Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI:10.21474/IJAR01/12656
DOI URL: http://dx.doi.org/10.21474/IJAR01/12656

ISSN NO. 2320-5407

RESEARCH ARTICLE

COMPLEX-ROTATION CALCULATIONS FOR DOUBLY EXCITED STATES (Nℓ N'ℓ') $^{2S+1}L^{\pi}$ [(2≤N≤ 3) ; (3 ≤N'≤4); (0 ≤ ℓ ≤ 2) AND (0 ≤ ℓ' ≤ 2)] OF HELIUM ISOELECTRONIC SERIES

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Manuscript Info

Manuscript History
Received: 05 February 2021
Final Accepted: 10 March 2021
Published: April 2021

Key words:-
Complex Rotation Method,Variational Calculations, Correlated Wavefunction, resonance energies and widths, Doubly Excited States, Helium Isoelectronic Series

Abstract

In this paper, we report the energies and resonant widths of the [(2s3s $^1S^e$ and 2s3s $^3S^e$); (2s4s $^1S^e$ and 2s4s $^3S^e$); (2s3p $^1P^0$ and 2s3p $^3P^0$); (2s4p $^1P^0$ and 2s4p $^3P^0$); (2p3p $^1D^e$ and 2p3p $^3D^e$); (2p4p $^1D^e$ and 2p4p $^3D^e$); (3s4s $^1S^e$ and 3s4s $^3S^e$); (3s4p $^1P^0$ and 3s4p $^3P^0$); (3p4p $^1D^e$ and 3p4p ^3De) ; (3d4d $^1G^e$ and 3d4d $^1G^e$)] Doubly Excited States of Helium isoelectronic series with nuclear charge Z ($2 \le Z \le 10$).Calculations are performedusing the Complex Rotation Method (CRM) in the framework of a variational procedure. The purpose of this study required a new correlated hydrogenic radial wave function combined with a Hylleraas wave function. The study leads to analytical expressions which are carried out under special MAXIMA computational program. This proposed variational procedure, leads to accurate results in good agreement with available other theoretical results.The present accurate data may be a useful guideline for future experimental and theoretical studies in the (Nℓ N'ℓ') $^{2S+1}L^{\pi}$systems.

Introduction:-

On the theoretical level, the usual use of the independent particle model is not appropriate for the description of the properties of the autoionizing states because it does not take into account the mixing of the configurations as well at the level of the discrete states as of the states of the continuum. The main features of these theoretical studies are the difficulties of constructing a new wave functions that can describe doubly excited states with good precision. Thus, to remove this equivocation and contribute to a better understanding of the phenomena of electronic correlations, new theoretical approaches have emerged such as the diagonalization method [1 – 3], the variational method [4 – 10], the method of the complex rotation [11 – 19] etc... The advantage of complex rotation coordinates in the study of resonances is the obtaining of the energy and the width of resonances at the same time. For the complex rotation method, the Hylleraas wave functions were valid only for electrons of the same valence or of level less than or equal to 3. Thus some authors [11 – 19] were obliged to use the functions of Slater wave for the calculation of energies and resonance widths of doubly excited states. As part of our study, a new Incomplete Hydrogenic Radial Wave function combined with a Hylleraas wave function was developed in place of wave functions using Slater orbitals

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valid in the case electrons of different valencies or levels greater than 3. In the case of two-electron atom systems, the exact resolution of the Schrödinger equation was impossible because of the electronic correlation term. A development of the correlation factor on the basis of the hypergeometric wave functions has been designed not to make the same approximations.We can add that the restriction to non-relativistic dynamics and the neglection of spin-orbit and nuclear-spin coupling as well as quantum-electrodynamic (QED) effects, does not allow tests at the highest level of precision.

Theory

The non-relativistic complex Hamiltonian operator for He-like ions is given by (in *Rydberg*):

$$H(r,\theta) = -(\Delta_1 + \Delta_2)e^{-2i\theta} - 2Ze^{-i\theta}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \frac{2e^{-i\theta}}{r_{12}} \quad (1)$$

This relation (1) can be broken down into a sum of three terms as follows:

$$H(r,\theta) = T(r,\theta) + C(r,\theta) + W(r,\theta) \quad (2)$$

Where:

- ✓ the kinetic energy operator of electrons:

$$T(r,\theta) = -(\Delta_1 + \Delta_2).e^{-2i\theta} \quad (3)$$

The Laplacian is written as(i =1, 2):

$$\Delta_i = \frac{1}{r_i^2}\frac{\partial}{\partial r_i}\left(r_i^2\frac{\partial}{\partial r_i}\right) + \frac{1}{r_i^2\sin\theta_i}\frac{\partial}{\partial\theta_i}\left(\sin\theta_i\frac{\partial}{\partial\theta_i}\right) + \frac{1}{r_i^2\sin^2\theta_i}\frac{\partial^2}{\partial\varphi_i^2} \quad (4)$$

