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***RESEARCH ARTICLE***

# A SIMPLE MATHEMATICAL METHOD TO ANALYZE AND PREDICT THE SPREAD OF COVID-19 IN INDIA

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## *Abstract*

As of 10 May 2020, the pandemic of COVID-19 claimed 274,488 lives globally. Since March 2020, the number of infections has been rising rapidly in India. To prevent and control its transmission, one needs to estimate how fast the number rises in the country. The objective of the present study is to provide the policymakers of the country with an easy mathematical tool to make proper predictions regarding the spread of the disease. This article is based on a simple algebraic structure, rather than a set of coupled differential equations which are used by the conventional mathematical models of epidemiology. An estimation of the asymptomatic patients, using this method, leads to the cumulative count of symptomatic patients. Data of COVID-19 cases in India, for the period from 01 March 2020 to 03 May 2020, have been used. The predictions made by this formulation are in reasonable agreement with observations, for a certain set of values of the parameters associated with it. Using these values, more predictions have been made regarding the time evolution of the number of infections, within and beyond the lockdown period. This method enables one to estimate the number of asymptomatic carriers who play a major role in transmitting the disease. The effectiveness of the imposition of lockdown has been clearly demonstrated here. All calculations are based on very simple mathematical expressions that can be understood, used and modified easily by those having an elementary knowledge of algebra. This model can be applied for the prediction of COVID-19 spread in any country of the world, through a proper tuning of the associated parameters.



## Introduction:-

An enormously infectious pneumonia was reported in Wuhan, China in December 2019 and it was named coronavirus disease 2019 (COVID-19) by the World Health Organization (WHO) [1, 2]. They declared this outbreak of COVID-19 a "public health emergency of international concern". As of 10 May 2020, there were 3,925,815 confirmed cases of COVID-19 infection, with 274,488 deaths, as reported to WHO from the entire world [3]. So far, the World Health Organization (WHO) has not found any pharmaceutical product to be safe and effective for the treatment of the disease caused by novel coronavirus. An immune-modulant drug, called chloroquine, which is generally used for malaria, has proved effective in reducing viral replication in infections like SARS (Severe Acute Respiratory Syndrome) and MERS (Middle East Respiratory Syndrome) [4, 5]. The respiratory symptoms of

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COVID-19 are very much similar to those of SARS and MERS. It strongly indicates the droplet transmission and contact transmission of the virus. Apart from respiratory disorders, one finds diarrhea, nausea, vomiting and abdominal discomfort in different populations studied so far [6]. COVID-19 virus is known to spread through respiratory droplets and contact channels [7-11]. Droplet transmission occurs when a susceptible person is less than one metre away from a patient. These transmissions are also found to occur through fomites in the immediate vicinity of the infected person [12]. Transmission can take place by direct contact with an infected person or an indirect contact with surfaces around the patient. The mode of airborne transmission is caused by the microbes within the droplet nuclei. These are released from droplets by evaporation or exist within dust particles. These particles are known to remain in the air and they can be transmitted over a distance greater than one metre [13].

The first infection of COVID-19 was diagnosed in India on 30 January 2020. As of 10 May 2020, a total of 62,939 cumulative cases of infection, including 41,472 active cases, 19,358 recoveries and 2,109 deaths were reported in India, as confirmed by the Ministry of Health and Family Welfare [14]. The government has taken several measures to spread awareness regarding COVID-19. It has issued necessary guidelines and also taken steps to ensure social distancing to break the chain of transmission of the disease. An announcement of a nationwide lockdown was made on 24 March 2020, which was to be effective initially from 25 March 2020 to 14 April 2020. This lockdown was later extended thrice, making it effective up to 31 May 2020.

We have come across some recent studies pertaining to COVID-19 infections in India. A detailed study, by Chatterjee *et al*, have accumulated evidence that can provide a direction to research activities towards the prevention and control of such a pandemic spreading so alarmingly in India [15]. In a different study, Agarwal *et al* have extensively discussed the characteristics of the necessary medical infrastructure to be built up to tackle the huge flow of patients and also to ensure the safety of the healthcare workers [16]. An elaborate mathematical analysis, by Mandal *et al*, has highlighted the policies required to resist the spread of the virus through community transmission [17]. Several other models, based on different theories, predict the trend of COVID-19 infections in India with sufficient accuracy [18-25]. These models can help the policymakers of the nation, at all levels, to make proper plans to control the pandemic.

The effect of lockdown has been shown by us through a previously published dynamical model [26]. Unlike that method, the present study is based on a simple algebraic structure. Its objective is to make the formulation useful to readers of diverse academic backgrounds. Although the calculus-based models make accurate predictions, they appear to be difficult to those who are not adequately trained in mathematics. Generally, the conventional models do not give us a mathematical expression to make predictions. Numerical calculations have to be done, rather than analytical, to make a prediction. In the emergency of a pandemic, one needs mathematical models that can be easily used by those responsible for infrastructural arrangements. The calculations in the present article are much simpler than solving a set of coupled differential equations that constitute the conventional mathematical models of epidemiology.

The present article shows an algebraic structure that determines the time evolution of the number of asymptomatic patients of COVID-19, from which the symptomatic cases can be estimated. It has been assumed that the number of infected persons, recorded by the government, is actually the number of symptomatic patients who are generally quarantined after diagnosis and thereby prevented from spreading the disease. This article discusses three models that can be used to generate data as close as possible to the data of COVID-19 infections in India, by tuning the parameters. The cumulative numbers of COVID-19 infections, over the period from 01 March 2020 to 03 May 2020, have been collected from government sources [14]. The impact of lockdown has been graphically depicted. Predictions have been made regarding the time evolution of COVID-19 infections within lockdown and beyond its withdrawal. The most important finding is that a high degree of social distancing must be maintained to reduce the rate of transmission considerably.

