RESEARCH ARTICLE

MODAL DISPERSION CURVES OF A METAL COATED OPTICAL WAVEGUIDE WITH TREFOIL CROSS-SECTION UNDER WEAK GUIDANCE APPROXIMATION

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Manuscript Info

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Abstract

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In this paper the author investigated a special type of core cross-section bounded by metallic boundary. In this structure three cores embedded in a common metallic boundary. Characteristic equation in derived for the guided modes under weak guidance approximation by choosing an appropriate coordinate system. This study in done by using analytical method. From the characteristic equation, dispersion curves are obtained and interpreted.

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Introduction:-

In the beginning, only two type of waveguides were used in optical range -waveguides with circular cross-section [1-2] and planner waveguides [3-4]. Now, optical waveguides with various non-circular cross-sections like rectangular and elliptical, triangular, have been studied by many researchers. The analysis of waveguides with non circular cross-sections is generally difficult, and usually approximate or numerical methods are employed [5-8] in such investigations. In the present paper author treat a very special type of cross-section of the shape of a trefoil. If three cores with such a cross-section is embedded in a common boundary, the structure is similar to a waveguide with three circular cores are embedded in a common boundary, each circular loop represents the cross-section of one core. Due to the presence of neighbouring circular loops, these circular loops are distorted into flatten shape. In this proposed waveguide the boundary in highly conducting. In order to obtain the number of modes, which can propagate through such a fiber for a given 'V' parameter, characteristics equation for the guided modes in derived. We obtain the dispersion curves for some lower order modes by the investigating of this characteristic equation. It is found that the dispersion curves are of the expected shapes, and these dispersion curves can be used to determine the number of modes which a fiber of this type can sustain at a known operating frequency for a given size parameter and fixed value of care refractive index.

Theory

We consider an optical fiber with trefoil like cross-section bounded by highly conducting material, represent by given equation in polar coordinates, as

$$r^3 = \xi^3 \sin 3\theta \qquad \text{...... (1)}$$

Its normal curve equation is

$$r^3 = \eta^3 \cos 3\theta \qquad \text{...... (2)}$$

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The basic structure and its normal curve are shown in fig. 1 and fig. 2. For the analysis and description of different modes in such a fiber, the choice of cylindrical polar coordinates in not suitable. To investigate of proposed waveguide, we choose a new coordinate set (ξ, η, z) which is appropriate for this geometry.

The scalar factors can easily be derived.

$$h_1 = \frac{\eta^4}{\left(\xi^6 + \eta^6\right)^{2\backslash 3}} \qquad \text{....... (3)}$$

$$h_2 = \frac{\xi^4}{\left(\xi^6 + \eta^6\right)^{2\backslash 3}} \qquad \text{....... (4)}$$

$$h_3 = 1 \qquad \text{....... (5)}$$

Now assuming a harmonic time dependence of the electric field, the vector wave equation for the electric field $\vec{E}$ is

$$\nabla^2 \vec{E} + \frac{n^2 \omega^2}{c^2} \vec{E} = 0 \qquad \text{....... (6)}$$

Using the weak guidance approximation, equation (6) can be reduced to a scalar wave equation.

$$\nabla^2 \psi + \omega^2 \mu_0 \varepsilon \psi = 0 \qquad \text{........ (7)}$$

Where μ_0 and ε are permeability of free-space and the permittivity of the medium.

The scalar wave equation in new coordinates is

$$\frac{\left(\xi^6 + \eta^6\right)^{4/3}}{\eta^4 \xi^4} \frac{\partial}{\partial \xi}\left\{\frac{\xi^4}{\eta^4} \frac{\partial \psi}{\delta \xi}\right\} + \frac{\left(\xi^6 + \eta^6\right)^{4/3}}{\eta^4 \xi^4} \frac{\partial}{\partial \eta}\left\{\frac{\eta^4}{\xi^4} \frac{\partial \psi}{\partial \eta}\right\}$$

$$+ \frac{\left(\xi^6 + \eta^6\right)^{4/3}}{\eta^4 \xi^4} \frac{\partial}{\partial z}\left\{\frac{\xi^4 \eta^4}{\left(\xi^6 + \eta^6\right)^{4/3}} \frac{\partial \psi}{\partial z}\right\} + \omega^2 \mu_0 \varepsilon \psi = 0 \qquad \text{........ (8)}$$

