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RESEARCH ARTICLE

DEVELOPMENT OF MATHEMATICAL MODEL ON PLUG FLOW APPLICATION INFLUENCED BY LINEAR PHASE VELOCITY IN HOMOGENEOUS SAND GRAVEL PORT HARCOURT METROPOLIS.

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Abstract

The behaviour of velocity in predominant sand gravel has been expressed; the concept of this study was to monitor the rate of fluids deposition in sand gravel formation, the velocity of flow experiences slight fluctuations in some deposited depths of the formations, while linear flow were found predominant, variation of velocity were observed base on the stratification as it is expressed on the lithology, the formation experiences low velocity in few simulated results, while predominant of linear velocity were observed in larger numbers of the study location, developed model generated theoretical values through simulation, these values were compared with experimental values, both parameters compared favourably well, experts in soil and water engineering will definitely use this concept as a tool in design for various purpose.

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Introduction:-

Most designs of deep geological repository for high level radioactive wastes (HLW) are based on the multi-barrier concept with isolation of the waste from the environment. The multi-barrier concept includes the natural geological barrier (host rock), engineered barriers made up of compacted sand-Bentonite mixtures (placed around waste containers or used as buffer and sealing elements) and metal canister. Compacted Bentonite-based materials are relevant materials for this purpose thanks to their low permeability, high swelling and high radionuclide retardation capacities (Pusch, 1979; Yong et al., 1986; Villar and Lloret, 2008; Komine and Watanabe, 2010; Cui et al., 2011). Engineered barriers are often made up of compacted bricks. When bricks are placed around waste canisters or to form sealing buffers, the so-called technological voids either between the bricks themselves or between bricks, canisters and the host rock are unavoidable. As an example, 10 mm thick gaps between Bentonite blocks and canister and 25 thick mm gaps between the Bentonite blocks and the host rock have been considered in the basic design of Finland (Juvankoski, 2010). These technological voids appeared to be equal to 6.6 % of the volume of the gallery in the FEBEX mock-up test (Martin et al., 2006). Once placed in the galleries, engineered barriers are progressively hydrated by pore water infiltrating from the host-rock. This water infiltration is strongly dependent on the initial state of the compacted material (water content, suction and density, e.g. Cui et al. 2008). Indeed, it has been shown that water transfer in unsaturated swelling compacted Bentonite or sand Bentonite mixtures is strongly dependent on the imposed boundary conditions in terms of volume change. As shown in Yahia-Aissa et al. (2001), Cui et al. (2008) and Ye et al. (2009), the degree of swelling allowed significantly affects the amount of infiltrated water, with much water absorbed when swelling is allowed and a minimum amount of water absorbed when swelling is prevented. Volume change conditions also appeared to have, through microstructure changes, significant

influence on the hydraulic conductivity. In this regard, the degree of swelling allowed by the technological voids described above has a significant influence on the hydro-mechanical behaviour of the compacted Bentonite and their effects need to be better understood. Swelling results in a decrease in dry density that may lead to a degradation of the hydro-mechanical performance of engineered barriers (Komine et al., 2009, Komine, 2010). As a result, the safety function expected in the design may no longer be properly ensured. Therefore, a better understanding of the effects of the technological voids is essential in assessing the overall performance of the repository.

3. Developed Governing Equation:-

$$\bar{V} \frac{\partial q}{\partial t} = \frac{\phi}{ne} \frac{\partial q}{\partial z} - K_x \frac{\partial q}{\partial z} \quad \dots\dots\dots (1)$$

The expression here is the is the governing equation that will assess aquiferous thickness in the study location, the study through the developed equation will monitor aquiferous thickness in the study area, lots of variation has been observed in the study area, the rate of aquiferous deposition has express lots of insufficient design and construction practices that has resulted to lots of abortive wells in the study location, therefore the developed governing considered this condition to developed the expression in [1].

Nomenclature

q	=	Aquifer height [L]
\bar{V}	=	Homogeneous velocity [LT^{-1}]
K_x	=	Permeability coefficient [LT^{-1}]
ϕ	=	Flow rate [LT^{-1}]
ne	=	Porosity [-]
T	=	Time [T]
Z	=	Depth [L]

$$\frac{\partial q}{\partial t} = S^1 C(t) - C(o) \quad \dots\dots\dots (2)$$

$$\frac{\partial q}{\partial z} = S^1 C(z) - C(o) \quad \dots\dots\dots (3)$$

$$\frac{\partial q}{\partial z} = S^1 C(z) - C(o) \quad \dots\dots\dots (4)$$

Substituting equation (2), (3) and (4) into equation (1) yields:

$$S^1 C(t) - \bar{V} [\bar{V} S^1 C(t) - C(o)] + \frac{\phi}{ne} + K_x [S^1 C(x) - C(o)] \quad \dots\dots\dots (5)$$