- ✓ the operator of the electron-nucleus interaction energy:

$$C(r,\theta) = -2Ze^{-i\theta}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad (5)$$

- ✓ the operator of the interaction energy between electrons:

$$W(r,\theta) = \frac{2e^{-i\theta}}{r_{12}} \quad (6)$$

In the complex rotation method, the resonance parameters (position and resonances width) are obtained by solving the complex eigenvalue equation below:

$$\left\langle \psi_\mu(\vec{r}_1,\vec{r}_2)\middle|\hat{H}(\vec{r},\theta) - E_{res}\middle|\psi_{\mu'}(\vec{r}_1,\vec{r}_2)\right\rangle = 0 \quad (7)$$

where $E_{res} = \dfrac{\left\langle \psi_\mu(\vec{r}_1,\vec{r}_2)\middle|H\middle|\psi_{\mu'}(\vec{r}_1,\vec{r}_2)\right\rangle}{\left\langle \psi_\mu(\vec{r}_1,\vec{r}_2)\middle|\psi_{\mu'}(\vec{r}_1,\vec{r}_2)\right\rangle} = E_r - \dfrac{\Gamma_r}{2}i$ (8)

Thereal part E_r gives the resonance energy and the imaginary part Γ_r gives the resonance width and $\psi_\mu(\vec{r}_1,\vec{r}_2)$ is a trial wave function.

This new trial wave function that we used in this work is obtained by a combination of incomplete hydrogenic radial wave functions with Hylleraas wave functions. Itis in the form:

$$\psi_{jkmnln'l'}(\vec{r}_1,\vec{r}_2) = \left\langle (2r_1 2r_2)^l \times \sum_{v=0}^{v=n-l-1}\left(n^2 r_0^2\lambda^2 2r_1 2r_2\right)^v + (2r_1 2r_2)^{l'} \times \sum_{v'=0}^{v'=n'-l'-1}\left(n'^2 r_0^2\lambda'^2 2r_1 2r_2\right)^{v'}\right\rangle \quad (9)$$
$$\times r_{12}^m \times \left[r_1^j r_2^{k+L} Y_{00}^{(1)} Y_{L0}^{(2)} + (-1)^S r_2^j r_1^{k+L} Y_{L0}^{(1)} Y_{00}^{(2)}\right] \times \exp(-\lambda r_1 - \lambda' r_2)$$

The set consisting of the three parameters j, k and m (with j, k and $m \geq 0$) represents a state base of the two-electron system where $j + k + m \leq \Omega$, (with Ω being an integer> 0).

j takes into account the distance of the two electrons from the nucleus.
k takes into account the approximation of the two electrons from the nucleus.
m takes into account the distance between the two electrons.

- ℓ and ℓ' are the orbital angular moments of the two electrons,
- L = ℓ + ℓ': is the total angular momentum as for the terms L = 0, $k \geq m$,
- S: the total spin of the atomic system,
- $\vec{r}_1$ and $\vec{r}_2$ correspond to the positions of the two electrons,
- r_0 : is the radius of Bohr,
- *n* and *n*': are major quantum numbers,
- $Y_{00}^{(1)}$, $Y_{L0}^{(2)}$, $Y_{L0}^{(1)}$ and $Y_{00}^{(2)}$: represent spherical harmonics of electrons 1 and 2.

The exponential parameters are given by:

$$\begin{cases} \lambda = \dfrac{Z}{\alpha n r_0} \\ \lambda' = \dfrac{Z}{\alpha' n' r_0} \end{cases} \quad (10)$$

Where Z is the nuclear charge; α and α'are the variational parameters,

r_{12}^n : represents the correlation factor defined by [20]:

For $r_1<r_2$ we have: $r_{12}^n = 4\pi \sum_{l=0}^{\infty} \frac{1}{2l+1} a_l^n(r_1, r_2) \times \left(Y_l(\Omega_1) Y_l(\Omega_2) \right)$ (11)

Where $a_l^n(r_1, r_2) = \frac{-\left(\frac{n}{2}\right)_l}{\left(\frac{1}{2}\right)_l} r_2^n \left(\frac{r_1}{r_2}\right)^l F\left(l-\frac{n}{2},\ -\frac{1}{2}-\frac{n}{2},\ l+\frac{3}{2},\ \frac{r_1^2}{r_2^2}\right)$ (12)

$F\left(l-\frac{n}{2},\ -\frac{1}{2}-\frac{n}{2},\ l+\frac{3}{2},\ \frac{r_1^2}{r_2^2}\right)$ is the hypergeometric wave function defined by [21]:

$$F\left(l-\frac{n}{2},\ -\frac{1}{2}-\frac{n}{2},\ l+\frac{3}{2},\ \frac{r_1^2}{r_2^2}\right) = 1 - \frac{(2l-n)(1+n)}{2(2l+3)} \times \frac{r_1^2}{r_2^2} \quad (13)$$