Using separation of variables technique we obtain

$$\frac{\left(\xi^6 + \eta^6\right)^{4/3}}{\eta^8}\left[\frac{4}{\xi} \cdot \frac{1}{F_1} \frac{dF_1}{d\xi} + \frac{1}{F_1} \frac{d^2 F_1}{d\xi^2}\right] + \frac{\left(\xi^6 + \eta^6\right)^{4/3}}{\xi^8}\left[\frac{4}{\eta} \frac{1}{F_2} \frac{dF_2}{d\eta} + \frac{1}{F_2} \frac{d^2 F_2}{d\eta^2}\right]$$

$$+ \omega^2 \mu_0 \varepsilon - \beta^2 = 0 \qquad \text{....... (9)}$$

Where β is unknown constant. To separate η and ξ, we consider two special cases. Firstly, we assume $\xi \gg \eta$ and we get following two equations.

$$\frac{d^2 F_1}{d\xi^2} + \frac{4}{\xi} \frac{dF_1}{d\xi} + \frac{m}{\xi^8} F_1 = 0 \qquad \text{........ (10)}$$

$$\frac{d^2 F_2}{d\eta^2} + \frac{4}{\eta} \cdot \frac{dF_2}{d\eta} + \left[\omega^2 \mu_0 \varepsilon - \beta^2 - \frac{m}{\eta^8}\right] F_2 = 0 \qquad \text{....... (11)}$$

For $\eta \gg \xi$

$$\frac{d^2F_1}{d\xi^2} + \frac{4}{\xi}\frac{dF_1}{d\xi} + \left[\omega^2\mu_0\varepsilon - \beta^2 + \frac{\alpha}{\xi^8}\right]F_1 = 0 \qquad \text{....... (12)}$$

$$\frac{d^2F_2}{d\eta^2} + \frac{4}{\eta}\frac{dF_2}{d\eta} - \frac{\alpha}{\eta^8}F_2 = 0 \qquad \text{....... (13)}$$

Here '**m**' and 'α' are constants we observe. When $\alpha = 0$, $m = 0$ equation (11) and (12) coincide and becomes

$$\frac{d^2F_1}{d\xi^2} + \frac{4}{\xi}\frac{dF_1}{d\xi} + \left(\omega^2\mu_0\varepsilon - \beta^2\right)F_1 = 0 \qquad \text{....... (14)}$$

Equation (14) can be solved by using Frobenius method, we get two independent series solutions,

$$F_{11} = 3C_0\left[\frac{1}{3} - \frac{\delta}{2\cdot3\cdot5}\xi^2 + \frac{\delta^2}{2\cdot3\cdot4\cdot5\cdot7}\xi^4 - \frac{\delta^3}{2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot9}\xi^6 + ...\right] \qquad \text{....... (15)}$$

$$F_{12} = C_0\xi^{-3}\left[1 + \frac{\delta}{2}\xi^2 - \frac{\delta^2}{2\cdot4}\xi^4 + \frac{\delta^3}{2\cdot3\cdot4\cdot6}\xi^6 - ....\right] \qquad \text{...... (16)}$$

Where $\delta = \omega^2\mu_0 \in -\beta^2$

Out of these two series solutions F_{12} approaches infinity as $\xi \to 0$. This solution is not physically acceptable. Hence the solution in core region given by eq.(15) can be written as

$$F_{11}(\xi) = 3C_0\sum_{n=0}^{\infty}\frac{(-1)^n\left\{\left(\sqrt{\delta}\xi\right)^2\right\}^n}{2^n n!\prod_{l=0}^{n}(2l+3)} \qquad \text{....... (17)}$$

Now we apply the boundary conditions to determine the characteristic equation. According to this the wave function ψ must be zero at boundary, as boundary is metallic.

$$\left[F_{11}(\xi)\right]_{\xi=a} = 0 \qquad \text{...... (18)}$$

$$\Delta = F_{11}(a)$$

$$\Delta = \sum_{n=0}^{\infty}\frac{(-1)^n\left\{\left(n_1^2k^2 - \beta^2\right)a^2\right\}^n}{2^n n!\prod_{l=0}^{n}(2l+3)} = 0 \qquad \text{...... (19)}$$

This equation (19) is known as modal characteristic equation of the waveguide under consideration.

