned}
\frac{dS_h}{dt} &= \mu_h N_h - (1-p_1)\frac{b\beta_h}{N_h} S_h I_v - \mu_h S_h \\
\frac{dI_h}{dt} &= (1-p_1)\frac{b\beta_h}{N_h} S_h I_v - [\alpha\, p_2\, \gamma_h + (1-p_2)\gamma_h] I_h - \mu_h I_h \\
\frac{dI_v}{dt} &= (1-p_3)\frac{b\beta_v}{N_h}\left(\frac{A}{\mu_v} - I_v\right) S_v I_h - \mu_v I_v
\end{aligned} \tag{2}$$

Setting $s_h = \frac{S_h}{N_h}, i_h = \frac{I_h}{N_h}$, $i_v = \frac{I_v}{A/\mu_v}$, $\eta = (1-p_1)\frac{Ab\beta_h}{\mu_v N_h}$, $\beta = \mu_h + [\alpha p_2 + (1-p_2)]\gamma_h, \gamma = (1-p_3)b\beta_v$, we find the following system of equations

$$\begin{aligned} \frac{ds_h}{dt} &= \mu_h(1-s_h) - \eta s_h i_v \\ \frac{di_h}{dt} &= \eta s_h i_v - \beta i_h \\ \frac{di_v}{dt} &= \gamma(1-i_v)i_h - \mu_v i_v \end{aligned} \tag{3}$$

Disease Free Equilibrium Point (DFE):-

Proposition 1:-
The system of equations (3) has a disease free equilibrium point (1, 0, 0).

Proof:-
Equilibrium points are obtained by setting $\frac{ds_h}{dt} = 0$, $\frac{di_h}{dt} = 0$ and $\frac{di_v}{dt} = 0$. In disease free situation $i_h = 0$ and $i_v = 0$. From the system of equations (3), we obtain $\mu_h(1-s_h) = 0$. Which implies that $s_h = 1$. Hence, the disease free equilibrium point is (1, 0, 0).

Basic Reproduction Number:-

Basic reproduction number R_0 is defined as "The average number of secondary infections caused by a single infectious individual during their entire infectious lifetime". It predicts whether a disease will die out or becomes endemic. Using the next generation method [2, 19], we computed

$$F = \begin{bmatrix} 0 & \eta \\ \gamma & 0 \end{bmatrix},\ V = \begin{bmatrix} \beta & 0 \\ 0 & \mu_v \end{bmatrix} \text{ and } R_0 = \rho\{FV^{-1}\}.$$

Therefore,

$$\mathrm{R}_0 = \sqrt{\frac{\eta\gamma}{\mu_v\beta}} = \sqrt{\frac{Ab^2\beta_h\beta_v(1-p_1)(1-p_3)}{N_h\mu_v^2[\alpha\, p_2\, \gamma_h + (1-p_2)\gamma_h + \mu_h)]}} \tag{4}$$

Stability of Disease Free Equilibrium (DFE):-

Proposition 2:-
The DFE is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof:-
Suppose that $R_0 < 1$. The Jacobian matrix of the system of equations (3) at the DFE is obtained as

$$J = \begin{bmatrix} -\mu_h & 0 & -\eta \\ 0 & -\beta & \eta \\ 0 & \gamma & -\mu_v \end{bmatrix}$$

The corresponding characteristic equation is

$$(\lambda + \mu_h)\,[\lambda^2 + (\mu_v + \beta)\lambda + \beta\mu_v(1 - R_0^2)] = 0$$

This implies that

$$\lambda = -\mu_h,$$

$$\lambda^2 + (\mu_v + \beta)\lambda + \beta\mu_v(1 - R_0^2) = 0 \tag{5}$$

Equation (5) is of the form $\lambda^2 + a_1\lambda + a_2 = 0$, where $a_1 = \mu_v + \beta$ and $a_2 = \beta\mu_v(1 - R_0^2)$. By the Routh Hurwitz Test [1], all the roots of equation (5) will have negative real parts precisely when a_1 and a_2 are positive. Here, a_1 is positive since μ_v and β are positive. Also, $a_2 > 0$ if $R_0 < 1$. So, all the eigenvalues of the equation (5) have negative real parts if $R_0 < 1$. Hence, the DFE is locally asymptotically stable if $R_0 < 1$. Again, if $R_0 > 1$, then $a_2 < 0$ which shows that the DFE is unstable.