The correlation factor is then written as follow:

$$r_{12}^n = 4\pi \sum_{l=0}^{\infty} \frac{1}{2l+1} \times \frac{\left(-\frac{n}{2}\right)_l}{\left(\frac{1}{2}\right)_l} \times \frac{r_1^l}{r_2^{l-n+2}} \times \left(r_2^2 - \frac{(2l-n)(1+n)}{2(2l+3)} \times r_1^2 \right) \left(Y_l(\Omega_1) Y_l(\Omega_2) \right) \quad (14)$$

With $Y_l(\Omega_1) Y_l(\Omega_2) = \sum_{m=-l}^{l} Y_{l,m}^*(\Omega_1) Y_{l,m}(\Omega_2)$ and $\Omega_i = (\theta_i, \varphi_i)$

In our calculations, we have set the dimension Ω of the base to 5 defined suchaccording to the values of the real parameters (*j*, *k* and *m*> 0) that $j + k + m \leq \Omega$, and the rotation angle θ fixed to 0.35 radian. The restriction α= α' has been adopted. To obtain the results of energies and resonance widths, the analytical expressions are carried out under special MAXIMA computational program.

MAXIMA is a computer algebra system for the manipulation of symbolic and numerical expressions, including differentiation, integration, ordinary differential equations, and matrix elements. MAXIMA yields high precision numeric results by using exact fractions, arbitrary precision integers, and variable precision floating point numbers. Our MAXIMA source code is compiled on windows systems.

To obtain the minimum eigenvalue of the [2s3s $^{1,3}S^e$; 2s4s $^{1,3}S^e$; 2s3p $^{1,3}P^0$;2s4p $^{1,3}P^0$;2p3p $^{1,3}D^e$;2p4p $^{1,3}D^e$; 3s4s $^{1,3}S^e$;3s4p $^{1,3}P^0$; 3p4p $^{1,3}D^e$ and 3d4d $^{1,3}G^e$] doubly excited states of Helium isoelectronic serieswith nuclear charge Z ($2 \leq Z \leq 10$) in which we are interested and quoted in tables 1-10, the nonlinear parameters α and α'are slightly varied that exhibit a plateau for the energy. At each minimum value of the energy we are interested, corresponds a minimum value of the resonance width given at near 0.001. Here 0.001 is not the precision of the result but a minimum limit of comparison between results.

Results and Discussions:-

The main results of our calculations for the energies and the resonance widths of the [2s3s $^{1,3}S^e$; 2s4s $^{1,3}S^e$; 2s3p $^{1,3}P^0$;2s4p $^{1,3}P^0$;2p3p $^{1,3}D^e$;2p4p $^{1,3}D^e$; 3s4s $^{1,3}S^e$;3s4p $^{1,3}P^0$; 3p4p $^{1,3}D^e$ and 3d4d $^{1,3}G^e$] doubly excited states of Helium isoelectronic serieswith nuclear charge Z ($2 \leq Z \leq 10$) are reported in Tables 1 – 10. The results are reported in Rydberg (Ry) for direct comparison. Our results are compared with other theoretical results available in the literature.For the calculation of the resonance parameters (energies and resonance widths) of He-like systems, several authors have used different methods. We noted, Ho [13 – 16] used the complex rotation method and a Hylleraas-type wave function, Biaye and al. [6 – 8] used the variational method of Hylleraas, Seminario and al. [22] used the method of "Z" perturbation in the formalism of Feshbach projection operators [23], Konté and al. [24] used the variational method of stabilization, Sakho and al. [25 – 27] used the SCUNC (Screening Constant by Unit Nuclear Charge) method, Bachau and al. [28] also used the Feschbach method of operations, Sow and al. [29] used the Modified Atomic Orbital Theory (MAOT), Ivanov [30, 31] who used the method of complex rotation.

In Tables 1 and 2, we compared our results respectively with the theoretical results of Ho [13], Dieng and al. [10] for 2s3s $^1S^e$ state; Dieng and al. [10],Ivanov and al. [30] for 2s3s $^3S^e$ state; the results of Sakho and al. [27],Ivanov and al. [31] for 2s4s $^{1,3}S^e$ state. We note a good agreement between our results and those of the authors mentioned above.In Tables 3 and 4, we compare our results with those of Ho [13],Sakho and al. [27] for 2s3p $^{1,3}P^\circ$ state and those of Dieng and al. [10], Sakho and al. [27] for 2s4p $^{1,3}P^\circ$ state. Generally, we note a good agreement between our results and those of Ho [13], Dieng and al. [10],Sakho and al. [27].

