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RESEARCH ARTICLE

LINEAR VISCOELASTIC MODEL OF BORASSUS WOOD: RHEOLOGICAL PARAMETERS

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Abstract

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The aim of the presentworkis to study the viscoelastic model of Borassus Aethiopum Mart whichis relevant for the characterisation of a Borassus Pre-stressedConcretebeam. Mechanical tests werecarried out; mainly the creep test on typical Borassus Aethiopum Mart specimensfrom the Pahou-Ahozon forestgallery in southern Benin, Africa, subjected to a constant bending stress along the beam. The moisture content of the specimensis set at 12% and isobtained by kilndrying at ATC du Bois. The loadinglevel of the specimensis set at 20% of the failureload. A methodologystartingfrom the determination of the compressive and bending stresses of Borassus Aethiopum Mart for the identification of the optimal values of the dynamicelasticmodulus E and the dynamicviscosity constant η of Borassus Aethiopum Mart with the non-linear least squares methodapplied to the Kelvin Voigt rheological model. The instantaneousmodulus of elasticity E0 wasdeterminedfromHooke'slaw. A series of validation tests wereperformed on the numerical model. Finally, a comparison of the numerical model with the rheologicalrepresentationconfirmed the previousresults.

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Introduction:-

In the field of construction, woodis the materialwhose use ismostbeneficial to the environmentbecauseits production requireslessenergythanothermaterials. It is one of the naturalproductswhose use is essential for the environment. It isused in manyways to meet the differentneeds of society. It is a renewable and biodegradablematerial and the process of obtainingitisless time-consumingthanthat of someothermaterials. The sustainable use of woodallows the storage of excesscarbon in the atmosphere. Wood isalso a materialthatcanbeusedstructurally, especially in the construction field. It is a naturalmaterialwhosephysical and mechanicalcharacteristicscanbesignificantlyinfluenced by the level of stress and duration of loading. Therefore, a good knowledge of itsphysical and mechanicalpropertiesis essential. However, studies on the instantaneous and delayedbehaviour of wood are less extensive, sothat the information isalmostsuperficial. It isnecessary to test itunderdifferent types of loading. According to somepreviousstudies on wood, several aspects are not takenintoaccountfrompurelyelasticmodellingeven in the case of lowloads. This being the case, several investigations have been undertaken and have led to the conclusion that the deferredbehaviour of woodislinearviscoelastic for stresses in the range]0% ;35%] of the failure stress. Thus the deferredbehaviour of woodcanberepresented by a

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Kelvin Voigt model. The aim of thisstudyistherefore to investigate the viscoelastic model of Borassus Aethiopum Mart (roastwood) from Benin.

Material and Methods:-
Material:-
The equipmentused for the creep test ismainlycomposed of a bracket and a suspension plate. This equipmentisidentical to thatproposed by Professor A. FOUDJET in hisworkin 1986.