Proposition 3: (Global Stability of DFE):-
The disease free equilibrium is globally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof:-
We use comparison theorem [8, 17] to prove the global stability of disease free equilibrium. Since $s_h \leq 1$ and $1 - i_v \leq 1$ from the system of equations (3), we can write

$$\frac{di_h}{dt} \leq \eta i_v - \beta i_h$$
$$\frac{di_v}{dt} \leq \gamma i_h - \mu_v i_v \quad (6)$$

Let us consider the following linear system of equations

$$\frac{di_h}{dt} = \eta i_v - \beta i_h$$
$$\frac{di_v}{dt} = \gamma i_h - \mu_v i_v \quad (7)$$

Above system of equations can be written as

$$\vec{u}'(t) = A\vec{u} \quad (8)$$

Here, $\vec{u} = [i_h, i_v]^T$, $A = F - V$. Here, F is a non-negative matrix and V is a non-negative M-matrix (F and V are defined in (4), so by [19]

Spectral abscissa, $s(F - V) = s(A) < 0 \Leftrightarrow \rho\{FV^{-1}\} < 1$

Thus, if $R_0 = \rho\{FV^{-1}\} < 1$, then $s(A) < 0$. Hence, if $R_0 < 1$, then

$$\lim_{t\to\infty} \vec{u} = 0 \text{ and } \lim_{t\to\infty} i_h = 0, \lim_{t\to\infty} i_v = 0$$

All the variables in the system of equations (3) are non-negative, so the use of comparison theorem, leads to

$$\lim_{t\to\infty} i_h = 0, \qquad \lim_{t\to\infty} i_v = 0$$

Hence, the system of equations (3) has the qualitative dynamics of the following limit system

$$\frac{ds_h}{dt} = \mu(1 - s_h)$$

Thus, if $R_0 < 1$, $t \to \infty \Rightarrow s_h \to 1$, $i_h \to 0$, $i_v \to 0$. Hence, the disease free equilibrium (1, 0, 0) of the system (3) is globally asymptotically stable if $R_0 < 1$.

When $R_0 > 1$, $s(F - V) > 0 \Leftrightarrow R_0 = \rho\{FV^{-1}\} > 1$ ([19]). So, the disease free equilibrium is unstable if $R_0 > 1$.

Existence of the Endemic Equilibrium Point (EEP):-

Proposition 4:-
The system of equations (3) has an endemic equilibrium point if $R_0 > 1$.

proof:-
Suppose that $R_0 > 1$. From the system of equations (3), we have

$$s_h^* = \frac{\mu_h\gamma + \beta\mu_v}{\gamma(\mu_h + \eta)}, \qquad i_h^* = \frac{\mu_h\,(\eta\gamma - \beta\mu_v)}{\beta\gamma(\mu_h + \eta)}, \; i_v^* = \frac{\mu_h(\eta\gamma - \beta\mu_v)}{\eta(\mu_h\gamma + \beta\,\mu_v)}$$

If $R_0 > 1$ then $\eta\gamma > \beta\mu_v$ and $i_h > 0$, $i_v > 0$. If $R_0 < 1$, then $i_h < 0$, $i_v < 0$. Hence, the endemic equilibrium point exists if $R_0 > 1$.

Table 2:- Model parameters and their values

Parameters	Values	References
N_h	10000	[3]
μ_h	0.0000457	[3]
μ_v	0.14286	[7]
γ_h	0.14286	[3]
b	0.5	[18]
β_h	0.75	[11]
β_v	0.75	[11]
A	5000	[3]

Sensitivity Analysis:-

It is necessary to identify the relative importance of different factors responsible for the disease transmission and prevalence. We have performed sensitivity analysis to determine the most sensitive awareness parameter among the parameters p_1, p_2, p_3 and α that has high impact on the basic reproduction number R_0. To perform sensitivity analysis we have assumed that 10% of susceptible hosts do not come in contact with mosquitoes, 10% of infected host recover fast by recovery rate improving factor $\alpha = 1.1$ and 10% of infected hosts are isolated.