## Methods:-

After being infected with COVID-19, the patients remain asymptomatic for a few days, although they can transmit the disease to others [9, 17]. Taking $w_1$ and $w_2$ to be the numbers of asymptomatic patients in the country, in the 1$^{st}$ and the 2$^{nd}$ day respectively, of the period being studied, we propose the following relation between them.

$$w_2 = w_1 + \alpha_1 w_1 - \beta_1 w_1 \qquad (1)$$

Here, $\alpha_1 w_1$ is the number of persons to whom the disease is transmitted by $w_1$ carriers on the first day and $\beta_1 w_1$ is the number of asymptomatic persons who become symptomatic on the same day. It has been assumed that, immediately after diagnosis, a symptomatic patient is put into complete isolation and thereby prevented from spreading the disease. Another assumption is that no new carrier has entered the geographical region under consideration, over the period being studied.
The subscript of $\alpha_1$ corresponds to the serial number of the day concerned (i.e., the 1st day here). During lockdown, the socializing pattern is likely to change significantly, causing a change in the number of persons coming into contact with a patient. Let us consider three successive periods of time, respectively of $d_1$, $d_2$ and $d_3$ days, during which we have *no-lockdown*, *lockdown* and again *no-lockdown* situations in the country. The time dependence of $\alpha_n$ (i.e., its dependence upon $n$), can be expressed as,

$$\alpha_n = g_1(n)a + g_2(n)b + g_3(n)a \tag{2}$$

Here, $g_1(n) = 1$ for $1 \leq n \leq d_1$ and $g_1(n) = 0$ otherwise; $g_2(n) = 1$ for $d_1 < n \leq d_2$ and $g_2(n) = 0$ otherwise; $g_3(n) = 1$ for $d_2 < n \leq d_3$ and $g_3(n) = 0$ otherwise.
Thus, the values of $\alpha_n$ are the constants $a$, $b$ and again $a$, respectively, during these three periods.
The constants, $a$ and $b$, are the indicators of social distancing during *no-lockdown* and *lockdown* periods. Their smaller values indicate greater social distancing.
Like $\alpha_1$, the subscript of $\beta_1$ corresponds to the serial number of the day concerned (i.e., the 1st day here). The reason for its dependence upon $n$ is based on a realistic assumption that the number of asymptomatic patients may increase so rapidly (due to high population density) that, the fraction of them turning into symptomatic ones on the $n^{th}$ day, cannot have a fixed value. One of the three forms of $\beta_n$ in the present paper has been chosen to be a constant (denoted by $\gamma$ in eqn. 22 of Model-3), which can be regarded as a time-average of $\beta_n$ for the entire span of study.
Equation (1) can be generalized to the following form.

$$w_n = (1 + \alpha_{n-1} - \beta_{n-1})w_{n-1} \tag{3}$$

For $n = 2$, equation (3) reduces to equation (1).
Putting $n = n - 1$ in equation (3) one obtains,

$$w_{n-1} = (1 + \alpha_{n-2} - \beta_{n-2})w_{n-2} \tag{4}$$

Substituting for $w_{n-1}$ from equation (4), in equation (3), one gets,

$$w_n = (1 + \alpha_{n-1} - \beta_{n-1})(1 + \alpha_{n-2} - \beta_{n-2})w_{n-2} \tag{5}$$

Proceeding in this fashion, equation (3) can be expressed as,

$$w_n = w_1 \prod_{j=1}^{n-1}(1 + \alpha_j - \beta_j) \tag{6}$$

The number of cases ($x_k$) that turn into the symptomatic type from the asymptomatic ones, on the $k^{th}$ day, is then written as,

$$x_k = \beta_k w_k = \beta_k w_1 \prod_{j=1}^{k-1}(1 + \alpha_j - \beta_j) \tag{7}$$

The total number of symptomatic cases ($s_n$), reported up to the $n^{th}$ day, is therefore expressed as,

$$s_n = \sum_{k=1}^{n} x_k = w_1 \sum_{k=1}^{n} \beta_k \prod_{j=1}^{k-1}(1 + \alpha_j - \beta_j) \tag{8}$$

In equations (6), (7) and (8), we have $\alpha_j = g_1(j)a + g_2(j)b + g_3(j)a$, according to equation (2). If the third phase (i.e., the phase of duration $d_3$) is absent, one should write $\alpha_j = g_1(j)a + g_2(j)b$. For a chain of such processes, with $L$ number of phases, $\alpha_j$ can be expressed as,

$$\alpha_j = \sum_{m=1}^{L} g_m(j) f_m \tag{9}$$

In equation (9), we have $f_m = b$ for the even values of *m* and $f_m = a$ for the odd values.
Here, $g_m(j) = 1$ for $d_{m-1} < j \leq d_m$ and $g_m(j) = 0$ otherwise, with $d_0 = 1$.

The value of $s_n$, as given by equation (8), has to be compared with the number of confirmed COVID-19 cases registered in the country till the $n^{th}$ day. It is the cumulative number of symptomatic patients recorded till that day.
If no lockdown is imposed we have $\alpha_j = a$, in accordance with equation (2). In that case, equation (8) will take the following form.

$$s_n = \sum_{k=1}^{n} x_k = w_1 \sum_{k=1}^{n} \beta_k \prod_{j=1}^{k-1} \left(1 + a - \beta_j\right) \quad (10)$$

The numbers of the symptomatic and asymptomatic cases, as percentages of the total number of patients, are denoted here by $P(s_n)$ and $P(w_n)$ respectively. They can be expressed as,

$$P(s_n) = \frac{s_n}{w_n + s_n} \times 100 = \frac{100 \sum_{k=1}^{n} \beta_k \prod_{j=1}^{k-1}(1+\alpha_j-\beta_j)}{\prod_{j=1}^{n-1}(1+\alpha_j-\beta_j) + \sum_{k=1}^{n} \beta_k \prod_{j=1}^{k-1}(1+\alpha_j-\beta_j)} \quad (11)$$

$$P(w_n) = \frac{w_n}{w_n + s_n} \times 100 = \frac{100 \prod_{j=1}^{n-1}(1+\alpha_j-\beta_j)}{\prod_{j=1}^{n-1}(1+\alpha_j-\beta_j) + \sum_{k=1}^{n} \beta_k \prod_{j=1}^{k-1}(1+\alpha_j-\beta_j)} \quad (12)$$

As a rough estimate we may say that, for each symptomatic case there are $w_n/s_n$ or $P(w_n)/P(s_n)$ number of asymptomatic cases, on the $n^{th}$ day. Generally, the asymptomatic cases remain undetected in India due to the fact that much smaller number of tests is conducted than required.