$$S^1 C(t) - \bar{V} \left[\bar{V} S^1 C(t) - \frac{\phi}{ne} S C(x) + K_x S^1 C(x) \right] \quad \dots\dots\dots (6)$$

$$C(t) = \frac{1}{S} \left[\bar{V} S^1 C(t) - \frac{\phi}{ne} S^1 C(x) + K_x S^1 C(x) \right] \quad \dots\dots\dots (7)$$

$$C(t) = \frac{1}{S^1} \left[\bar{V} S^1 C(t) - \frac{\phi}{ne} S^1 C(x) + K_x C(x) \right] \quad \dots\dots\dots (8)$$

$$C(t) = \frac{\bar{V} - \frac{\phi}{ne} + Kx}{S^1} \dots\dots\dots (9)$$

$$C(t) = C(t) - \bar{V} + \frac{\phi}{ne} V + Kx \dots\dots\dots (10)$$

$$C(t) = S^1 C(t) = \bar{V} C(t) + \frac{\phi}{ne} + Kx C^1 \dots\dots\dots (11)$$

$$C(o) = \left[\bar{V} C(t) + \frac{\phi}{ne} + Kx \right] C(t) \dots\dots\dots (12)$$

$$S^1 C(t) = \left[\bar{V} - \frac{\phi}{ne} + Kx \right] C(t) \dots\dots\dots (13)$$

$$C(t) = \frac{S^1 C(t)}{\bar{V} + \frac{\phi}{ne} + Kx} \dots\dots\dots (14)$$

$$C(t) = \frac{S^1(t)}{\bar{V} + \frac{\phi}{ne} + Kx} \dots\dots\dots (15)$$

Looking at directions of flow in ground water, several variables that generate aquiferous deposition should be thorough evaluated, this implies that in [15], these stages of the derived solution should monitor the condition of phreatic zone, including its thickness base on the deposited hydraulic conductivity, these are some of the major factors that should determined the rate of aquifers thickness in any formation, therefore the expressed solution monitor the ability of the develop aquifers that are determined by formation characteristic, the deposition of the expressed model at these phase of the derived solution assess the parameters and express its relationships with the time of flow in such unconfined bed.

Furthermore, considering the boundary condition, we have at

$$t = 0 \quad C^1(o) = C(o) = 0$$

$$C(t) = \left[\bar{V} S^1 C(t) - \frac{\phi}{ne} C(x) + Kx C(x) \right] = 0 \dots\dots\dots (16)$$

$$\frac{0}{\bar{V} - \frac{\phi}{ne} - Kx} = 0 \quad \dots\dots\dots (17)$$

Considering the following boundary condition in the equation

$$C(t) - Co - \bar{V}S^1 C(t) - \bar{V}Co - S^1(t) + \frac{\phi}{ne} C(x) + \frac{\phi}{ne} Co S^1 C(x) \\ + KxS^1(x) + KxCo + S^1(x) \quad \dots\dots\dots (18)$$

$$C(t) = \bar{V} C(t) = SC(t) Co - \bar{V} + \frac{\phi}{ne} + KxCo \quad \dots\dots\dots (19)$$

Considering the denominator in the equation, we have

$$C(t) = \left[\bar{V} + \frac{\phi}{ne} + Kx \right] Co \quad \dots\dots\dots (20)$$

$$\text{Considering } \frac{\phi}{ne} = \frac{1}{\bar{V}}$$

$$C(t) = \left| \frac{1}{\bar{V}} + \bar{V} + Kx \right| Co \quad \dots\dots\dots (21)$$

$$C(t) = \left| \frac{1+V^2 + VKx}{\bar{V}} \right| Co \quad \dots\dots\dots (22)$$

$$C(t) = \left| (1+V^2 + VKx) \frac{1}{V} \right| Co \quad \dots\dots\dots (23)$$

$$C(t) = \left| (1+V^2 + VK) \frac{\phi}{ne} \right| Co \quad \dots\dots\dots (24)$$

$$C(t) = \lambda \quad \dots\dots\dots (25)$$

$$\lambda = \left| (1+V^2 + VK) \frac{\phi}{ne} \right| Co \quad \dots\dots\dots (26)$$

$$\lambda = \left| \frac{\phi}{ne} + \frac{\phi}{ne} V^2 + \frac{\phi}{ne} VKx \right| Co \quad \dots\dots\dots (27)$$

$$\frac{\phi}{ne} V^2 + \frac{\phi}{ne} KxV_{Co} + \left[\frac{\phi}{ne} Co - \lambda \right] = 0 \quad \dots\dots\dots (28)$$