Comparisons are made in Tables 5 and 6 with the results of Ho [16], Dieng and al. [12], Sakho and al. [26] for 2p3p $^{1,3}D^e$, with the results of Dieng and al. [10], Sakho and al. [27] as well as for 2p4p $^{1,3}D^e$ state. Here again we note a good agreement.In Tables 7 and 8 the values of the energies we calculated have been compared with those of Dieng and al. [10], Bachau and al. [28] for the 3s4s $^{1,3}S^e$ and 3s4p $^{1,3}P^\circ$ states. For these states, we note a good agreement.Tables 9 and 10, show that our values and those of Dieng and al. [10], Ho [13], Bachau [28], and Ivanov [31] are in good agreement.

For all the states studied, the results obtained are in good agreement with the theoretical results we have consulted. The small difference noted between our results and those of the authors mentioned above is explained by the fact that Dieng and al. [10] used the variational method of Hylleraas considering only the radial part of the Hamiltonian then that Sakho and al. [27] used the semi-empirical procedure of the Screening Constant by Unit Nuclear Charge method. Moreover, this small difference with the results of Ivanov and al. [30, 31] using the same method of complex rotation is also due to the choice of nonlinear parameters (α and α'), the size of the basis Ω and the angle of rotation θ. However, it should also be noted that our results agree better with those of Ho [13, 16], because we used the same method of complex rotation and the same angular part of the wave function.

For the calculation of the resonance widths, our results were compared with those of Ho [13, 16] for doubly excited states 2s3s $^1S^e$, 2s3p $^{1,3}P^\circ$, 2p3p $^{1,3}D^e$ and 2p4p $^{1,3}D^e$. We also compare the results of our calculations for resonance widths with the values of Bachau [28] for the 3s4s $^{1,3}S^e$, 3s4p $^{1,3}P^\circ$, 3p4p $^{1,3}D^e$ states. The results we obtained, were also compared to those of Ivanov and al. [31] for 3d4d $^{1,3}G^e$ state. We then note a good agreement in the whole between the values obtained on the widths with the values of other authors, Ho [13, 16], Ivanov and al. [30, 31] and Bachau [28].

Summary and Conclusion:-

In this work, the variational procedure of the complex rotation method has been applied to the calculations of energies and resonance widths of the [(2s3s $^1S^e$ and 2s3s $^3S^e$); (2s4s 1Se and 2s4s $^3S^e$); (2s3p $^1P^0$ and 2s3p $^3P^0$);(2s4p $^1P^0$ and 2s4p $^3P^0$); (2p3p $^1D^e$ and 2p3p $^3D^e$); (2p4p $^1D^e$ and 2p4p $^3D^e$); (3s4s $^1S^e$ and 3s4s $^3S^e$); (3s4p $^1P^0$ and 3s4p $^3P^0$); (3p4p $^1D^e$ and 3p4p $^3D^e$) ; (3d4d $^1G^e$ and 3d4d $^1G^e$)]Doubly Excited States of Helium isoelectronic series with

nuclear charge Z ($2 \leq Z \leq 10$). It has demonstrated the possibilities to construct a new correlated wave function adapted to the correct description of the electron-electron correlations phenomena in the (nln'l') 2S+1LπDoubly Excited States of the He-like systems. These very important results obtained in this work indicate the possibility to apply the complex rotation variational procedure to the treatment of atomic spectra in two electron systems and probably in more complex atomic systems. The good results give also the possibility to analyze resonance energies and width via a very complex rotation flexible method, in contrast to the complex procedures of experimental and theoretical methods based on the determination of the photoionization cross-section.In summary, the manuscript reports on new calculations for key atomic-structure parameters of important fundamental few-body systems (helium and helium-like ions). While not allowing precision tests of physics due to the neglect of relativistic, spin, and QED effects, such results can still be helpful in the future development of theories to describe more complex atoms, or may be further developed to study the time-dependent evolution of atoms in external (e.g. laser) fields.

Table 1:- Energies (– E) and resonances widths (Γ) of (2s3s $^1S^e$ and 2s3s $^3S^e$) doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**2s3s $^1S^e$**						**2s3s $^3S^e$**				
	α = α'	$-E^{Present}$	$-E^{Ho}$ [13]	$-E^{Dieng}$ [10]	$\Gamma^{Present}$	Γ^{Ho} [13]	α = α'	$-E^{Present}$	$-E^{Dieng}$ [10]	$-E^{Sakho}$ [27]	$\Gamma^{Present}$
2	2.200	1.179510257	1.179850	1.179902	0.002970	0.002970	2.460	1.226589571	1.225489	1.202960	0.0010709
3	2.077	2.831193484	2.831150	2.823779	0.004420	0.004330	2.350	2.909279973	2.903172	2.876740	0.0025566
4	2.017	5.204587321	5.204100	5.189878	0.005460	0.005330	2.300	5.302389261	5.303077	5.274440	0.0046709
5	1.981	8.299709509	8.299000	8.278200	0.006140	0.005993	2.260	8.455298238	8.425204	8.395040	0.0074724
6	1.957	12.118407994	12.11600	12.088743	0.006443	0.006337	2.245	12.270167761	12.269553	12.238160	0.0108543
7	1.940	16.046441499	16.655000	16.621509	0.006868	0.006970	2.233	16.805423079	16.836124	16.803700	0.0148780
8	1.928	21.913270359	21.916400	21.876498	0.006921	0.006990	2.217	22.134305159	22.124918	22.091580	0.0196165
9	1.920	27.865758232	27.899800	27.853708	0.007274	0.007010	2.210	28.117025 8647	28.135934	28.101760	0.0249303
10	1.910	34.607680863	34.605400	34.553141	0.007319	0.007225	2.200	34.896165068	34.869172	34.834200	0.0309618