The sum of $w_n$ and $s_n$ gives us the total cumulative count of patients (symptomatic and asymptomatic) on the $n^{th}$ day. The percentage ($F_n$) of the total population (*N*) infected, either symptomatically or asymptomatically, is expressed as,

$$F_n = \frac{w_n + s_n}{N} \times 100 = \frac{100\, w_1 \left[\prod_{j=1}^{n-1}(1+\alpha_j-\beta_j) + \sum_{k=1}^{n} \beta_k \prod_{j=1}^{k-1}(1+\alpha_j-\beta_j)\right]}{N} \quad (13)$$

The present population (*N*) of India is $1.36 \times 10^9$.

**Model - 1: An exponential expression for $\beta_j$**

Here we have the following expression for $\beta_j$.

$$\beta_j = \beta_1 Exp[\lambda(j-1)] \quad (14)$$

In the above equation, $\lambda$ and $\beta_1$ are constants. The symbol *j* denotes time (in days). The value of the parameter $\lambda$ determines how fast $\beta_j$ changes as a function of time (*j*). Substituting for $\beta_j$ in equations (6), (8) and (10), respectively, from equation (14), one obtains,

$$w_n = w_1 \prod_{j=1}^{n-1} \left(1 + \alpha_j - \beta_1 Exp[\lambda(j-1)]\right) \quad (15)$$

$$s_n = w_1 \sum_{k=1}^{n} \beta_1 Exp[\lambda(k-1)] \prod_{j=1}^{k-1} \left(1 + \alpha_j - \beta_1 Exp[\lambda(j-1)]\right) \quad (16)$$

$$s_n = w_1 \sum_{k=1}^{n} \beta_1 Exp[\lambda(k-1)] \prod_{j=1}^{k-1} (1 + a - \beta_1 Exp[\lambda(j-1)]) \quad (17)$$

**Model - 2: A Power-law expression for $\beta_j$**

Here we have the following expression for $\beta_j$.

$$\beta_j = \beta_1 j^{\mu} \quad (18)$$

In the above equation, $\mu$ and $\beta_1$ are constants. The symbol $j$ denotes time (in days). The value of the parameter $\mu$ determines how fast $\beta_j$ changes as a function of time ($j$). Substituting for $\beta_j$ in equations (6), (8) and (10), respectively, from equation (18), one obtains,

$$w_n = w_1 \prod_{j=1}^{n-1}(1 + \alpha_j - \beta_1 j^{\mu}) \quad (19)$$

$$s_n = w_1 \sum_{k=1}^{n} \beta_1 k^{\mu} \prod_{j=1}^{k-1}(1 + \alpha_j - \beta_1 j^{\mu}) \quad (20)$$

$$s_n = w_1 \sum_{k=1}^{n} \beta_1 k^{\mu} \prod_{j=1}^{k-1}(1 + a - \beta_1 j^{\mu}) \quad (21)$$

**Model - 3: A Constant Value for $\beta_j$**

Here we have the following expression for $\beta_j$.

$$\beta_j = \gamma \quad (22)$$

Here $\gamma$ is a constant. Substituting for $\beta_j$ in equations (6), (8) and (10), respectively, from equation (22), one obtains,

$$w_n = w_1 \prod_{j=1}^{n-1}(1 + \alpha_j - \gamma) \quad (23)$$

$$s_n = w_1 \sum_{k=1}^{n} \gamma \prod_{j=1}^{k-1}(1 + \alpha_j - \gamma) \quad (24)$$

$$s_n = w_1 \sum_{k=1}^{n} \gamma (1 + a - \gamma)^{k-1} \quad (25)$$

In the present article we have used the following functional forms of $g_1(n)$, $g_2(n)$ and $g_3(n)$ respectively, which have been incorporated into equation (2) for all calculations.

$$g_1(n) = \frac{(1+\tanh \kappa n)[1+\tanh \kappa(d_1-n)]}{4} \quad (26)$$

$$g_2(n) = \frac{\{1+\tanh \kappa(n-d_1)\}[1+\tanh \kappa(d_1+d_2-n)]}{4} \quad (27)$$

$$g_3(n) = \frac{\{1+\tanh \kappa(n-d_1-d_2)\}[1+\tanh \kappa(d_1+d_2+d_3-n)]}{4} \quad (28)$$

In accordance with the definitions of $g_1(n)$, $g_2(n)$ and $g_3(n)$, they must be rectangular pulses of unit height (when plotted against $n$), with widths $d_1$, $d_2$ and $d_3$ respectively. To obtain this behaviour, one must select a value of the constant $\kappa$ which is very large compared with the values of $d_1$, $d_2$ and $d_3$.

The data, regarding the cumulative number of COVID-19 cases registered in India, during the period from 01 March 2020 to 03 May 2020, are listed in Table-1. Figures 1-3, 10-12 show the plots of these data along with the plots of data generated by the three mathematical models formulated in the present study.

## Results:-

Equations (15), (19) and (23) enable one to determine the time evolution of the asymptomatic cases, based on Models-1, 2, 3 respectively. Using equations (16), (20) and (24), one can determine the time evolution of the symptomatic cases from Models-1, 2, 3 respectively. In both sets of equations, the effect of lockdown has been incorporated.

Time evolution of the symptomatic patients ($s_n$), based on Models-1, 2, 3, are depicted by Figures 1, 2, 3 respectively, along with the cumulative numbers of confirmed cases recorded in India during the span from 01 March 2020 to 03 May 2020 (Table 1). The numbers predicted by each model are in reasonable agreement with observations for a certain set of parameter values. Their values for Model-1 are, $w_1 = 100$, $a = 0.2$, $b = 0.125$, $\beta_1 = 0.022$ and $\lambda = -0.008$. Their values for Model-2 are, $w_1 = 100$, $a = 0.2$, $b = 0.118$, $\beta_1 = 0.03$ and

$\mu = -0.1$. Their values for Model-3 are, $y_1 = 100$, $a = 0.2$, $b = 0.116$, $\gamma = 0.03$. These parameter values have been used here for making predictions based on these three models.

Equations (17), (21) and (25) enable one to determine the time evolution of the symptomatic cases, from Models-1, 2, 3 respectively, if no lockdown is imposed. Using these equations (along with eqns. 16, 20, 24), one can determine the time dependence of $s_n$, *with* and *without* a lockdown being imposed. Figures 4, 5, 6 show these plots, based on Models-1, 2, 3 respectively, for the days from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64). The parameter values, obtained from Figures 1, 2, 3 (mentioned above), have been used.

