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RESEARCH ARTICLE

STUDY OF SELF-SUPERPOSABLE FLUID MOTIONS IN CONFOCAL PARABOLOIDAL DUCTS

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Abstract

In this paper, studies have been made on some self-superposable motion of incompressible fluid is confocal Paraboloidal ducts. The boundary conditions have been neglected therefore the solutions contain a set of constants. Pressure distribution and the nature of vorticity are discussed. Tendency of irrotationality of the fluid flow is also determined. The aim of the paper is to introduce a method for solving the basic equations of fluid dynamics in confocal paraboloidal coordinates by using the property of self superposability.

Introduction:-

Formulation of Problem:-

For determining a flow of liquid in confocal Paraboloidal ducts let us consider the flow in confocal Paraboloidal coordinates (u, v, w). If q_u, q_v and q_w be the components of $\vec{q}$ at any point (u, v, w) in confocal Paraboloidal coordinates, then in order to make equation of Pressure distribution integrable we may consider the following cases.

Case: I- Let $q_u = 0,$ In this case equation of pressure distribution will be satisfied by a solution.

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{A\,U(u)W(w)}{\sqrt{(v-w)(v-u)}} \\ q_w &= \frac{BU_1(u)V(v)}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \qquad \ldots\ldots\ldots (1)$$

Where, U(u), $U_1(u)$ are integrable functions of $u, V(v)$ and $W(w)$ the integrable functions of v and w respectively, and A,B the constants.

For this fluid velocity, it can be shown that $\vec{p}$ can be represented by the gradient of a scalar quantity θ given by

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$$q=\frac{A^2\left[W(w)\right]^2}{(w-v)}\int\frac{U(u)U'(u)}{(v-u)}du-\frac{B^2\left[V(v)\right]^2}{(w-v)}\int\frac{U_1(u)U'_1(u)du}{(w-u)}+\frac{B^2\left[U_1(u)\right]^2}{(w-u)}\int\frac{V(v)V'(v)dv}{(w-v)}$$

$$-\frac{2ABU(u)U_1(u)W'(w)\sqrt{(a^2-w)(b^2-w)}}{(w-u)}\int\frac{V(v)dv}{(W-v)\sqrt{(a^2-v)(b^2-v)}}+\frac{ABU(u)U_1(u)V'(v)(a^2-v)(b^2-v)}{2(v-u)}$$

$$\int\frac{W(w)dw}{(v-w)\sqrt{(a^2-w)(b^2-w)}}-\frac{A^2\left[U(u)\right]^2}{(v-u)}\int\frac{W(w)W'(w)dw}{(v-w)} \qquad \ldots\ldots\ldots\ldots (2)$$

Now, onwards we will represent $U(u), U_1(u), V(v)$ by U, U_1, V, W respectively and U', U'_1, V', W' represent their differentials. By choosing different suitable sets of values of U, U_1, V and W we may get a number of self-superposable fluid velocities. One of such Velocities can be obtained by taking

$$\left.\begin{aligned} U_1 &= U=\sqrt{(a^2-u)(b^2-u)} \\ V &= \sqrt{(a^2-v)(b^2-v)} \\ W &= \sqrt{(a^2-w)(b^2-w)} \\ B &= 2A \end{aligned}\right\} \qquad \ldots\ldots\ldots (3)$$

The fluid velocity will become

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= A\sqrt{\frac{(a^2-u)(b^2-u)(a^2-w)(b^2-w)}{(v-w)(v-u)}} \\ q_w &= 2A\sqrt{\frac{(a^2-u)(b^2-u)(a^2-v)(b^2-v)}{(w-u)(w-v)}} \end{aligned}\right\} \qquad \ldots\ldots (3a)$$

And

$$q=A^2\left[\frac{(a^2-w)(b^2-w)}{(w-v)}\left\{\left(\frac{a^2+b^2}{2}-v\right)\log(v-u)-u\right\}\right.$$
$$-\frac{4(a^2-v)(b^2-v)}{(w-v)}\left\{\left(\frac{a^2+b^2}{2}-w\right)\log(w-u)-u\right\}$$
$$\left.+\frac{(a^2-u)(b^2-u)(w^2-4v^2+4uv-uw)}{(w-u)(v-u)}\right]\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If U, U_1, V, W are constants then

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{A}{\sqrt{(v-w)(v-u)}} \\ q_w &= \frac{B}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \qquad ....(5)$$

and $\theta = \text{constant}$ …… (6)

Case II:-(i) When $q_v = 0$ In this case the self-superposable flows may be

$$\left.\begin{aligned} q_u &= \frac{A_1V_1W_1}{\sqrt{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= \frac{B_1U_2V}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \qquad …….. (7)$$