Table 2:- Energies (– E) and resonances widths (Γ) of (2s4s $^1S^e$ and 2s4s $^3S^e$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10).Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**2s4s $^1S^e$**					**2s4s $^3S^e$**				
	α = α'	$-E^{Present}$	$-E^{Sakho}$[27]	$-E^{Ivanov}$[30]	$\Gamma^{Present}$	α = α'	$-E^{Present}$	$-E^{Sakho}$ [27]	$-E^{Ivanov}$ [30]	$\Gamma^{Present}$
2	1.870	1.085082474	1.088999	1.070599	0.00093093	2.072	1.082536521	1.098200	1.082400	0.000940977
3	1.815	2.525630596	2.552039	2.525598	0.00217524	1.992	2.548362335	2.571120	2.547000	0.002227218
4	1.775	4.605416885	4.640338	4.605598	0.00397826	1.952	4.637543375	4.669640	4.636600	0.004647005
5	1.750	7.313375241	7.353717	7.310597	0.00632945	1.928	7.350371283	7.393420	7.351200	0.006453752
6	1.734	10.642346787	10.692175	10.640596	0.00922194	1.911	10.693418223	10.742320	10.690800	0.009400881

7	1.723	14.591200963	14.655633	14.595594	0.00126546		1.900	14.652432575	14.716280	14.655200	0.012892022
8	1.714	19.172428460	19.244132	19.175592	0.00166396		1.891	19.243317588	19.315280	19.244800	0.016942895
9	1.707	24.379038034	24.457630	24.379990	0.00211702		1.884	24.459626473	24.539320	24.458000	0.021547135
10	1.701	30.219179979	30.296128	30.209988	0.00252542		1.879	30.290158453	30.388380.	30.298000	0.026693639

Table 3:- Energies (– E) and resonances widths (Γ) of ($2s3p\ ^1P^0$ and $2s3p\ ^3P^0$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**$2s3p\ ^1P^0$**							**$2s3p\ ^3P^0$**					
	α = α'	$-E^{Present}$	$-E^{Ho}$[13]	$-E^{Sakho}$[27]	$\Gamma^{Present}$	Γ^{Ho}[13]		α= α'	$-E^{Present}$	$-E^{Ho}$[13]	$-E^{Sakho}$[27]	$\Gamma^{Present}$	Γ^{Ho}[13]
2	2.113	1.12839382	1.128000	1.124780	0.00098091	0.000 60		2.059	1.15893150 82	1.158 06	1.172 00	0 .0001 0156	
3	1.991	2.72186241 5	2.722860	2.721400	0.0013857	0.001 32		1.939	2.79766460 1	2.797 028	2.809 60	0.00011 285	
4	1.926	5.03671591 9	5.036500	5.040400	0.0024359	0.002 00		1.882	5.15732382 6	5.158 510	5.169 42	0.00014 002	0 .00 010
5	1.888	8.07154272 4	8.070700	8.081290	0.0025296	0.002 36		1.848	8.24921243 7	8.242 240	8.251 46	0.00014 385	0 .00 010
6	1.864	11.82855 0327	11.826 300	11.844 580	0.0026 685	0.002 62		1.830	12.04736 5684	12.04 8 180	12.05 573	0.00014 771	0 .00 011
7	1.849	16.30230 4647	16.303 500	16.330 080	0.0029 439	0.002 82		1.818	16.57593 5608	16.57 6 330	16.58 221	0.00014 791	0 .00 011
8	1.839	21.50244 7409	21.502 600	21.537 810	0.0030 721	0.002 95		1.809	21.85347 2899	21.82 6 690	21.96 690	0.00015 126	0 .00 012
9	1.833	27.42245 0768	27.423 800	27.467 750	0.0031 384	0.003 07		1.805	27.83764 3419	27.79 9 263	27.80 185	0.00015 355	0 .00 012
10	1.829	34.08172 4989	34.066 900	34.119 920	0.0032 639	0.003 15		1.805	34.50986 0659	34.49 4 050	34.49 501	0.00015 445	0 .00 012