On the basis of Models-1, 2, 3 respectively, Figures 7, 8, 9 depict the time evolution of the numbers of asymptomatic and symptomatic cases, for the days from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64). The parameter values, obtained from Figures 1, 2, 3 (mentioned above), have been used. The slopes of both curves in each figure decrease during lockdown as an effect of social distancing.

Figures 10, 11, 12 show the time evolution of the number of symptomatic patients, for three phases: pre lockdown, lockdown and post lockdown, from Models-1, 2, 3 respectively (assuming the lockdown to continue till 17 May 2020). The parameter values, obtained from Figures 1, 2, 3 (mentioned above), have been used. The cumulative counts of COVID-19 infections (Table 1), for 64 days since 01 March 2020, have also been plotted here.

Using equations (11) and (12), relative proportions of asymptomatic and symptomatic patients have been plotted as functions of time in Figure 13. For three models, they are shown as percentages of the total number of patients, for a span of 100 days starting from 01 March 2020.

Using equation (13), the time evolution of the percentage of Indian population carrying the COVID-19 infection (as asymptomatic or symptomatic patients), has been plotted in Figure 14, for Models-1, 2, 3. It has been shown here for a span of 100 days starting from 01 March 2020.

Figures 15, 17, 19 show the time evolution of the number of asymptomatic patients, based on Models-1, 2, 3 respectively, for three values of the parameter *b*, for a span of 100 days starting from 01 March 2020.

Figures 16, 18, 20 show the time evolution of the number of symptomatic patients, based on Models-1, 2, 3 respectively, for three values of the parameter *b*, for a span of 100 days starting from 01 March 2020.

## Discussion:-

The values of the parameters, associated with the three models, have been obtained by tuning the parameters to make the predicted results as close as possible to the observations regarding the cumulative counts of confirmed COVID-19 cases in India (Table 1). The set of parameter values, corresponding to any of these models, is not unique. We have found that, only very small deviations from these values lead to a sufficiently reasonable agreement between predictions and observations, as shown by the first three plots (Figures 1, 2, 3). Predictions, based on a different mathematical method, are consistent with the findings of our simple algebraic models [20].

It is evident from some figures that the number of patients ($s_n$) would have been nearly 10 times its recorded value, on the 64$^{th}$ day, without the imposition of lockdown (Figures 4, 5, 6). This result gives an idea regarding the importance of lockdown as a mechanism to slow down the transmission of the disease. The findings of our study, that clearly illustrate the effectiveness of lockdown, are consistent with the findings of several other studies based on different mathematical formulations [19, 22, 23, 25].

Some plots of the present article enable one to estimate the difference between the numbers of asymptomatic ($w_n$) and symptomatic ($s_n$) cases, on any day (i.e., for any value of $n$) during the period of study (Figures 7, 8, 9). Similar findings, regarding undetected and detected cases, have been depicted graphically by another recent mathematical study [21]. From our study, the difference between $w_n$ and $s_n$ is found to be the largest for Model-1 and smallest for Model-3. The asymptomatic ($w_n$) cases have mostly remained undetected in India.

We have shown our predictions graphically regarding the time evolution of the number of symptomatic patients ($s_n$) *during* and *after* the lockdown period (Figures 10, 11, 12). Here we have also plotted the data of 64 days, from 01

March 2020 to 03 May 2020 (Table 1), obtained from government sources [14]. Predictions from a different mathematical model are consistent with these findings [20]. The slope of the curve in each figure (indicating the rate of rise in $s_n$) is clearly smaller during the lockdown period in comparison with the periods without lockdown. According to Models-1, 2, 3, the average of the predicted cumulative numbers of infection on 17 May 2020 (i.e., $n$ = 78 in the figures), is 1,11,666. The actual cumulative count of infections on that day was 90,927, as obtained from government sources [14]. It indicates a reasonable agreement of theoretical findings with the observations.

The results, depicted by Figures 1-6 & 10-12, are consistent with the findings of our previous study in this field, which was carried out by a completely different mathematical method [26].

The present study allows one to estimate the relative proportions of asymptomatic and symptomatic carriers of the disease (Figure 13). From these plots, the largest ratio of symptomatic to asymptomatic cases is found to be approximately 2/3. In a country like India, most of the asymptomatic cases remain undetected due to the lack of testing facilities in sufficient numbers. So, the confirmed cases of COVID-19, as declared by the government, are mostly symptomatic. Therefore, one may say that, for every two such cases, there are at least three cases that remain undetected. According to a recent study, the ratio of undetected to detected cases decreases with time during the lockdown period [21]. If the ratio of $w_n/s_n$ decreases with time, $P(s_n)$ increases according to equation (11), and $P(w_n)$ decreases according to equation (12). This is exactly what we have found from our models (Figure 13).

An estimate of the total number of persons infected in the country, including both symptomatic and asymptomatic patients, can be obtained from some plots (Figure 14). On the 78th day (i.e., 17 May 2020), around 0.1% of the population were to be infected, as predicted by Model-1. Models-2, 3 predict smaller percentages of infection. For this calculation, the present population of India has been taken to be $1.36 \times 10^9$.

The present article shows the effect of social distancing in controlling the speed of transmission of the disease (Figures 15-20). It is illustrated by the plots of $w_n$ versus $n$ and $s_n$ versus $n$, for three values of the parameter $b$, which decreases as social distancing increases. Parameter values, other than $b$, are taken from Figures 1-3.

It is found that, for a sufficiently low value of the parameter $b$, $w_n$ starts decreasing with time immediately after the lockdown is imposed (Figures 15, 17, 19). Once $w_n$ becomes zero, there is almost no transmission of the disease, since the transmission is believed to be caused mainly by the undetected carriers.

For a sufficiently low value of the parameter $b$, the slope of $s_n$ starts decreasing immediately after the lockdown is imposed (Figures 16, 18, 20). Here $s_n$ seems to be approaching a constant value with time, which means that the number of new cases of infection, reported on each day, gradually decreases with time. Similar findings (about SARS-2003) have been graphically depicted by another recent study [18].