Applying quadratic expression to equation (28), we have

$$V^2 + \frac{\phi}{ne} Kx + \left[\frac{\phi}{ne} - \lambda \right] = 0$$

$$\text{Where } a = \frac{\phi}{ne} V^2, b = \frac{\phi}{ne} KxVCo \text{ and } c = \frac{\phi}{ne} \lambda$$

$$V = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \dots\dots\dots (29)$$

$$V = \frac{-\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \dots\dots\dots (30)$$

$$V_1 = \frac{-\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \dots\dots\dots (31)$$

$$\phi\Lambda_2 = \frac{-\frac{\phi}{ne} KxVCo - \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \dots\dots\dots (32)$$

Since we have $A\ell^{st} + B\ell^{st}$, it implies that

$$qt = A \exp \frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \dots\dots\dots (33)$$

If $A = B = 1$

$$q(t) = \exp \left[-\frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} t \right] + \ell^L \left[-\frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} t \right] \dots\dots\dots (34)$$

The behaviour of these system are to express various parameters that transmits water in different phreatic zone, looking at this condition it has to express the phreatic zone that transmit ground water, thus those formations that express high degree of hydraulic conductivity in deltaic formation. the expression system from [16-34] establish their various functions, these condition provided a platform for parameters to institute their various functions in the storage of ground water in phreatic zone, the application of quadratic method were to integrate various parameters base on their relationship to evaluate their functions, because theses will always pressure the deposition of ground water in the formation. The expressed deriving solution to this stage implies that the relation between those parameters shows the rate of integrated influences despite their variations in depositions. Applying inverse Laplace of the equation yield

$$q(t) = \left[t + \frac{\phi}{ne} KxVCo \right] Co \left[- \frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 + 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} t \right]$$

$$\left[- \frac{\frac{\phi}{ne} KxVCo \sqrt{\frac{\phi}{ne} KxVCo^2 + 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \right] t - \left[\frac{\frac{\phi}{ne} KxVCo - \sqrt{\frac{\phi}{ne} KxVCo^2 + 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} t \right]$$

(35)

$$qt = \left[\frac{\frac{\phi}{ne} KxVCo}{t^2} Co \right] \left[- \frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V \phi \Lambda} \right]$$

$$\left[- \frac{\frac{\phi}{ne} VKxCo \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \right] \ell \left[- \frac{\frac{\phi}{ne} KxVCo \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda \phi \Lambda V \beta}}{2 \frac{\phi}{ne} V} t \right]$$

$$- \left[- \frac{\frac{\phi}{ne} KxVCo \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} t \right]$$

(36)

At this point $Co = \phi t \neq 0$

for equation (34) at $t = 0$ $C(o) = Co$, we have

$$Co = \left[\frac{\phi}{ne} KxVCo + \frac{\phi}{ne} V + \frac{\phi}{ne} x \right] Co [1+1+1] = 0$$

$$= \left[\frac{\phi}{ne} KxVCo + \frac{\phi}{ne} V + \frac{\phi}{ne} \lambda \right]$$

Hence $\frac{\phi}{ne} KxV + \frac{\phi}{ne} V + \frac{\phi}{ne} \lambda = 0$

Equation (37) can be written as:

$$qx = Co \left[t + 2 \left[\frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \right] t \right. \\ \left. \left[\frac{\frac{\phi}{ne} KxVCo - \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2 \frac{\phi}{ne} V} \right] t \right] \quad (37)$$

If $t = \frac{d}{v}$

$$C(z) = Co \left[\frac{d}{v} + 2 \left[\frac{\frac{\phi}{ne} KxVCo + \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2V\phi\Lambda} \right] \frac{d}{v} \right. \\ \left. \left[\frac{\frac{\phi}{ne} KxVCo - \sqrt{\frac{\phi}{ne} KxVCo^2 - 4 \frac{\phi}{ne} V \frac{\phi}{ne} \lambda}}{2V\phi\Lambda} \right] \frac{d}{v} \right] \quad (38)$$

Materials and method:-

Standard laboratory experiment where performed to monitor the rate of velocity of flow deposited at different formation, the soil deposition of the strata were collected in sequences base on the structural deposition at different locations, this samples collected at different location generate variation at different depth producing different migration of velocity of flow through its pressure flow at different strata, the experimental result are applied to compare with the theoretical values to determined the validation of the model.