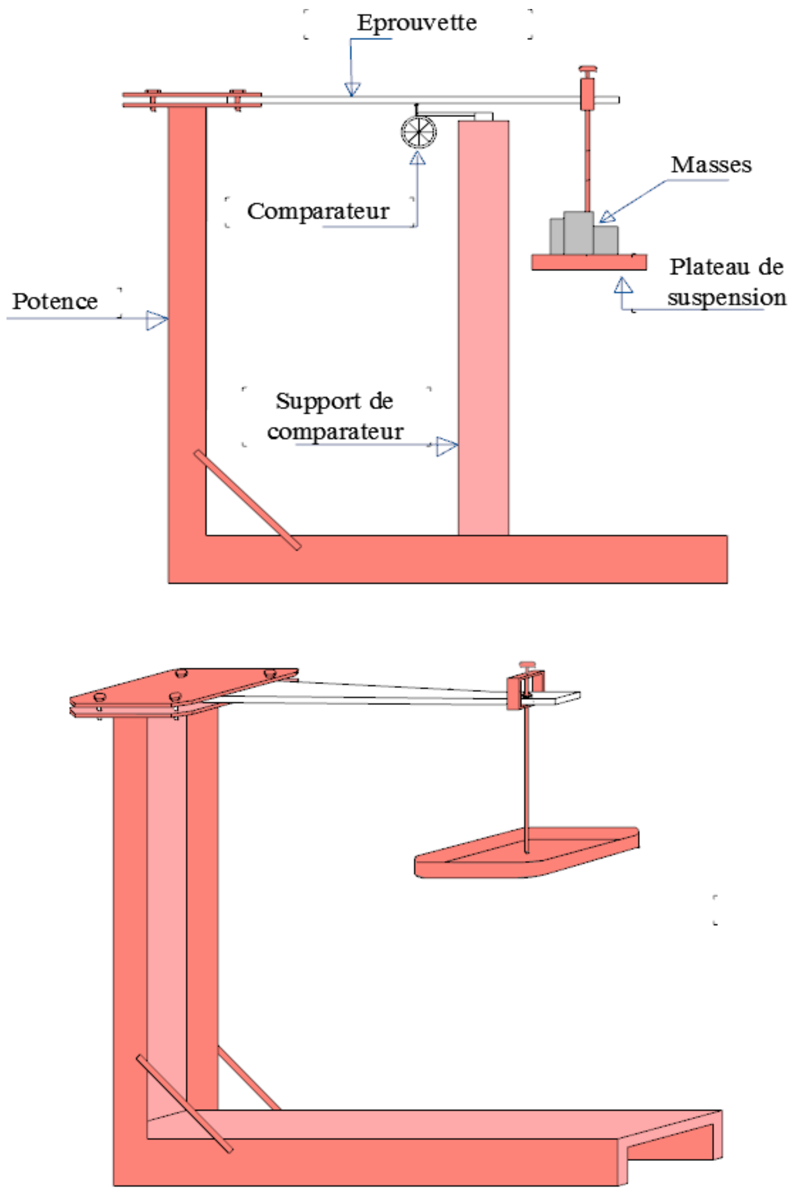

Figure 1:- Creep test material.

Plant material
The specimensdesigned for the creep tests weretakenfrom the outer crown of Borassus logs in the longitudinal direction. Thus, nine (09) specimens of iso stress formwerecollectedaccording to the configuration in Figure 2. For ourstudy, the specimens have a moisture content of 12% obtained by drying in the ATC du Bois company in Allada (Benin) and are carefullywrapped in aluminium foil. During the entire test period, whichlasted 15 hours, the specimensweregweighedimmediately. The test specimensweregweighedimmediatelybefore and after the test. To ensure a smoothoperation, weadopted the practicalformpresentedbelow.

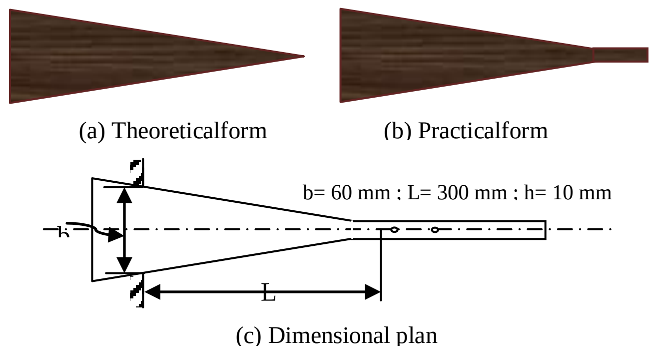

Figure 2:-Iso stress specimen of two-point creep test.

Methods:-

Principle of the test

The test consists of subjecting Borassus specimens to two-point bendingwith a uniform maximum stress on the fibres. During the test, a concentratedloadisapplied to the free end of the specimen. The extreme fibre stress isthenmaintained at 20% of the bendingfailure stress, i.e. 18.90MPa. The deflectionisreadevery 30 minutes at a distance of 150 mm from the end on which the loadisplaced, by means of a dial indicator, whoseresolution and travel are 0.01 mm and 10 mm respectively.

Kelvin-Voigt model

The model consists of a series of Kelvin-Voigt models and a spring. The springcharacterises the instantaneousdeformation, hence the instantaneouselasticmodulus E0; and the Kelvin-Voigt model characterises the creep, hence the dynamicelasticmodulus E and the constant dynamicviscosity η.