The incubation period for COVID-19 is 1 to 14 days [14]. The mean incubation period is 6.4 days [15]. The value of the parameter $a$, for three models, has been found to be 0.2. By definition, this is the average number of persons infected by an asymptomatic carrier per day. Thus, the average number of infections, caused by an asymptomatic carrier, is 1.28 (i.e., $a \times$ *average incubation period*). The maximum number of infections caused by such a carrier is 2.8 (i.e., $a \times$ *maximum incubation period*). This parameter ($a$) seems to have a relation with the basic reproduction number ($R_0$). Estimates of $R_0$ are found to be lying in the range from 1.4 to 3.5 [15]. In a recent mathematical study on COVID-19, $R_0$ has been taken to be 1.5 (optimistic scenario) and 4.0 (pessimistic scenario) [17].

The lockdown period in India began on 25 March 2020. For the present study, the lockdown phase was assumed to continue till 17 May 2020. Although it was later extended till 31 May 2020, the graphs in the present article are still very relevant in the sense that they demonstrate very clearly the effect of enforcing social distancing, through the imposition of lockdown, in reducing the speed of transmission of the disease significantly. They show the characteristics of how the number of infections changes with time, during and beyond the period of lockdown.

## Conclusion:-

An underlying assumption of this formulation is that, the symptomatic patients can’t spread the disease, since they are quarantined immediately after diagnosis. But there must be some symptomatic patients who remain undetected due to the similarity of their symptoms with those caused by influenza viruses. Insufficient number of tests, conducted in the country, is also a reason. One must consider the role of the symptomatic patients in spreading the disease, to improve this mathematical method. The nature of dependence of $\beta_j$ upon time ($j$), is an important feature of this study which has ample scopes for modifications. Apart from the three expressions of $\beta_j$, chosen for the present study, one may select many other forms for greater accuracy. The social distancing parameters, $a$ and $b$, have been assumed to have fixed values during *no-lockdown* and *lockdown* periods respectively. But, during the outbreak of such a pandemic, the characteristics of social mixing may change with time. Despite these shortcomings, the predictions made by these models are in reasonable agreement with the recorded observations. The results demonstrate that only a high degree of social distancing can reduce the transmission rate appreciably, as shown by the Figures 15-20. By a proper tuning of parameters, this method can be used to make predictions regarding the spread of COVID-19 in any country of the world.

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**Table 1:- Cumulative Numbers of COVID-19 Cases in India from 01 March 2020 to 03 May 2020**

| Serial Number of the day ($n$) | Date | Number of COVID-19 Cases ($s_n$) | Serial Number of the day ($n$) | Date | Number of COVID-19 Cases ($s_n$) |
|---|---|---|---|---|---|
| 1 | 01.03.2020 | 3 | 33 | 02.04.2020 | 2069 |
| 2 | 02.03.2020 | 5 | 34 | 03.04.2020 | 2547 |
| 3 | 03.03.2020 | 6 | 35 | 04.04.2020 | 3072 |
| 4 | 04.03.2020 | 28 | 36 | 15.04.2020 | 3577 |
| 5 | 05.03.2020 | 30 | 37 | 06.04.2020 | 4281 |
| 6 | 06.03.2020 | 31 | 38 | 07.04.2020 | 4789 |
| 7 | 07.03.2020 | 34 | 39 | 08.04.2020 | 5274 |
| 8 | 08.03.2020 | 39 | 40 | 19.04.2020 | 5865 |
| 9 | 09.03.2020 | 44 | 41 | 10.04.2020 | 6761 |
| 10 | 10.03.2020 | 50 | 42 | 11.04.2020 | 7529 |
| 11 | 11.03.2020 | 60 | 43 | 12.04.2020 | 8447 |
| 12 | 12.03.2020 | 74 | 44 | 13.04.2020 | 9352 |
| 13 | 13.03.2020 | 81 | 45 | 14.04.2020 | 10815 |
| 14 | 14.03.2020 | 84 | 46 | 15.04.2020 | 11933 |
| 15 | 15.03.2020 | 110 | 47 | 16.04.2020 | 12759 |
| 16 | 16.03.2020 | 114 | 48 | 17.04.2020 | 13835 |
| 17 | 17.03.2020 | 137 | 49 | 18.04.2020 | 14792 |
| 18 | 18.03.2020 | 151 | 50 | 19.04.2020 | 16116 |
| 19 | 19.03.2020 | 173 | 51 | 20.04.2020 | 17656 |
| 20 | 20.03.2020 | 223 | 52 | 21.04.2020 | 18985 |
| 21 | 21.03.2020 | 315 | 53 | 22.04.2020 | 20471 |
| 22 | 22.03.2020 | 360 | 54 | 23.04.2020 | 21700 |
| 23 | 23.03.2020 | 468 | 55 | 24.04.2020 | 23452 |
| 24 | 24.03.2020 | 519 | 56 | 25.04.2020 | 24942 |
| 25 | 25.03.2020 | 606 | 57 | 26.04.2020 | 26917 |
| 26 | 26.03.2020 | 694 | 58 | 27.04.2020 | 28380 |
| 27 | 27.03.2020 | 834 | 59 | 28.04.2020 | 29974 |
| 28 | 28.03.2020 | 918 | 60 | 29.04.2020 | 31787 |
| 29 | 29.03.2020 | 1024 | 61 | 30.04.2020 | 33610 |
| 30 | 30.03.2020 | 1251 | 62 | 01.05.2020 | 35365 |
| 31 | 31.03.2020 | 1397 | 63 | 02.05.2020 | 37776 |
| 32 | 01.04.2020 | 1834 | 64 | 03.05.2020 | 40263 |

**Source:** Ministry of Health and Family Welfare, Government of India. (https://www.mohfw.gov.in).

**Figures:**

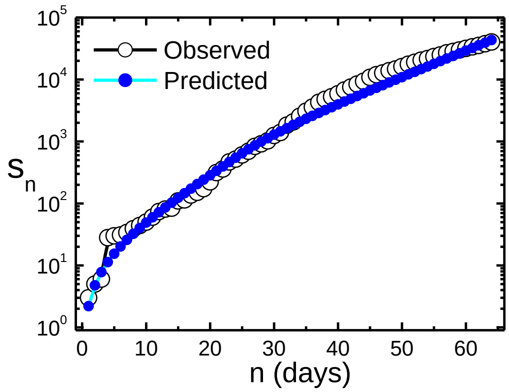


**Figure 1:** Time evolution of the number of symptomatic patients. The data, from Model-1, are represented by blue circles. Black circles represent cumulative COVID-19 counts from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64). The parameter values, required for the best match between theory and observations, are, $y_1 = 100$, $a = 0.2$, $b = 0.125$, $\beta_1 = 0.022$ and $\lambda = -0.008$.