And $$\theta = \frac{2A_1B_1V_1V_2W'_1\sqrt{(a^2-w)(b^2-w)}}{(w-v)}\int\frac{Udu}{(w-u)\sqrt{(a^2-u)(b^2-u)}}$$

$$-\frac{B_1^2V_2^2}{(w-v)}\int\frac{U_2U'_2\,du}{(w-u)}+\frac{A^2W_1^2}{(w-u)}\int\frac{V_1V'_1\,dv}{(u-v)}+\frac{B_1^2U_2^2}{(w-u)}\int\frac{V_2V'_2\,dv}{(w-v)}$$

$$+\frac{A^2V_1^2}{(u-v)}\int\frac{W_1W'_1\,dw}{(u-w)}-\frac{A_1B_1V_1V_2U'_2}{2(u-v)}\sqrt{(a^2-u)(b^2-u)}\int\frac{W_1dw}{(u-w)\sqrt{(a^2-w)(b^2-w)}} \qquad …….. (8)$$

(ii)

$$\left.\begin{aligned} q_u &= A_1\sqrt{\frac{(a^2-v)(b^2-v)(a^2-w)(b^2-w)}{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= 2A_1\sqrt{\frac{(a^2-u)(b^2-u)(a^2-v)(b^2-v)}{(w-u)(w-v)}} \end{aligned}\right\} \qquad ……..(9)$$

and

$$\theta = A^2\left[\frac{4(a^2-u)(b^2-u)}{(w-u)}\left\{\left(\frac{a^2+b^2}{2}-w\right)\log(w-v)-v\right\}+\frac{(a^2-v)(b^2-u)(4u^2-w^2-4uv-vw)}{(w-v)(u-v)}\right.$$

$$+\frac{(a^2-w)(b^2-w)}{(w-u)}\left\{\left(\frac{a^2+b^2}{2}-u\right)\log(u-v)-v\right\}\Bigg] \quad \text{........ (10)}$$

(iii)

$$\left.\begin{aligned} q_u &= \frac{C_2}{\sqrt{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= \frac{D_2}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \quad \text{........ (11)}$$

and $\theta = \text{Constant}$ .... (12)

Case III: When $q_w = 0$ Some self-superposable flow may be

$$\left.\begin{aligned} q_u &= \frac{A_2V_3W_2}{\sqrt{(u-v)(u-w)}} \\ q_v &= \frac{B_2U_3W_3}{\sqrt{(v-w)(v-u)}} \\ q_w &= 0 \end{aligned}\right\} \quad \text{........ (13)}$$

and
$$\theta = \frac{B_2^2W_3^2}{(v-w)}\int\frac{U_3U'_3\,du}{(v-u)} - \frac{A_2B_2V_3'W_2W_3\sqrt{(a^2-v)(b^2-v)}}{(v-w)}\int\frac{U_3du}{(v-u)\sqrt{(a^2-u)(b^2-u)}}$$
$$+\frac{A_2B_2U'_3W_3W_2\sqrt{(a^2-u)(b^2-u)}}{(u-w)}\int\frac{V_3dv}{(u-v)\sqrt{(a^2-v)(b^2-v)}}$$
$$-\frac{A_2^2W_2^2}{(u-v)}\int\frac{V_3V'_3\,dv}{(u-v)} + \frac{B_2^2U_3^2}{(v-u)}\int\frac{W_3W'_3\,dw}{(w-v)} - \frac{A_2^2V_3^2}{(v-u)}\int\frac{W_2W'_2\,dw}{(w-u)} \quad \text{....... (14)}$$

(ii)

$$\left.\begin{aligned} q_u &= A_2\sqrt{\frac{(a^2-v)(b^2-v)(a^2-w)(b^2-w)}{(u-v)(u-w)}} \\ q_v &= A_2\sqrt{\frac{(a^2-u)(b^2-u)(a^2-w)(b^2-w)}{(v-w)(v-u)}} \\ q_w &= 0 \end{aligned}\right\} \quad \text{....... (15)}$$

and $$\theta = A_2^2\left[\frac{(a^2-w)(b^2-w)(v-u)(u+v-w)}{(u-v)(v-w)}+\frac{(a^2-u)(b^2-v)}{(v-u)}\left\{w-\left(\frac{a^2+b^2}{2}-v\right)\log(w-v)\right\}\right.$$
$$\left.-\frac{(a^2-v)(b^2-v)}{(v-u)}\left\{w-\left(\frac{a^2+b^2}{2}-u\right)\log(w-u)\right\}\right] \qquad \text{....... (16)}$$

(iii)

$$\left.\begin{aligned} q_u &= \frac{c_3}{\sqrt{(u-v)(u-w)}} \\ q_v &= \frac{D_3}{\sqrt{(v-u)(v-w)}} \\ q_w &= 0 \end{aligned}\right\} \qquad \text{.......(17)}$$

and $\theta = \text{Constant}$ .... (18)

Case IV: When $q_u = q_v = 0$ a possible solution of equation of pressure distribution is given by

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= 0 \\ q_w &= \frac{A_3U_4V_4}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \qquad \text{...... (19)}$$