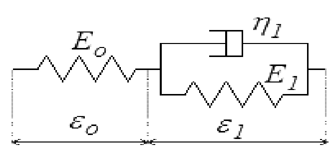

Figure 3:- Kelvin-Voigt model.

Wewillthereforewritewith the natural notations :$\varepsilon = \varepsilon_0 + \varepsilon_1$; $\sigma_0 = \varepsilon_0 E_0 = \varepsilon_1 E + \eta \frac{d\varepsilon_1}{dt}$ (1)

Method for determining the deformation ε

The expression for the longitudinal deformation of the beamisestablishedfrom the geometry of the deformedbeam and isdescribed as follows : $\varepsilon = -\frac{2fy}{L^2+f^2}$ (2)

Where: ε: beamdeflection, L: beamlength, y: distance from the tension fibre to the neutral axis.

The position of the neutral axis with respect to the tension fibre isdeterminedusing the trapezoidal normal stress distribution model on a complete surface proposed by PRAGER.

Based on the Navier-Bernoulli assumption and assumingthat the squared moment of the beam section is constant. The followingrelationshipisestablished :$\frac{y}{h} = \frac{2.\sigma_{uc}\sigma_{ut}}{(\sigma_{uc}+\sigma_{ut})^2}$ (3)

Where$\sigma_{ut}$is the tensile stress; $\sigma_{uc}$is the compressive stress; h is the height of the beam; y is the distance from the neutral axis of a tensioned fibre.

Withequations (2) and (3) we have :$\varepsilon = 4h \frac{f}{f^2+L^2} \frac{\sigma_{ut}\sigma_{uc}}{(\sigma_{ut}+\sigma_{uc})^2}$ (4)

The method of identifyingparameters

Using the non-linear least squares method and based on equation (5), the parameters η and E of the linearviscoelastic behavior law : $F(t,a,b) = \frac{\sigma_0}{a}\left(1 - e^{-\frac{a}{b}t}\right)$ Aveca $= E$, $b = \eta$ett ≥ 0 (5)

Weadjust the measureddeformations in order to minimize the distances di as follows:$di = \sum_{i=1}^{n}[y_i - F(t,a,b)]^2$(6)

By means of equation (6) we have the iterativeformulae of equation (7) below:

$$\begin{cases} a_{j+1} = a_j + \dfrac{(\sum_{i=1}^{n} Ay_i - \sum_{i=1}^{n} AC)\sum_{i=1}^{n} B^2 - \sum_{i=1}^{n} AB(\sum_{i=1}^{n} By_i - \sum_{i=1}^{n} BC)}{\sum_{i=1}^{n} A^2 \sum_{i=1}^{n} B^2 - (\sum_{i=1}^{n} AB)^2} \\ b_{j+1} = b_j + \dfrac{\sum_{i=1}^{n} A^2(\sum_{i=1}^{n} By_i - \sum_{i=1}^{n} BC) - (\sum_{i=1}^{n} Ay_i - \sum_{i=1}^{n} AC)\sum_{i=1}^{n} AB}{\sum_{i=1}^{n} A^2 \sum_{i=1}^{n} B^2 - (\sum_{i=1}^{n} AB)^2} \end{cases}$$

With :

$$A = \frac{\partial}{\partial a}F(t_i, a_0, b_0) = \left[-\frac{1}{{a_0}^2}\left(1 - e^{-\frac{a_0}{b_0}t_i}\right) + \frac{t_i}{a_0 b_0}e^{-\frac{a_0}{b_0}t_i}\right]\sigma_0$$

$$B = \frac{\partial}{\partial b}F(t_i, a_0, b_0) = \left[-\frac{t_i}{{b_0}^2}e^{-\frac{a_0}{b_0}t_i}\right]\sigma_0$$

$$C = F(t_i, a_0, b_0) = \left[-\frac{1}{a_0}\left(1 - e^{-\frac{a_0}{b_0}t_i}\right)\right]\sigma_0$$

Withequation (7), and according to the iterationstoppingcriteriondefined for eachparameter, the optimal values a and b weredetermined.