Result and Discussion:-

Results and discussion are presented in tables including graphical representation of bacillus concentration

Table 1: Theoretical values of Bacillus concentration at Different Depths:-

Depth [M]	Theoretical values Conc.
3	0.99
6	1
9	3.03
12	4.07
15	5.12
18	6.19
21	7.27
24	8.37
27	9.48
30	10.61

Table 2: Comparison of Theoretical and Measured Values of Bacillus Concentration Different Depth:-

Depth [M]	predicted values Conc.	Measured Values
3	0.99	0.88
6	1	1.1
9	3.03	3.11
12	4.07	4.14
15	5.12	5.21
18	6.19	6.25
21	7.27	7.35
24	8.37	8.44
27	9.48	9.55
30	10.61	10.74

Table 3: Theoretical values of Bacillus concentration at Different Depth:-

Depth [M]	Theoretical values Conc.
3	8.42E-05
6	8.00E-05
9	1.26E-04
12	3.37E-04
15	4.36E-04
18	5.05E-04
21	5.90E-04
24	6.74E-04
27	7.59E-04
30	8.43E-04
33	9.27E-04
36	1.09E-03

Table 4: Comparison of Theoretical and Measured Values of Bacillus Concentration Different Time:-

Depth [M]	Predicted bacillus conc.	Measured values bacillus conc.
3	8.42E-05	8.55E-05
6	8.00E-05	8.22E-05
9	1.26E-04	1.33E-04
12	3.37E-04	3.45E-04
15	4.36E-04	4.56E-04
18	5.05E-04	5.21E-04
21	5.90E-04	5.98E-04
24	6.74E-04	6.88E-04
27	7.59E-04	7.77E-04
30	8.43E-04	8.66E-04
33	9.27E-04	9.33E-04
36	1.09E-03	1.18E-03

Table 4: Theoretical values of Bacillus concentration at Different Depth:-

Depth [M]	Theoretical values Conc.
3	4.97E-03
6	9.99E-03
9	1.50E-02
12	2.00E-02
15	2.50E-02
18	3.00E-02
21	3.50E-02
24	4.00E-02
27	4.60E-02
30	5.10E-02
33	5.70E-02
36	6.20E-02

Table 5: Comparison of Theoretical and Measured Values of Bacillus Concentration Different Depth:-

Depth [M]	Predicted Values bacillus con	Measured Values Bacillus Conc.
3	4.97E-03	5.05E-03
6	9.99E-03	9.98E-03
9	1.50E-02	1.57E-02
12	2.00E-02	2.11E-02
15	2.50E-02	2.66E-02
18	3.00E-02	3.14E-02
21	3.50E-02	3.61E-02
24	4.00E-02	4.11E-02
27	4.60E-02	4.72E-02
30	5.10E-02	5.24E-02
33	5.70E-02	5.83E-02
36	6.20E-02	6.26E-02