Table 4:- Energies (– E) and resonances widths (Γ) of ($2s4p\ ^1P^0$ and $2s4p\ ^3P^0$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present energies values are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**$2s4p\ ^1P^0$**						**$2s4p\ ^3P^0$**				
	α = α'	$-E^{Present}$	$-E^{Dieng[10]}$	$-E^{Sakho[27]}$	$\Gamma^{Present}$		α=α'	$-E^{Present}$	$-E^{Dieng[10]}$	$-E^{Sakho[27]}$	$\Gamma^{Present}$
2	1.908	1.06808384 3	0.975938 64	1.067410 0	0.000939 751		1.735	1.08567147 6	1.007528 06	1.086720	0.000957 797
3	1.779	2.37819749 7	2.378209 49	2.509720 0	0.002089 751		1.677	2.54109059 9	2.444897 46	2.546470	0.002252 985
4	1.916	4.40838616 1	4.408265 95	4.577040 0	0.003893 879		1.641	4.63311987 9	4.513855 29	4.631220	0.004121 096
5	1.777	7.07483775 8	7.070269 67	7.269360 0	0.006270 165		1.621	7.34361765 3	7.215274 27	7.340970	0.006544 066
6	1.654	10.367819 981	10.365921 40	10.586680 0	0.009209 023		1.608	10.674837 341	10.549462 90	10.675 720	0.009524 140
7	1.634	14.293582 822	14.295956 60	14.529000 0	0.012711 167		1.594	14.679124 332	14.516553 10	14.635 470	0.013114 122
8	1.6	18.858138	18.86072	19.0963	0.016799		1.5	19.229414	19.11661	19.220	0.017185

	20	024	050	100	244		90	106	120	220	822
9	1.596	24.288385562	24.06038970	24.2886300	0.076484218		1.585	24.427451591	24.34967590	24.429970	0.021841871
10	1.592	30.074430943	29.89506340	30.1059500	0.026861347		1.580	30.269595184	30.21577300	30.264720	0.027078624

Table 5:- Energies (– E) and resonances widths (Γ) of (2p3p $^1D^e$and 2p3p $^3D^e$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**2p3p $^1D^e$**						**2p3p $^3D^e$**					
	α = α'	$-E^{Present}$	$-E^{Ho[16]}$	$-E^{Sakho[26]}$	$\Gamma^{Present}$	$\Gamma^{Ho[16]}$	α = α'	$-E^{Present}$	$-E^{Ho[16]}$	$-E^{Sakho[26]}$	$\Gamma^{Present}$	$\Gamma^{Ho[16]}$
2	1.979	1.142000437	1.138441	1.13813994	0.001006100	0.001110	2.034	1.168444747	1.1675681	1.16673995	0.00076484	5.72 X10^{-8}
3	1.868	2.747764991	2.748201	2.74779994	0.002441337	0.002208	1.922	2.812388513	2.8111367	2.80705999	0.00076484	3.32 X10^{-8}
4	1.805	5.084699709	5.080575	5.07984000	0.002540973	0.002852	1.867	5.176649821	5.1766751	5.17081993	0.00076484	1.92 X10^{-8}
5	1.769	8.134697914	8.135532	8.13415965	0.003287123	0.003246	1.834	8.263619299	8.2642689	8.25728011	0.00762943	1.26 X10^{-8}
6	1.744	11.911459921	11.912922	11.91073990	0.003693705	0.003504	1.812	12.073685560	12.0739865	12.06621997	0.01116962	0.94 X10^{-8}
7	1.726	16.411917153	16.412662	16.40956010	0.003757681	0.003686	1.797	16.599070562	16.6058649	16.5974799	0.01537761	0.94 X10^{-8}
8	1.713	21.627700513	21.634700	21.63064090	0.003970554	0.003824	1.785	21.856353876	21.8599252	21.8510597	0.02027093	< 10 X10^{-8}
9	1.702	27.581212480	27.579020	27.57390030	0.004000000	0.003928	1.776	27.831264406	27.8361797	27.8269200	0.02583448	< 10 X10^{-8}
10	1.694	34.240647673	34.245600	34.23942010	0.004019086	0.004010	1.769	34.523890082	34.5346364	34.5250402	0.03206381	< 10 X10^{-8}

Table 6:- Energies (– E) and resonances widths (Γ) of (2p4p $^1D^e$and 2p4p $^3D^e$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present energies results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**2p4p $^1D^e$**					**2p4p $^3D^e$**				
	α=α'	$-E^{Present}$	$-E^{Dieng[10]}$	$-E^{Sakho[27]}$	$\Gamma^{Present}$	α=α'	$-E^{Present}$	$-E^{Dieng[10]}$	$-E^{Sakho[27]}$	$\Gamma^{Present}$
2	1.679	1.072656274	0.97594092	1.07289996	0.000948181	1.728	1.081671796	1.00752983	1.08149997	0.000972850
3	1.616	2.523328 0_	2.37821397	2.52064002	0.002243131	1.664	2.544613080	2.44490069	2.54134003	0.002302162
4	1.583	4.595324573	4.35081956	4.59343991	0.004097693	1.633	4.625645557	4.51385 83	4.62709994	0.004197402
5	1.562	7.293295085	7.07026952	7.29123990	0.006516633	1.612	7.339450459	7.21527228	7.33821994	0.006673735