Iterationstoppingcriterion :$\frac{a_{j+1} - a_j}{a_j} < 10^{-5}$ et$\frac{b_{j+1} - b_j}{b_j} < 10^{-5}$

Results And Discussion:-

Table 1:- Key parameters of the creep test.

N°	**Key parameters**	**Experimental values**	**Numerical values**
1	Moisture content (%)	12	12
2	Initial strain ε_0(%)	0,324	0,319
3	Final strainε_f(%)	0,341	0,348
4	Creep coefficient K	1,05	1,09
5	Truecreep (%)	0,017	0,029
6	Relative creep	0,052	0,091
7	Request rate (%)	20	20
8	Instantaneouselasticmodulus E_0 (MPa)	5835	5935

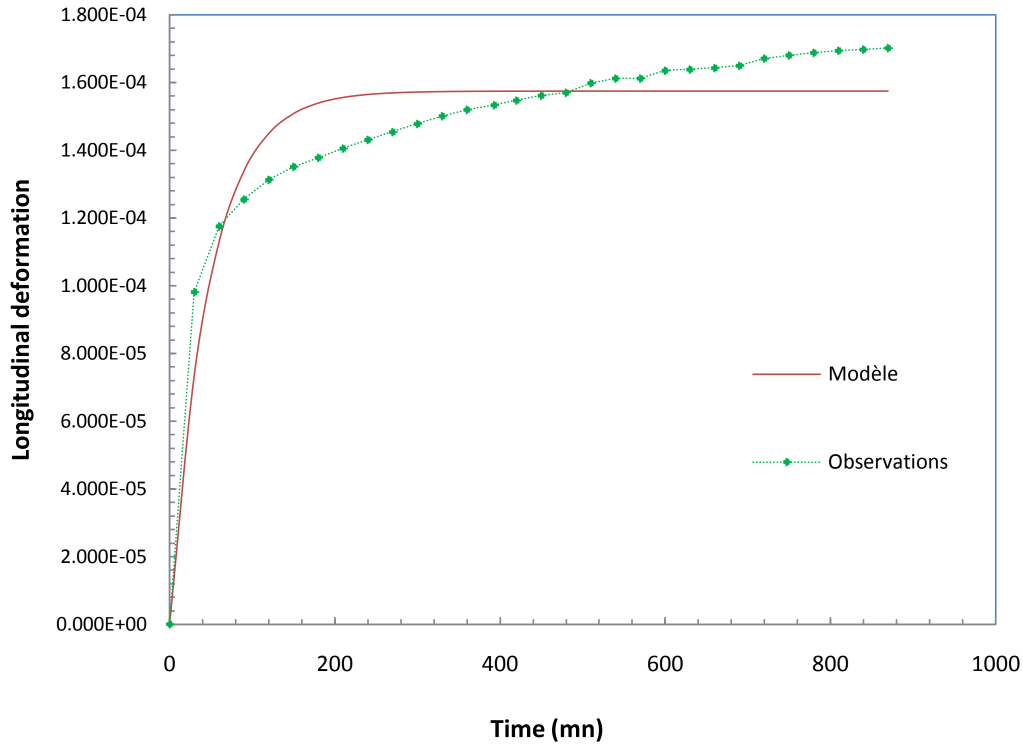

Figure 4:- Longitudinal deformationcurves of the Borassus over time.

In view of the resultsobtained, Table 1 presents the main characteristics of the creep test, such as the initial deformation ε0, the final deformation f, the creep coefficient K, the naturalcreep Δε and the instantaneousmodulus E0. The differentstudies are used to model the linearviscoelasticbehaviour of wood and to validate the model. Figure 4 shows the experimental and simulated curves of the studied Borassus woodwith respect to the evolution of the longitudinal deformation.

Table 2:- Optimal values of the rheologicalparameters of the model.

N°	Rheologicalparameters		Optimal values
1	Dynamicmodulus of elasticity E	MPa	120 000
2	Dynamicviscosity constant η	MPa.mn	5 700 000

The optimal values of the rheologicalparameters (E and η) of the model obtainedwith a precision of 10^{-7} by minimizing equation (6) and using Excel software werepresented in Table 2.