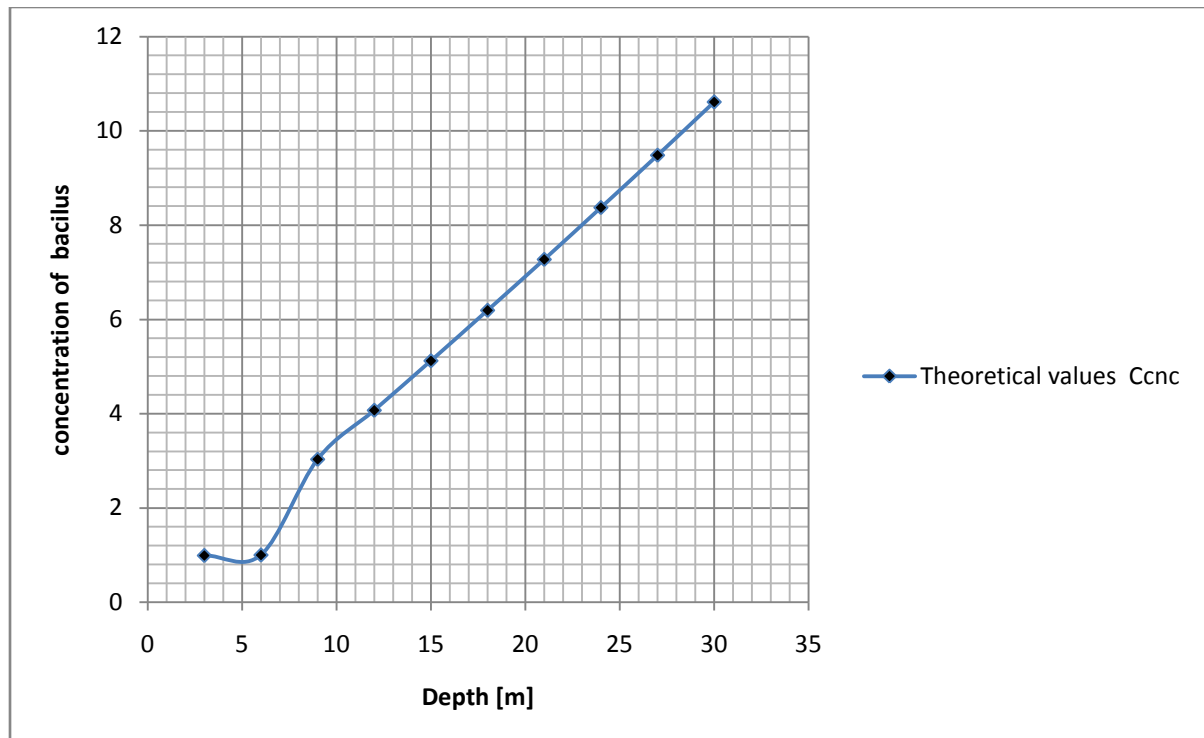


Figure 1: Theoretical values of Bacillus concentration at Different Depth:-

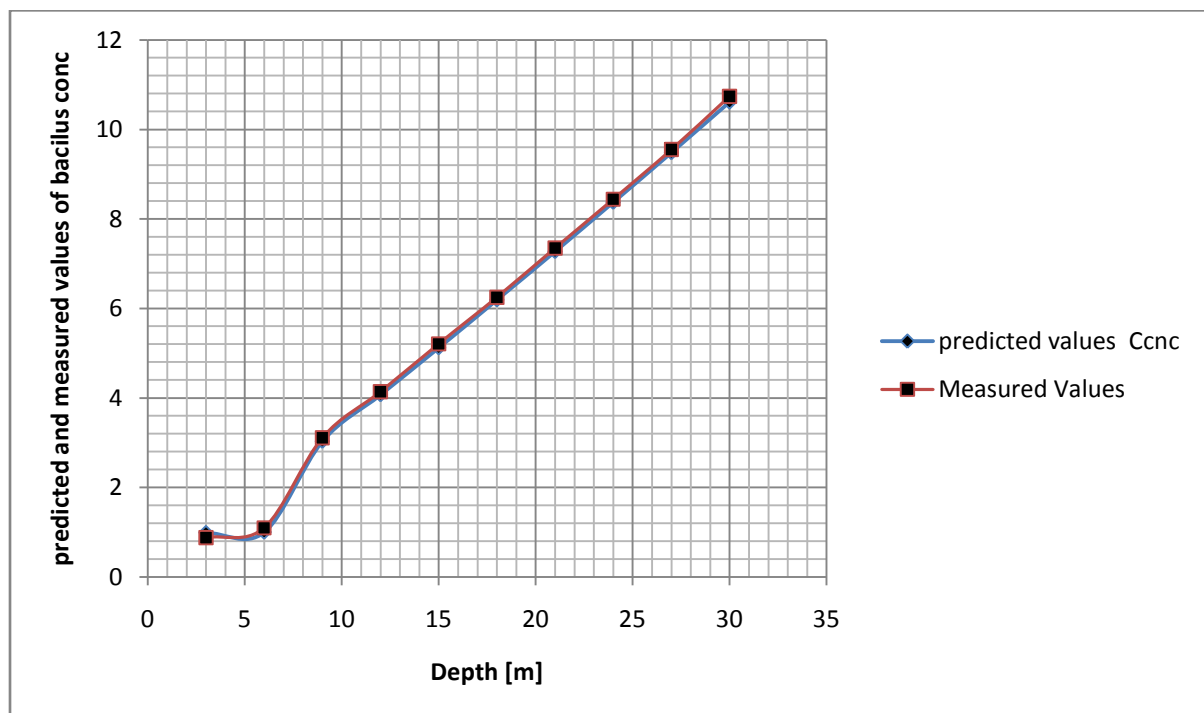


Figure 2: Comparison of Theoretical and Measured Values of Bacillus Concentration Different Depth:-