6	1.548	10.613739597	10.3659055	10.6140596	0.009496470		1.598	10.678941158	10.5494475	10.6745203	0.009723920
7	1.537	14.567834992	14.2959083	14.5618997	0.013048520		1.589	14.633208895	14.5165113	14.6359393	0.013336670
8	1.529	19.144298024	18.8606157	19.1347198	0.017161391		1.581	19.227941068	19.1165244	19.2224000	0.017538580
9	1.523	24.341659317	24.0601955	24.3325598	0.021833600		1.576	24.427343722	24.3495189	24.4341785	0.022292600
10	1.519	30.144441210	29.8947369	30.1554003	0.027049404		1.571	30.271609506	30.2155132	30.2704602	0.027640320

Table 7:- Energies (– E) and resonances widths (Γ) of (3s4s $^1S^e$and 3s4s $^3S^e$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**3s4s $^1S^e$**							**3s4s $^3S^e$**					
	α = α'	$-E^{Present}$	$-E^{Bachau}$ [28]	$-E^{Dieng}$ [10]	$\Gamma^{Present}$	Γ^{Bachau} [28]		α = α'	$-E^{Present}$	$-E^{Bachau}$ [28]	$-E^{Dieng}$ [10]	$\Gamma^{Present}$	Γ^{Bachau}[28]
2	2.500	0.564616063	0.563000	0.569242	0.00015026	0 .00013		2.779	0.573497945	0.574600	0.569242	0.00026758	0.000240
3	2.350	1.357567675	1.357400	1.363931	0.00018003	0.000176		2.626	1.372658307	1.378800	1.363931	0.00029532	0.000280
4	2.280	2.493263833	2.498000	2.505843	0.00019729	0.000194		2.540	2.530937602	2.530000	2.505843	0.00031184	0.000300
5	2.232	3.986086875	3.988000	3.994976	0.00020854	0.000200		2.500	4.024093339	4.028000	3.994976	0.00032812	0.000300
6	2.200	5.830100464	5.822000	5.831332	0.00219870	0.000200		2.468	5.877310173	5.874000	5.831332	0.00033872	0.000320
7	2.182	8.006123365	8.006000	8.014910	0.00021570	0.000220		2.449	8.067956329	8.068000	8.014910	0.00034132	0.000320
8	2.167	10.535198846	10.536000	10.545711	0.00022019	0.000200		2.435	10.604428119	10.608000	10.545711	0.00035080	0.000320
9	2.155	13.413877157	13.412000	13.423733	0.00023303	0.000220		2.423	13.494422949	13.496000	13.423733	0.00035413	0.000340
10	1.145	16.643830746	16.636000	16.648978	0.00024436	0.000220		2.413	16.735833123	16.730000	16.648978	0.00037941	0.000360

Table 8:- Energies (– E) and resonances widths (Γ) of (3s4p $^1P^0$and 3s4p $^3P^0$) doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**3s4p $^1P^0$**							**3s4p $^3P^0$**					
	α = α'	$-E^{Present}$	$-E^{Bachau}$ [28]	$-E^{Dieng}$ [10]	$\Gamma^{Present}$	Γ^{Bachau}[28]		α = α'	$-E^{Present}$	$-E^{Bachau}$ [31]	$-E^{Dieng}$[10]	$\Gamma^{Present}$	Γ^{Bachau}[28]
2	2.300	0.574460147	0.572000	0.521740	0.00026762	0.00026		2.350	0.560105947	0.560200	0.53858148	0.00115511	0.00108
3	2.175	1.374188508	1.374400	1.274919	0.00037463	0.00036		2.200	1.354382444	1.352600	1.30546541	0.00127742	0.00124
4	2.112	2.523929544	2.524000	2.373100	0.00381580	0.00380		2.130	2.495347169	2.490000	2.41786517	0.00174686	0.00166
5	2.075	4.022995817	4.020000	3.828345	0.00387805	0.00380		2.093	3.976554611	3.978000	3.88526633	0.00296759	0.00580
6	2.054	5.862026654	5.864000	5.653554	0.00388961	0.00380		2.065	5.815172905	5.812000	5.71872832	0.01078494	0.01080
7	2.0	8.05400	8.056	7.862	0.0039	0.003		2.0	7.98045	7.994	7.9310	0.0135	0.012

	38	7508	000	730	0065	80		51	1403	000	5129	7120	40
8	2.026	10.597570536	10.594000	10.470913	0.00390138	0.00380		2.034	10.528573202	10.522000	10.5365097	0.01467679	0.01300
9	2.018	13.484648640	13.480000	13.493813	0.00391927	0.00380		2.025	13.403696554	13.398000	13.5503737	0.01505133	0.01320
10	2.012	16.721033874	16.714000	16.968842	0.00392227	0.00380		2.018	16.629327158	16.620000	16.9883152	0.01571585	0.01320