The normalized forward sensitivity index ([14]) of R_0, that depends differentiably on a parameter ν is defined as;

$$\Upsilon_{\nu}^{R_0} = \frac{\partial R_0}{\partial \nu} \times \frac{\nu}{R_0}$$

Table 3:- Sensitivity indices of R_0

Parameter values	Baseline values	Sensitivity index
p_1	0.1	-0.056
p_2	0.1	-0.005
p_3	0.1	-0.056
α	1.1	-0.054

Table 3 with negative sensitivity index values shows that awareness parameters and basic reproduction number have inverse relation. Basic reproduction number decreases with the increase in values of awareness parameters. Also, the most sensitive parameter among the awareness parameters p_1, p_2, p_3 and α are p_1 and p_3, the fractions of susceptible and infected hosts who do not interact with mosquitoes. So, the present study suggests that we can reduce the disease prevalence by increasing the awareness about the disease transmission in host population. As there is no vaccine and effective treatment for the dengue disease, we should increase awareness in host population to bring the disease under control. Also, the tabulated results show that we should concentrate on increasing the awareness in the people (both susceptible and infected hosts) so that they do not come in contact with mosquitoes taking all possible precautions.

Numerical Results and Discussion:-

In the present work, we have investigated the impact of awareness in the transmission and control of dengue disease. So, we have made simulations with different values of awareness parameters. Other parameter values involved in the model are taken from the Table 2.

Fig. 1 is drawn for the parameter values $p_1 = p_2 = p_3 = 0$ and $\alpha = 1$ (normal situation) which depicts the dynamics of host and vector population of different compartments. The population size of susceptible host decreases with time due to interaction of susceptible hosts with infected mosquitoes and due to death of some susceptible hosts. The population sizes of infected host and infected vector increase initially due to interaction between susceptible and infective population and then start decreasing due to recovery from the disease and due to the death of some infective hosts and vectors.

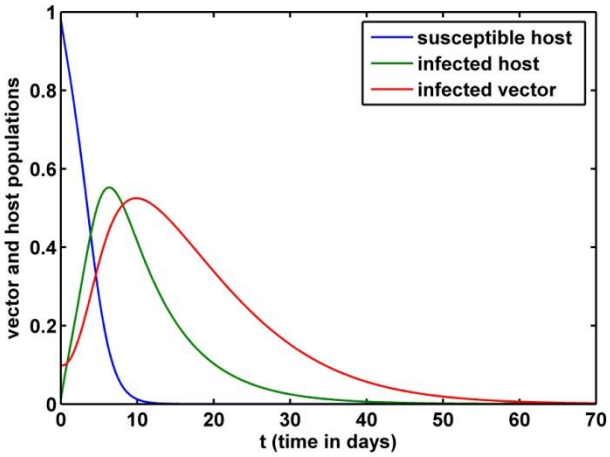

Figure1:- Dynamics of vector and host populations without awareness.

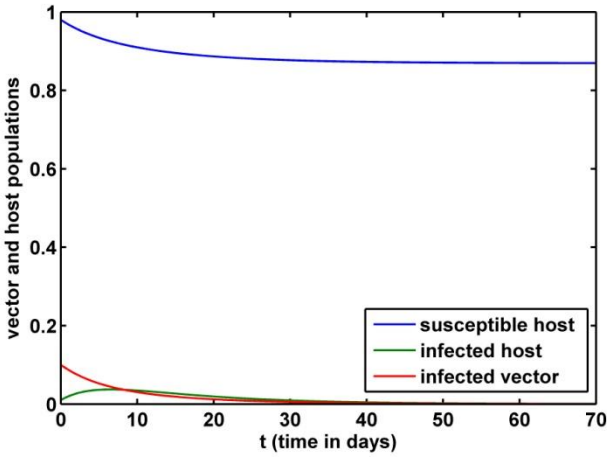

Figure 2:- Dynamics of the vector and host populations with awareness.

In the presence of awareness in host population (Fig. 2, $p_1 = p_2 = p_3 = 0.9,\ \alpha = 2$) small number of hosts get infected and greater number of hosts remain healthy. Thus, increase in the values of awareness parameters help in reducing the spread of the disease when compared to the normal situation. Thus, we can reduce the transmission rate of dengue disease by spreading awareness in human population.