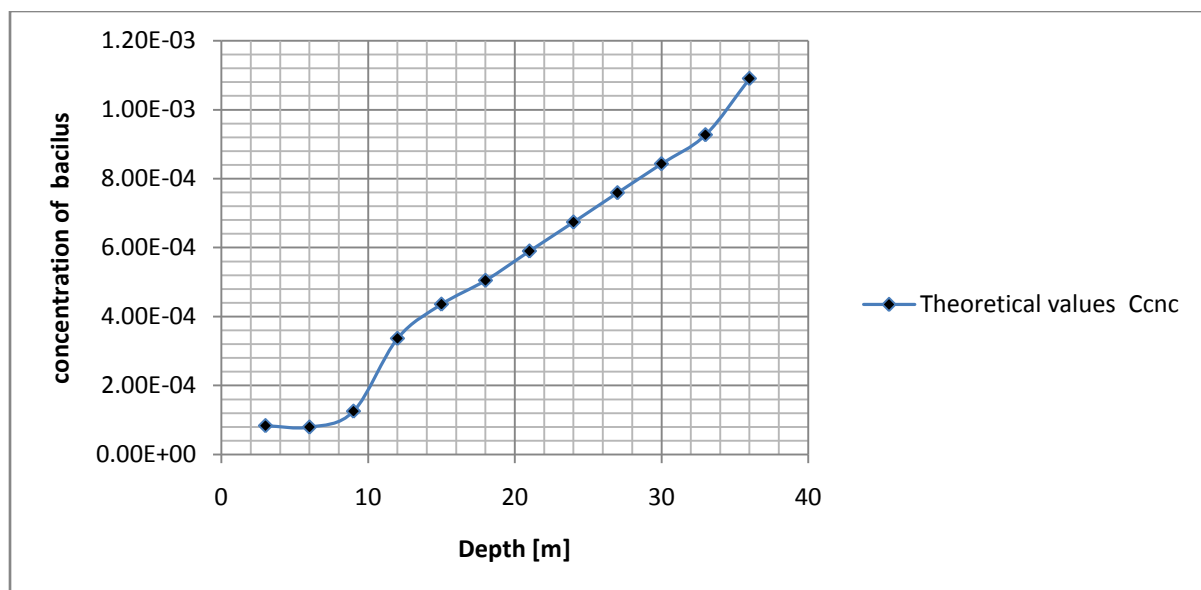


Figure 3: Theoretical values of Bacillus concentration at Different Depth:-

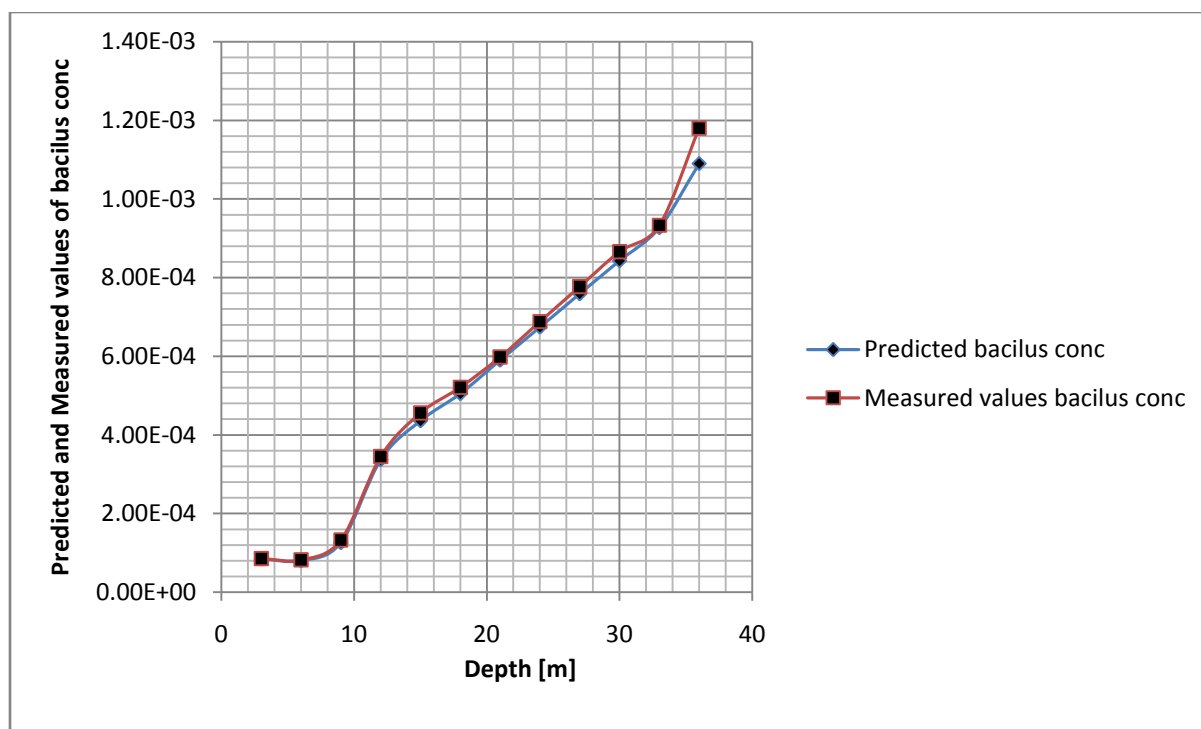


Figure 4: Comparison of Theoretical and Measured Values of Bacillus Concentration Different Depth:-