Tables 3 and 4 belowpresent the statistical values of the tests for eachparameter as well as the different t-distribution tests carried out using the t-distribution law and considering a 1% risk; in order to confirm the values obtained. It shouldbenotedthat the level of precision of the resultsdoes not require the determination of the confidence interval of the values

Table 3:- Statistical values.

Indicators			
Nomenclature	**Formula**	**Description**	**Result**
S_{xx}	$\sum_{i=1}^{n}(x_i - \bar{x})^2$	Sum of squares of x	2,022E+06

SS_E	$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$	Errorsum of squares	8,71058E-12
SS_R	$\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$	Regressionsum of squares	8,62718E-11
SS_T	$\sum_{i=1}^{n}(y_i - \bar{y})^2$	Total correctedsum of squares	8,62389E-11
$\hat{\sigma}^2$	$\frac{SS_E}{n-2}$	Unbiaisedestimator	3,11092E-13
se ($\hat{\beta}_2$)	$\sqrt{\frac{\hat{\sigma}^2}{Sxx}}$	Estimated standard error of the slope	3,92194E-10
se ($\hat{\beta}_1$)	$\sqrt{\hat{\sigma}^2(\frac{1}{n} + \frac{\bar{x}^2}{Sxx})}$	Estimated standard error of the intercept	1,98898E-07
$\hat{\beta}_1$	E	Estimator de E	12E+04
$\hat{\beta}_2$	H	Estimator de η	5,7E+06

The number n of observations isequal to 30.

Table 4:- Statistical validation of parameters$\hat{\beta}_1$ and $\hat{\beta}_2$

Type of parameter	Statistical test	Data
parameter$\hat{\beta}_2$	**Hypotheses**	$H_0: \hat{\beta}_2 = 0$; $H_1: \hat{\beta}_2 \neq 0$
	$\mathbf{t_0 = \frac{\hat{\beta}_2}{se(\hat{\beta}_2)}}$	1,4534E+16
	For α=1% and using a t-distribution, we have : $\mathbf{t_{\alpha/2,(n-2)}}$	2,763
	Conclusion	$t_0 > t_{\alpha/2,(n-2)}$ therefore we reject H_0.
		$\hat{\beta}_2$ =5,700E+06
parameter$\hat{\beta}_1$	**Hypotheses**	$H_0: \hat{\beta}_1 = 0$ $H_1: \hat{\beta}_1 \neq 0$
	$\mathbf{t_0 = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)}}$	6,0332E+11
	For α=1% and using a t-distribution, we have : $\mathbf{t_{\alpha/2,(n-2)}}$	2,763
	Conclusion	$t_0 > t_{\alpha/2,(n-2)}$thereforewerejecH_0.
		Hence$\hat{\beta}_1$ =120 000