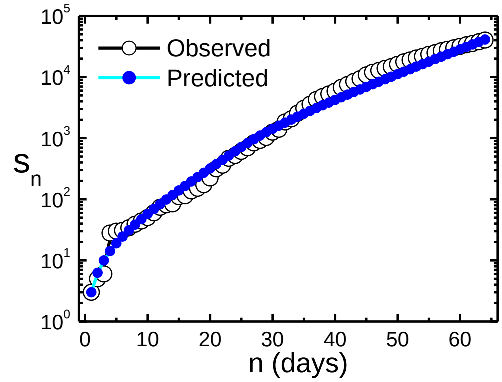


**Figure 2:** Time evolution of the number of symptomatic patients. The data, from Model-2, are represented by blue circles. Black circles represent cumulative COVID-19 counts from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64). The parameter values, required for the best match between theory and observations, are, $y_1 = 100$, $a = 0.2$, $b = 0.118$, $\beta_1 = 0.03$ and $\mu = -0.1$.

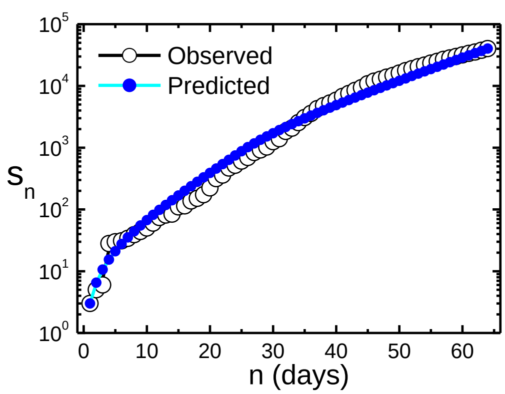


**Figure 3:** Time evolution of the number of symptomatic patients. The data, from Model-3, are represented by blue circles. Black circles represent cumulative COVID-19 counts from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e. $n$ = 64). The parameter values, required for the best match between theory and observations, are, $y_1 = 100$, $a = 0.2$, $b = 0.116$, $\gamma = 0.03$.

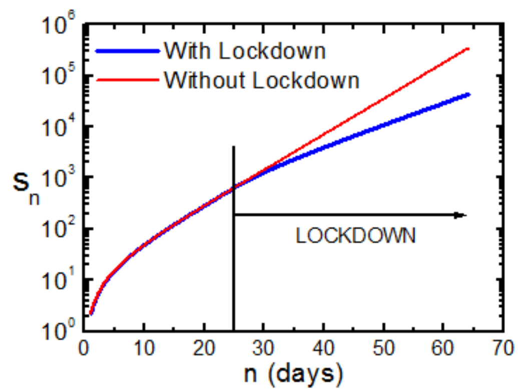


**Figure 4:** Time evolution of the number of symptomatic patients, with and without lockdown, from Model-1, for the period from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64), using the set of parameter values obtained from Figure 1. Lockdown was effective from 25 March 2020 (i.e., $n$ = 25).

**Figures:**

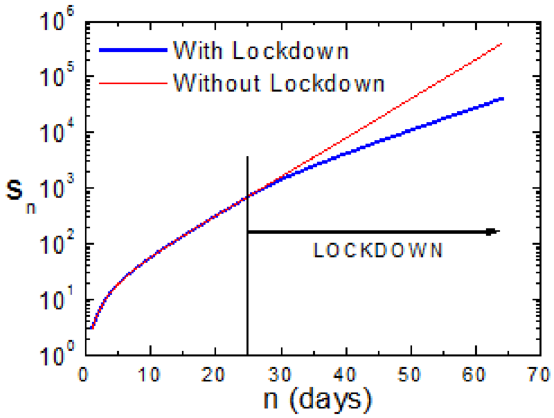


**Figure 5:** Time evolution of the number of symptomatic patients, with and without lockdown, from Model-2, for the period from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64), using the set of parameter values obtained from Figure 2. Lockdown was effective from 25 March 2020 (i.e., $n$ = 25).

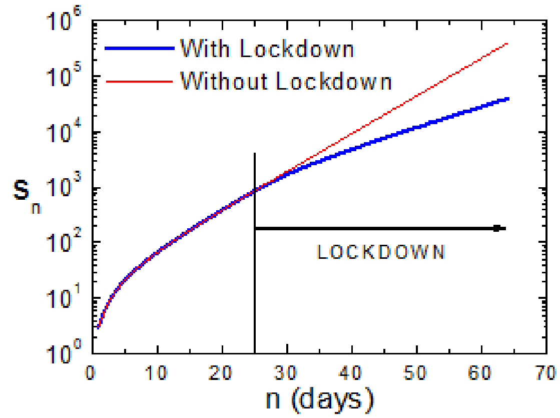


**Figure 6:** Time evolution of the number of symptomatic patients, with and without lockdown, from Model-3, for the period from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64), using the set of parameter values obtained from Figure 3. Lockdown was effective from 25 March 2020 (i.e., $n$ = 25).

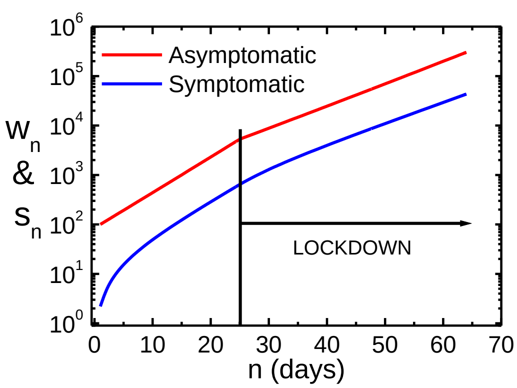


**Figure 7:** Time evolution of the numbers of symptomatic and asymptomatic cases ($s_n$ and $w_n$ respectively), for the period from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64), from Model-1, using the set of parameter values obtained from Figure 1. Lockdown was effective from 25 March 2020 (i.e., $n$ = 25).