Table 9:- Energies (– E) and resonances widths (Γ) of (3p4p $^1D^e$and 3p4p $^3D^e$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**3p4p $^1D^e$**							**3p4p $^3D^e$**					
	α = α’	– E Present	– E Dieng[10]	– E Ho [16]	Γ Present	Γ Bachau[28]		α = α’	– E Present	– E Dieng[10]	– E Ho [16]	Γ Present	Γ Bachau[28]
2	2.250	0.555829906	0.53888635	0.5394520	0.00012611	0.00011		2.300	0.562554523	0.55536100	0.5505202	0.00037319	0.00036
3	2.120	1.335026334	1.31308450	1.3197258	0.00028118	0.00026		2.150	1.365050629	1.34224326	1.3382968	0.00045748	0.00044
4	2.045	2.469454868	2.43426838	2.4473737	0.00028152	0.00026		2.090	2.505275682	2.47572059	2.4733509	0.00046174	0.00046
5	2.000	3.954516979	3.90689654	3.9222669	0.00028209	0.00024		2.050	4.000732059	3.95769969	3.9556339	0.00046244	0.00046
6	1.970	5.790507415	5.73503620	5.7443855	0.00201821	0.00198		2.030	5.825193057	5.79000180	5.7851393	0.00049176	0.00048
7	1.950	7.971089339	7.92398366	7.9137254	0.00204009	0.00154		2.020	7.973108549	7.97771205	7.9618660	0.00049335	0.00048
8	1.932	10.530865641	10.48181910	10.4302856	0.00208246	0.00126		2.008	10.484255169	10.5257726	10.4858134	0.00051101	0.00050
9	1.930	13.328126898	13.42089050	13.2940066	0.00211530	0.00112		1.985	13.443272085	13.4447739	13.3569823	0.00051343	0.00050
10	1.920	16.549556284	16.75979340	16.7506751	0.00211987	0.00106		1.978	16.663171518	16.5050677	16.5753727	0.00051557	0.00050

Table 10:- Energies (– E) and resonances widths (Γ) of (3d4d $^1G^e$and 3d4d $^3G^e$)doubly excited states of He-like ions with nuclear charge Z (Z = 2 – 10). Our present results are compared with other theoretical results. The results are reported in Rydberg (Ry), 1Ry = 13.6056925 eV.

Z	**3d4d $^1G^e$**							**3d4d $^3G^e$**						
	α = α’	– E Present	– E Bachau [28]	– E Ivanov [31]	Γ Present	Γ Ivanov [31]		α = α’	– E Present	– E Bachau [28]	– E Ivanov [31]	Γ Present	Γ Ivano [31]
2	2.300	0.528953581	0.524200	0.5236648	0.005148397	0.00508875		2.258	0.539758616	0.539400	0.539138523	0.00001512	0.00000145
3	2.154	1.281431437	1.282000	1.28235251	0.011595162	0.01144140		2.107	1.313829868	1.314000	1.313835751	0.00008517	0.00000827
4	2.066	2.388338941	2.388000	2.38903500	0.016052129	0.01532500		2.030	2.436527029	2.434000	2.434874885	0.00018348	0.00001654
5	2.014	3.841507003	3.840000	3.84340900	0.017811772	0.01768700		1.986	3.903265227	3.902000	3.902706820	0.00026263	0.00002416
6	1.978	5.646630596	5.640000	5.64525900	0.019879636	0.01923700		1.956	5.719164092	5.716000	5.717525390	0.00031720	0.00003069
7	1.954	7.793517077	7.790000	7.79447800	0.020745921	0.02032400		1.936	7.876361206	7.878000	7.879424050	0.00036934	0.00003621
8	1.936	10.287491888	10.284000	10.29100900	0.021915766	0.02112400		1.920	10.385592451	10.386000	10.38845287	0.00040905	0.00004088
9	1.9	13.1286	13.12	13.134	0.0219	0.0217		1.9	13.2468	13.24	13.244	0.0000	0.0000

	22	17823	8000	82300	29241	3700		07	73137	2000	64095	4575	4484
1	1.9	16.3247	16.31	16.325	0.0227	0.0222		1.8	16.4430	16.44	16.448	0.0000	0.0000
0	10	58006	8000	90000	73989	2200		98	08761	4000	00635	48937	4824

CrediT author statement
Youssou GNING : Conceptualization; Methodology, Software ;Formal analysis, validation
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Acknowledgments:-

The authors are grateful to the Orsay Institute of Molecular Sciences (OIMS), Paris, France and the Abdus Salam International Center for Theoretical Physics (ICTP), Trieste, Italy, for support.

Conflicts of Interest:
The authors declare no conflicts of interest regarding the publication of this paper.

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