CASE V: When $q_v = 0, q_w = 0$ the self-superposable flow is given by

$$\left.\begin{aligned} q_u &= \frac{A_4V_5W_4}{\sqrt{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= 0 \end{aligned}\right\} \qquad \text{...... (20)}$$

CASE VI: When $q_w = 0, q_u = 0$ the flow is

$$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{A_5U_5W_5}{\sqrt{(v-u)(v-w)}} \\ q_w &= 0 \end{aligned}\right\} \qquad \text{...... (21)}$$

In all the above cases $U_n, V_n, W_n\ (n = 1, 2, 3, 4, ...)$ are integrable functions of U, V and W respectively and $A_n, B_n, C_n, D_n\ (n = 1, 2, 3, 4, ...)$ are constants which may be determined by boundary conditions.

Superposable Fluid Motion

It has already been shown that the hydrodynamic flows given by equations (1) and (20) are self –superposable. It can also easily be shown that if, $\overrightarrow{q_1}$ and $\overrightarrow{q_2}$ be the two flows given by equations (1) and (20), then $\vec{p}$ for $\overrightarrow{q_1} \pm \overrightarrow{q_2}$ can also be represented by the gradient of scalar quantities. Thus $\overrightarrow{q_1}$ and $\overrightarrow{q_2}$ will be mutually superposable and a flow

$$\left.\begin{aligned} q_u &= \frac{AV(v)W(w)}{\sqrt{(u-v)(u-w)}} \\ q_v &= \frac{BU(u)W(w)}{\sqrt{(v-u)(v-w)}} \\ q_w &= \frac{CU(u)V(v)}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \quad \ldots\ldots (22)$$

is possible. The mass flow can be determined by mutually superposing the flows (7) and (21), (12) and (19).

Pressure Distribution:-

It is interesting to note that θ is nothing but Bernoulli function given by

$$\theta = \frac{q^2}{2g} + h + \frac{p}{g} \quad \ldots\ldots. (23)$$

Where $q, g, h,$ and p denote velocity, acceleration due to gravity height above some horizontal plane of reference and the pressure head. It is a well known fact that for an incompressible fluid the pressure head P is given by

$$p = \frac{p}{p_0} + \text{constant}$$

where p is the pressure distribution.

Also if the motion of the fluid be steady and slow then the value of h can be taken without much loss of generality as u for the flows (1), (4) and (7); v for the flows(9), (11) and (13) and w for (15), (17) and (19).Thus for the flow (7) the pressure distribution is

$$P = K_1 + \frac{1}{2(u-v)(v-w)(w-u)}\left[K_2(w-u) + K_3(u-v) + K_4 u(u-v)(v-w)(w-u)\right] \ldots\ldots (24)$$

Similarly for the flow (19) taking $C_3 = D_3$ we have

$$p = K_3 + \frac{1}{2(u-v)(v-w)(w-u)}\left[K_6(u-v) + K_7(w-v) + K_8 w(u-v)(v-w)(w-u)\right] \ldots\ldots. (25)$$

Where $K_n\left(n = 1,2,3,4,5,6,7,8......\right)$ are constants.

Similarly the pressure distribution for other flows can be determined.

Vorticity of the Flow:-

It was shown by Ballah [3] that for a self-superposable flow, vorticity is constant along its stream lines. If T is a unit tangent along a stream line.

Then $\vec{T} \times \vec{q} = 0$ ……(26)

By equations (16) and (25) it can be readily shown that

$$T = \left[2\sqrt{\frac{(a^2-w)(b^2-w)}{4(a^2-w)(b^2-w)+(a^2-u)(b^2-u)}},\ 0,\ \sqrt{\frac{(a^2-u)(b^2-u)}{4(a^2-w)(b^2-w)+(a^2-u)(b^2-u)}} \right] \text{.....(27)}$$

Hence, the vorticity of the flow (17) is constant along the curve represented by equation (27). Similarly, the curves of constant velocity can also be found for other flows.

Irrotationality:-

Vorticity $\vec{\xi}$ for the flow (15) can be calculated as

$$\vec{\xi} = 4A\sqrt{\frac{(a^2-u)(b^2-v)}{(w-u)(u-v)}}\hat{i}_1 + \frac{2A}{(w-u)}\sqrt{\frac{(a^2-v)(b^2-v)}{(u-v)(v-w)}}\left(a^2+b^2-2u\right)\hat{i}_2$$

$$-\frac{A}{(u-v)}\sqrt{\frac{(a^2-w)(b^2-w)}{(v-w)(w-u)}}\left(a^2+b^2-2u\right)\hat{i}_3 \qquad \text{...... (28)}$$

It is clear from equation (28) that flow (15) is not irrotational. For the flow (7) $\vec{\xi} = 0$

Hence, the flow (7) isirrotational throughout.Similar conclusions can be drawn for the other flows discussed earlier.

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