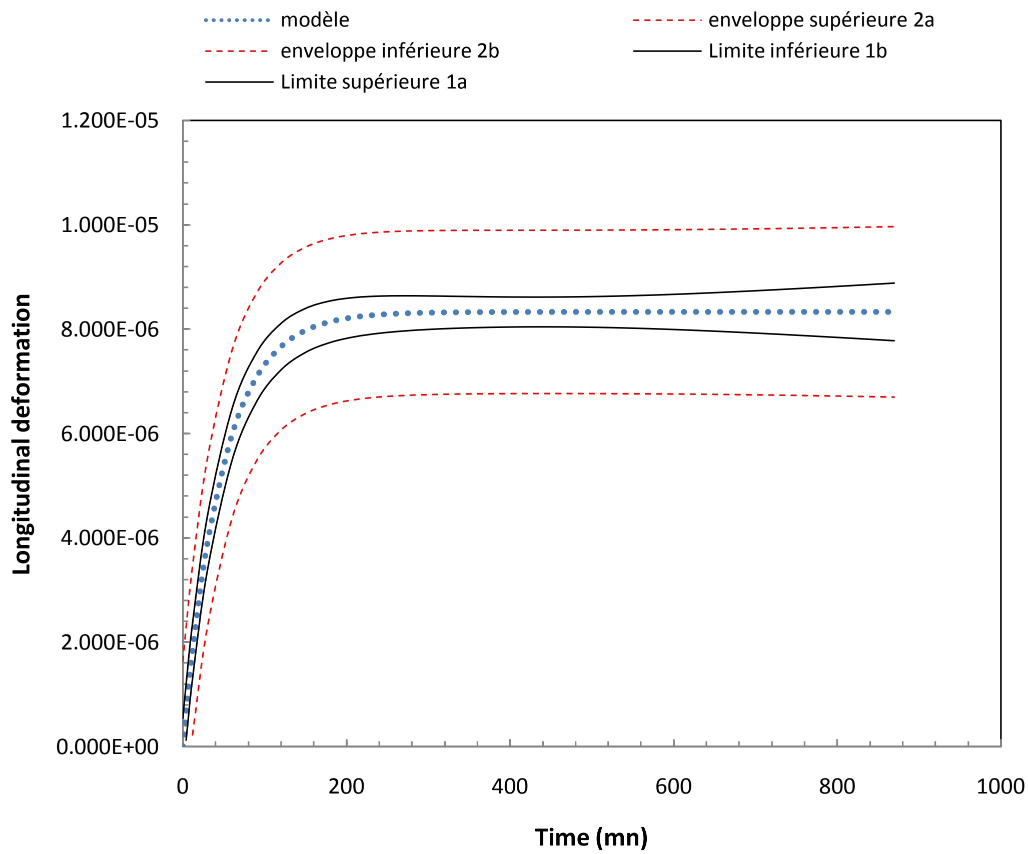

Figure 5:- Envelopecurves of the linearviscoelastic model prediction.

Table 5:- Boundary values of the rheologicalparameters of the predictionfields.

Curves	**Parameters**	
	E^*	η^*
1a	112 400	5 700E+03
1b	133 200	5 700E+03
2a	105 840	5 700E+03
2b	148 500	5 700E+03

Figure 5 shows the predictionfieldboundaries of the linearviscoelastic model which are plottedusing the statisticallaw. These are the 99% confidence interval on the solid line regression and the 99% predictioninterval for a dashed line future observation. Table 5 summarises all the rheologicalparameters characterizing eachlimit.

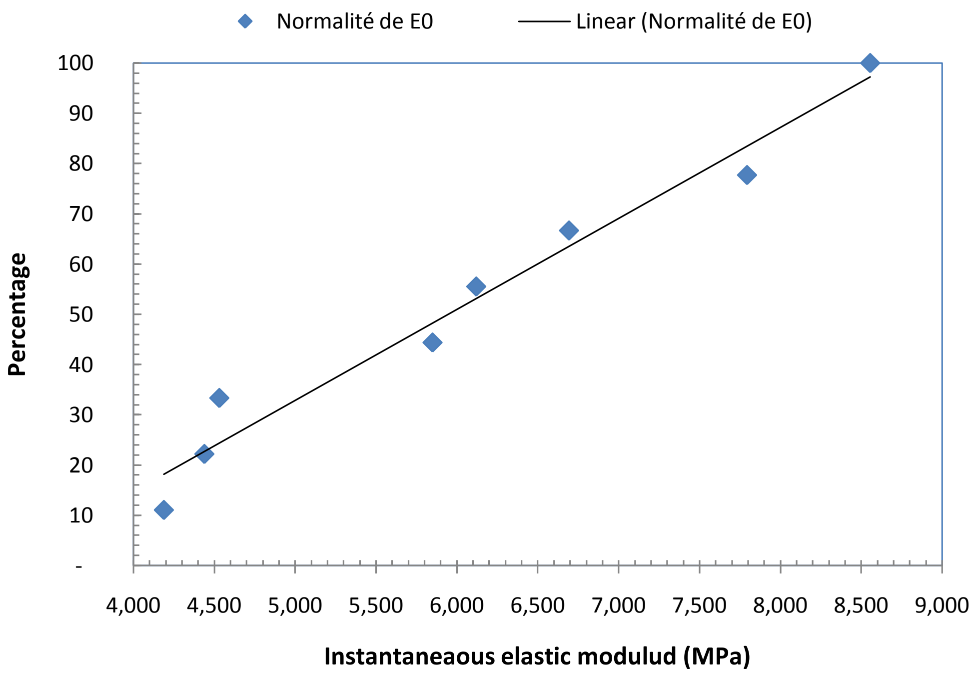

Figure 6:- Normalitycurve of the instantaneousmodulus of elasticity.