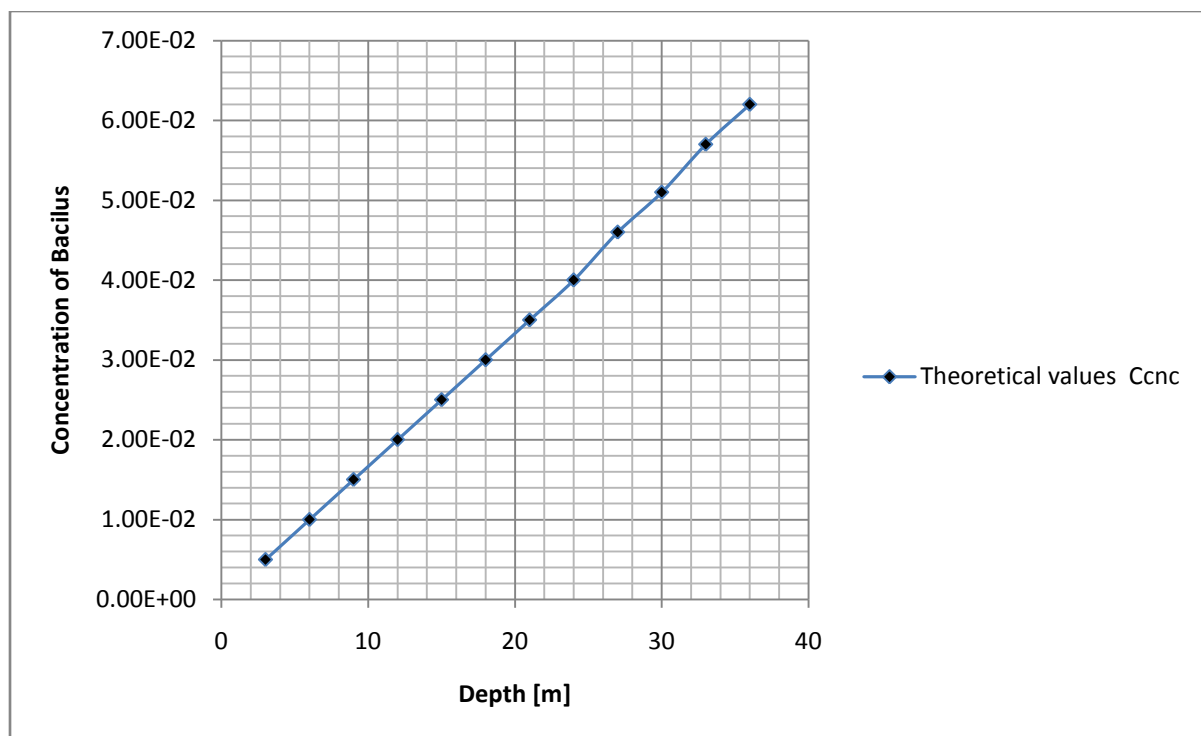


Figure 5: Theoretical values of Bacillus concentration at Different Depth:-

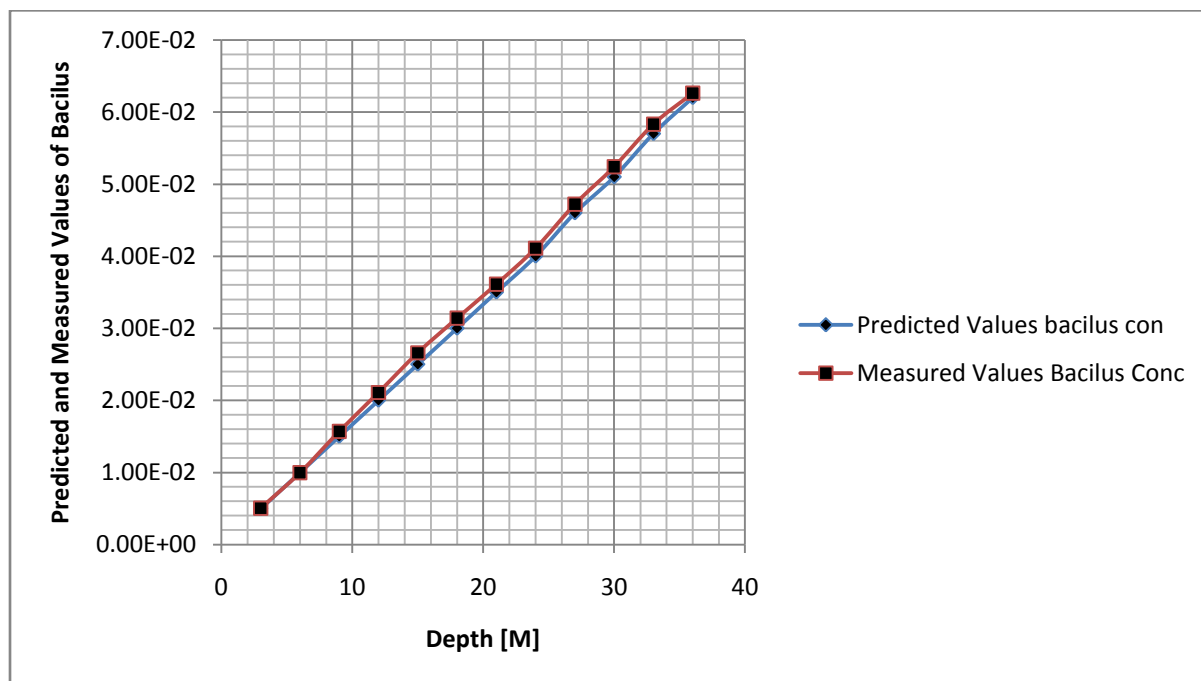


Figure 6: Comparison of Theoretical and Measured Values of Bacillus Concentration Different Depth:-