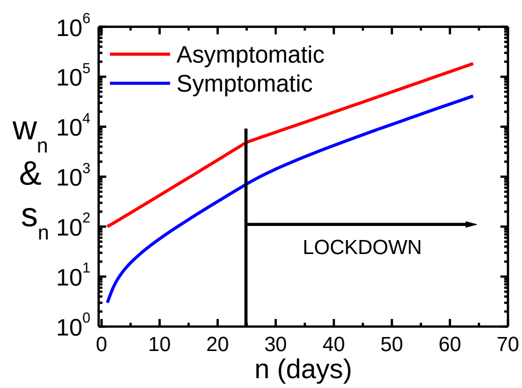


**Figure 8:** Time evolution of the numbers of symptomatic and asymptomatic cases ($s_n$ and $w_n$ respectively), for the period from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64), from Model-2, using the set of parameter values obtained from Figure 2. Lockdown was effective from 25 March 2020 (i.e., $n$ = 25).

**Figures:**

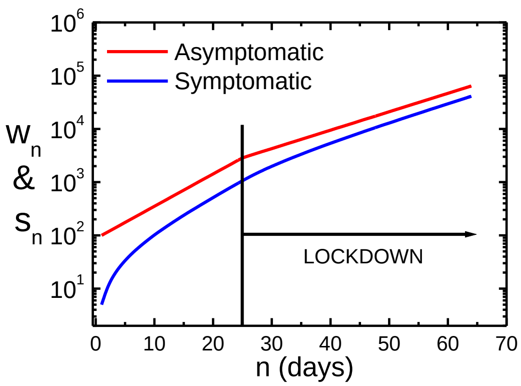


**Figure 9:** Time evolution of the numbers of symptomatic and asymptomatic cases ($s_n$ and $w_n$ respectively), for the period from 01 March 2020 (i.e., $n$ = 1) to 03 May 2020 (i.e., $n$ = 64), from Model-3, using the set of parameter values obtained from Figure 3. Lockdown was effective from 25 March 2020 (i.e., $n$ = 25).

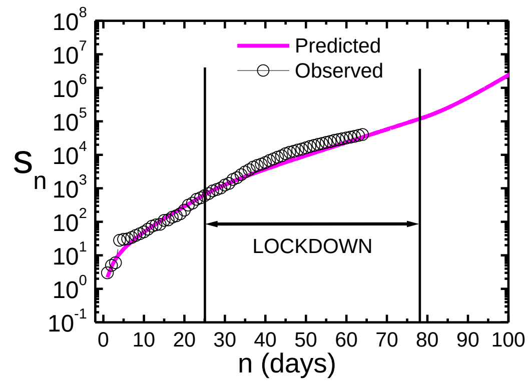


**Figure 10:** Time evolution of the number of symptomatic patients (predicted & observed), for three phases: 1) pre-lockdown, 2) lockdown, 3) post lockdown, based on Model-1, using the set of parameter values obtained from Figure 1. The lockdown has been assumed to continue till $n$ = 78 (17 May 2020). $n$ = 1 corresponds to 01 March 2020.

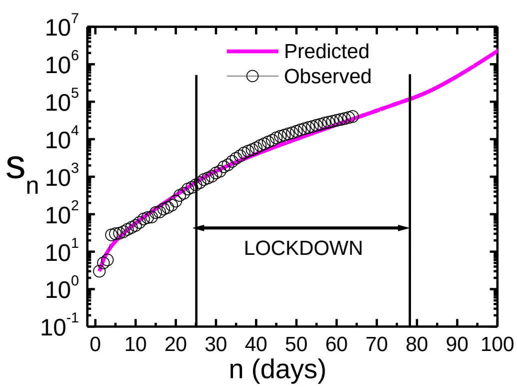


**Figure 11:** Time evolution of the number of symptomatic patients (predicted & observed), for three phases: 1) pre lockdown, 2) lockdown, 3) post lockdown, based on Model-2, using the set of parameter values obtained from Figure 2. The lockdown has been assumed to continue till $n$ = 78 (17 May 2020). $n$ = 1 corresponds to 01 March 2020.

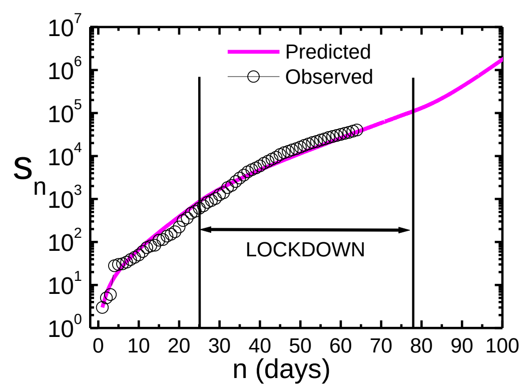


**Figure 12:** Time evolution of the number of symptomatic patients (predicted & observed), for three phases: 1) pre lockdown, 2) lockdown, 3) post lockdown, based on Model-3, using the set of parameter values obtained from Figure 3. The lockdown has been assumed to continue till $n$ = 78 (17 May 2020). $n$ = 1 corresponds to 01 March 2020.

**Figures:**

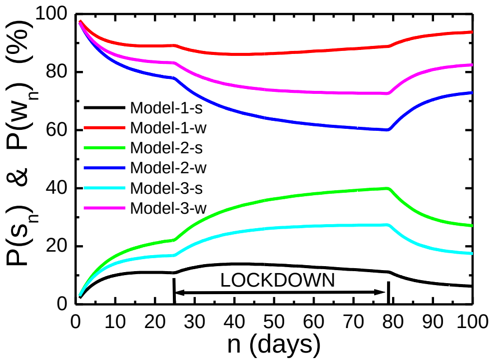


**Figure 13:** Time evolution of the proportions of symptomatic and asymptomatic cases, based on three models. They are expressed as percentages of the total number of patients ($s_n+w_n$). Lockdown has been assumed to continue till $n$ = 78 (17 May 2020). Parameter values, obtained from Figures 1, 2 & 3, have been used. $n$ = 1 corresponds to 01 March 2020.