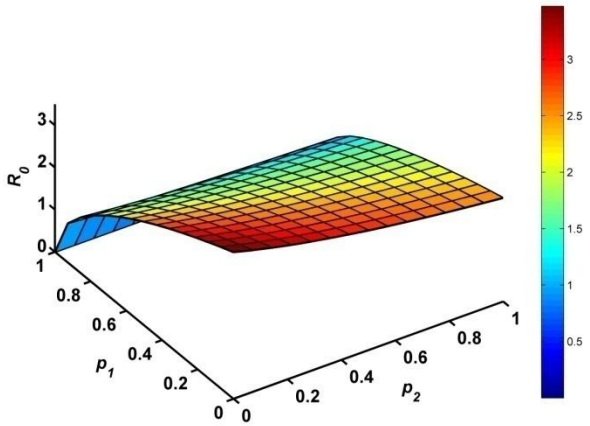

Figure 3:- Basic reproduction number with different values of p_1 and p_2.

Basic reproduction number determines whether the disease becomes endemic or dies out. Fig. 3 shows that both the values of p_1 and p_2 contribute in reducing the values of R_0. So, if the numbers of susceptible hosts who get isolated and infected hosts who approach the doctor soon are increased, the burden of the disease reduces.

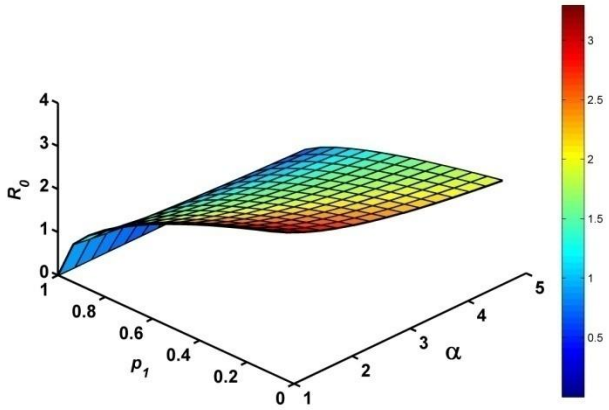

Figure 4:- Basic reproduction number with different values of p_1 and α.

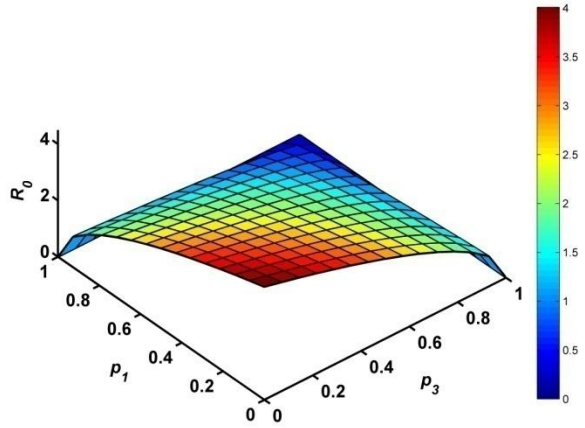

Figure 5:- Basic reproduction number with different values of p_1 and p_3.

Also, the value of basic reproduction number decreases with the increase in the values of p_1 and α (Fig. 4). The isolated fractions of hosts, p_1 and p_3 have similar impact on the basic reproduction number (Fig. 5). Basic reproduction number R_0 decreases with the increase in p_1 and p_3. All the awareness parameters are found to help in decreasing the value of R_0 (Fig. 3 - Fig. 5). So, we should increase awareness in both susceptible and infected host population to bring the dengue disease under control as there is no vaccine and effective treatment for dengue disease.

Conclusion:-

In the present paper, we have made a study on the transmission of dengue disease with awareness of the disease transmission in host population. We have introduced the awareness terms p_1, p_2, p_3 and α in the dengue disease model [3].

Present work provides an insight into the effects of awareness on the transmission of the dengue fever. The increase in awareness decreases the density of infectives and the value of basic reproduction number as well. We found that the disease dies out if $R_0 < 1$ and persists if $R_0 > 1$. Also, proposition 2 and 3 show that the disease free equilibrium of the model is locally and globally stable.

In the paper, the evaluation of the sensitivity indices of the basic reproduction number R_0 is made to determine the relative importance of the awareness parameters in the disease transmission. The results show that basic reproduction number and awareness parameters vary inversely. Thus, awareness parameters contribute in decreasing the value of basic reproduction number and hence decreasing the transmission of the disease. So, spread of awareness in human helps in controlling the spread of dengue disease. Since there is no vaccine and no effective treatment for the dengue disease, awareness can be effective measure to control the disease.

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