The study presented graphically shows the behaviour of velocity at different predominant sand gravel, the study were to determine the velocity rate of flow in different predominant sand gravel formation, from the graphical representation in figure one to six there is low deposition and slight fluctuation due to variation on the transition from one deposit to another, slight vacillation were observed, but the flow deposition were very low in some locations. Furthermore, other deposited area experiences higher velocity pressure in the system with the same slight fluctuation, the behaviour shows that the stratification from geological setting in the deltaic formations may

developed some homogeneous higher pressure flow in those formation, this implies that high yield aquifer may found to deposit, but cannot ascertain the quality of ground water depositions in such formations, the derived solution generated theoretical values from the simulation, these values were compared with experimental results for validation, both parameters compared favourable well thus express the authenticity of the model for the study.

Conclusion:-

The study of velocity of flow for sand gravel was to monitor the pressure of flow in such predominant deposition, the behaviour express the rate of stratification base on its geological setting in the study location, various flow in predominant deposited sand gravel can be applied for different purpose in soil and water engineering, monitoring of such velocity of flow determined its rate of fluids in the formation thus for different purpose can be considered in design for soil and water engineering, the study express the rate of these fluids in different depth between three and thirty six metres, application of mathematical modelling approach were found necessary to monitor the deposition of the fluids velocity at different depth in predominant sand gravel formation, experts in soil and water engineering will definitely find favour in these developed model for various construction purpose.

References:-

1. Cui, Y.J., Tang, A.M., Loiseau, C., Delage, P., 2008. Determining the unsaturated hydraulic conductivity of a compacted sand-bentonite mixture under constant-volume and free-swell conditions. *Physics and Chemistry of the Earth, Parts A/B/C*, 33 (Supplement 574 1), S462 - S471.
2. Cui, Y.J., Tang, A.M., Qian, L.X., Ye, W.M., Chen, B., 2011. Thermal-mechanical behavior of compacted GMZ Bentonite. *Soils and Foundations*, Vol. 51, No. 6, 1065-1074
3. Cui, Y.J., Loiseau, C. and Delage, P., 2002. Microstructure changes of a confined swelling soil due to suction controlled hydration. *Unsaturated soils: proceedings of the Third International Conference on Unsaturated Soils*, 10-13, March 2002, Recife, Brazil, 570 593-598.
4. Juvankoski, M., 2010. Description of basic design for buffer (working report 2009-131). 612 Technical report, EURAJOKI, FINLAND
5. Komine, H. and Yasuhara, K. and Murakami, S. 2009. Swelling characteristics of bentonites in artificial seawater. *Canadian Geotechnical Journal*. 46, 177-189
6. Komine, H., 2010. Predicting hydraulic conductivity of sand bentonite mixture backfill before and after swelling deformation for underground disposal of radioactive wastes. *Engineering Geology*. 114, 123-134
7. Komine, H., Watanabe, Y., 2010. The past, present and future of the geo-environment in Japan. *Soils and Foundations*, Vol. 50 (2010) No. 6 977-982.
8. Martin, P.L., Barcala, J.M., and Huertas, F., 2006. Large-scale and long-term coupled thermo-hydro-mechanic experiments with bentonite: the febex mock-up test. *Journal of Iberian Geology*, 32(2), 259-282.
9. Villar, M.V., Lloret, A., 2008. Influence of dry density and water content on the swelling of a 672 compacted bentonite. *Applied Clay Science*, 39(1-2), 38-49.
10. Ye, W.M., Cui, Y.J., Qian, L.X., and Chen, B., 2009. An experimental study of the water 679 transfer through confined compacted gmz bentonite. *Engineering Geology*, 108(3-4), 680 169-176.
11. Yahia-Aissa, M., Delage, P., & Cui, Y.J. 2001. Suction-water relationship in swelling clays. 676 *Clay science for engineering*, IS-Shizuoka International Symposium on Suction, Swelling, 677 Permeability and Structure of Clays, 65-68, Adachi & Fukue eds, Balkema
12. Pusch, R., 1979, Highly compacted sodium bentonite for isolating rock-deposited 645 radioactive
13. 646 waste products. *Nucl. Technol.:(United States)*, 45(2).
14. Yong, R.N., Boonsinsuk, P., and Wong, G., 1986. Formulation of backfill material for a 682 nuclear fuel waste disposal vault. *Canadian Geotechnical Journal*, 23(2), 216-228. Qiong Wang1, Anh Minh Tang Yu-Jun Cui1, Pierre Delage1, Jean-Dominique Barnichon Wei-Min Ye3