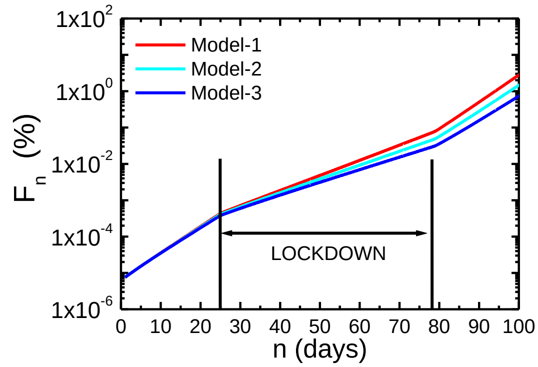


**Figure 14:** Time evolution of the percentage of the Indian population infected (symptomatic & asymptomatic), based on three models. Lockdown has been assumed to continue till $n$ = 78 (17 May 2020). Present population of India is $1.36 \times 10^9$. Parameter values, obtained from Figures 1, 2 & 3, have been used. $n$ = 1 corresponds to 01 March 2020.

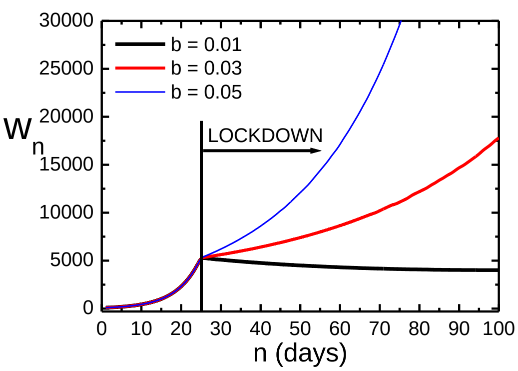


**Figure 15:** Time evolution (in a hypothetical situation) of the number of asymptomatic patients, from Model-1, for three values of the parameter $b$, for a span of 100 days where lockdown continues from the 25$^{th}$ day. Greater social distancing means smaller values of $b$. The values of the parameters, other than $b$, have been obtained from Figure 1.

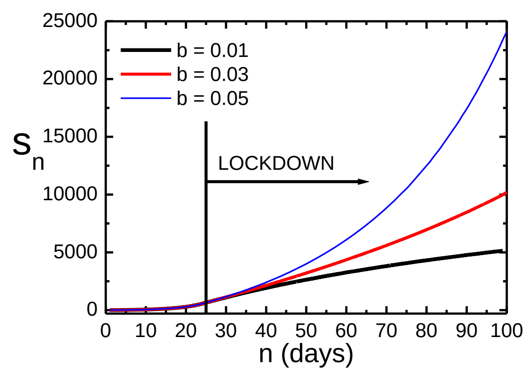


**Figure 16:** Time evolution (in a hypothetical situation) of the number of symptomatic patients, from Model-1, for three values of the parameter $b$, for a span of 100 days where lockdown continues from the 25$^{th}$ day. Greater social distancing means smaller values of $b$. The values of the parameters, other than $b$, have been obtained from Figure 1.

**Figures:**

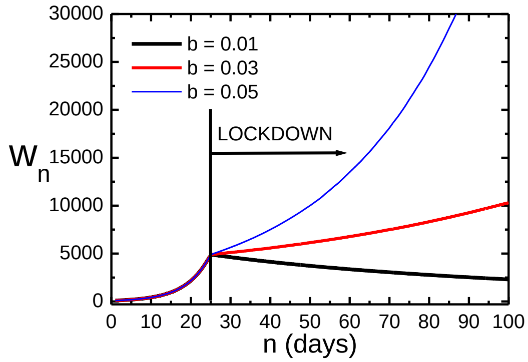


**Figure 17:** Time evolution (in a hypothetical situation) of the number of asymptomatic patients, from Model-2, for three values of the parameter *b*, for a span of 100 days where lockdown continues from the 25th day. Greater social distancing means smaller values of *b*. The values of the parameters, other than *b*, have been obtained from Figure 2.

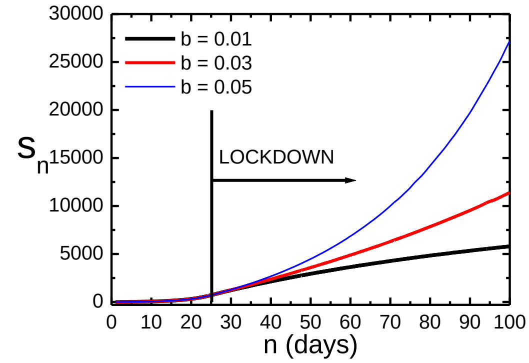


**Figure 18:** Time evolution (in a hypothetical situation) of the number of symptomatic patients, from Model-2, for three values of the parameter *b*, for a span of 100 days where lockdown continues from the 25th day. Greater social distancing means smaller values of *b*. The values of the parameters, other than *b*, have been obtained from Figure 2.

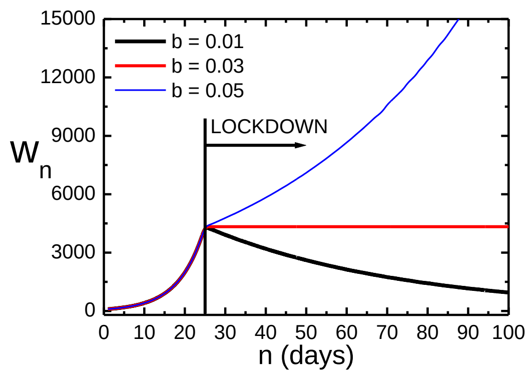


**Figure 19:** Time evolution (in a hypothetical situation) of the number of asymptomatic patients, from Model-3, for three values of the parameter *b*, for a span of 100 days where lockdown continues from the 25th day. Greater social distancing means smaller values of *b*. The values of the parameters, other than *b*, have been obtained from Figure 3.

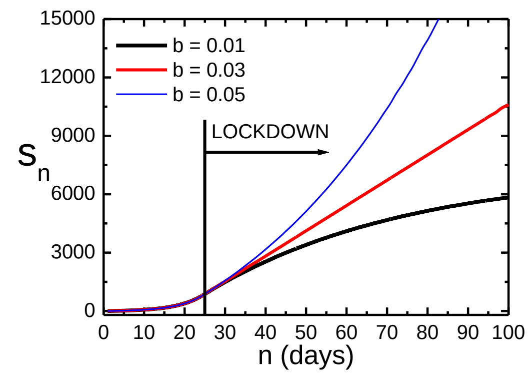


**Figure 20:** Time evolution (in a hypothetical situation) of the number of symptomatic patients, from Model-3, for three values of the parameter *b*, for a span of 100 days where lockdown continues from the 25th day. Greater social distancing means smaller values of *b*. The values of the parameters, other than *b*, have been obtained from Figure 3.

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