Figure 6 shows the normalitycurve of the instantaneouselasticmodulus. This probabilitydiagram of the distribution of the instantaneouselasticmodulus values calculatedusingHOOKE'slaw shows that the values are normallydistributed and validates the instantaneous elasticmoduluspresented in Table 1.

Table 6:- Coefficient of determination R^2

Statistical test		**Data**
Coefficient of determination R^2	**Formula**	$R^2 = 1 - \frac{SS_E}{SS_T}$
	Value	89,9%
Interpretation		The model helpsaccount for 89,9% of the explained values

Table 7:- Values of statisticalindicators.

Indicators			**Data**
Notation	**Expression**	**Designation**	**Calculated values**
S	$\sum_{i=1}^{n} E_{0i}$	Sum	56715,573
$\overline{E}_0$	$\frac{S}{n}$	Samplemean	6301,730303
S_{xx}	$\sum_{i=1}^{n} (E_{0i} - \overline{E}_0)^2$	Sum of the squares of sampledeviation	2022497
S^2	$\frac{S_{xx}}{n-1}$	Sample variance	252812,125
S	$\sqrt{S^2}$	Sample standard deviation	502,804

The number of observations n isequal to 9.

Table 8:- Validation of E0 and values of the lower and upperbounds of the confidence and predictioninterval.

	Statistical values	**Data**
VALIDATION OF E_0	**Hypotheses**	$H_0 : E_0 = \mu$ $H_1 : E_0 > \mu$, μ =5835
	$\mathbf{t_0} = \frac{E_0 - \mu}{\mathbf{S}/\sqrt{n}}$	2,785
	For α=1% and using t-distribution, we have : $\mathbf{t_{\alpha/2,(n-1)}}$= 3,106	
	Conclusion	$t_0 < t_{\alpha(n-1)}$ therefore we accept H_0.
		HenceE_0 =5835
CONFIDENCE INTERVAL	**For α=1% and using t-distribution, we have : $\mathbf{t_{\alpha/2,(n-1)}}$**	
	$\Delta\mathbf{E} = \mathbf{t_{\alpha/2,(n-1)}} \cdot \frac{S}{\sqrt{n}}$	521
	E_0	5835
	Lowerbound	5314
	Upperbound	6356
PREDICTION INTERVAL	**For α=1% and using t-distribution, we have : $\mathbf{t_{\alpha/2,(n-1)}}$**	
	$\Delta\mathbf{E} = \mathbf{t_{\alpha/2,(n-1)}} . S\sqrt{1 + \frac{1}{n}}$	1646
	E_0	5835
	Lowerbound	4189
	Upperbound	7481

Table 6 presents the first test of the model's fit by calculating the coefficient of determination for Borassus Aethiopum Mart. According to the results, the model accounts for 89.90% of the variability in the data.

Tables 7 and 8 present the values of the statisticalindicators on the determination of the mean, the confidence interval and a prediction of future observation. The validation of the mean value of the instantaneouselasticmoduluswasbased on the t-distribution lawwith a 1% risk for verification. The confidence and predictionintervalswerealsovalidatedusing the t-distribution table with a 1% risk of verification.

Conclusion:-

This studycarried out on Borassus woodthroughtwo-point bendingcreep tests made it possible to determine the rheologicalparameters of the linearviscoelasticbehaviour of the latter. The model used in thisstudywasthat of Kelvin Voigt, and the characteristics of the basic bricks, namely the spring and the damper, weredetermined, reflecting the instantaneous and delayedbehaviour of the materialrespectively. Fromthiswork, some key indicators of Borassus woodsuch as the creep coefficient, the instantaneousmodulus of elasticity etc. werealsofound. This study of the deferredbehaviour of woodwillthereforebenecessary for the prediction of future deformations of a structural element made of Borassus wood. Finally, the resultsshowedthat the free flow of Borassus woodcanbecharacterised in the evaluation of the linearviscoelasticbehaviour by adopting the Kelvin-Voigt rheological model with the bendingcreep test whenperformed for a maximum loading time of 15 hours.

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