Numerical Computation
We now make some numerical estimates of the modal properties of a fiber with trefoil cross-section having metallic boundary. The obvious step is to determine the value of propagation constant β by solving the equation (19). We now choose the refractive index of core $n_1 = 1.48$. The wavelength of wave is fixed at $\lambda_0 = 1.55\,\mu$. In order to obtain the dispersion curves we choose a given value of 'a' and then plot the L.H.S of eq.(19) against different value of β having range $n_1k_0 > \beta > 0$, where $k_0 = \frac{2\pi}{\lambda_0}$. The intersections of curve with the β axis give us possible β values of different modes. This procedure is repeated for different values of size parameter 'a'. We next plot the β

values for each mode against V, where 'V' is normalized frequency $V = \dfrac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2}$ and obtain the dispersion curves. They contain all the informations about the guiding modes.

Result And Discussion:-

In figure (3) one can see the dispersion curves for various modes sustained by a fiber with a trefoil cross-section. Here 'V' parameter is plotted along the abscissa and the normalised propagation constant 'b' is plotted along the ordinate. The possible number of modes can be determined by these curves for the proposed waveguide. From these curves we see that when $V < 5$. There is a only a single mode sustain by the guide and when $V = 7$ there are two guided modes. When the value of V parameter increases, the number of modes also increase, an expected. We have five modes when $V < 19$. The dispersion curve for the lowest mode in fig.(3) does not begin at $V = 0$, this anomaly appears because of the singularity of curve for $a = 0 (V = 0)$. For all the dielectric fiber, however, the fundamental mode always exists.
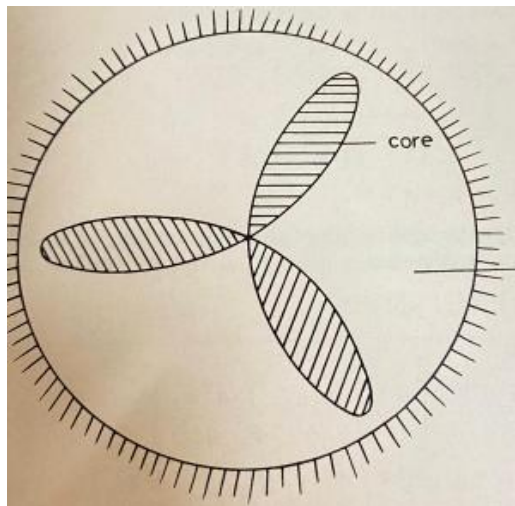

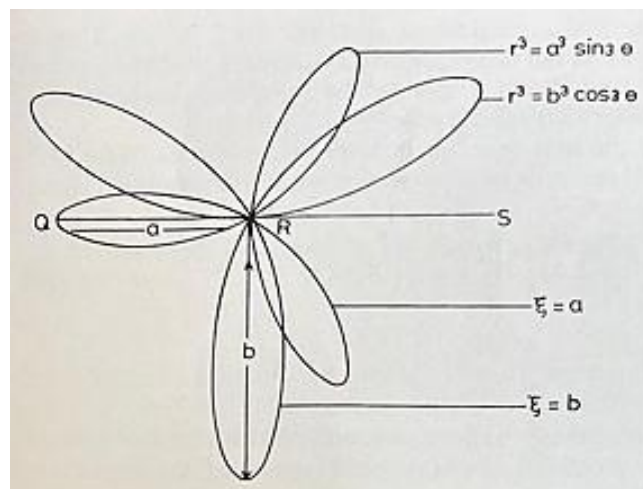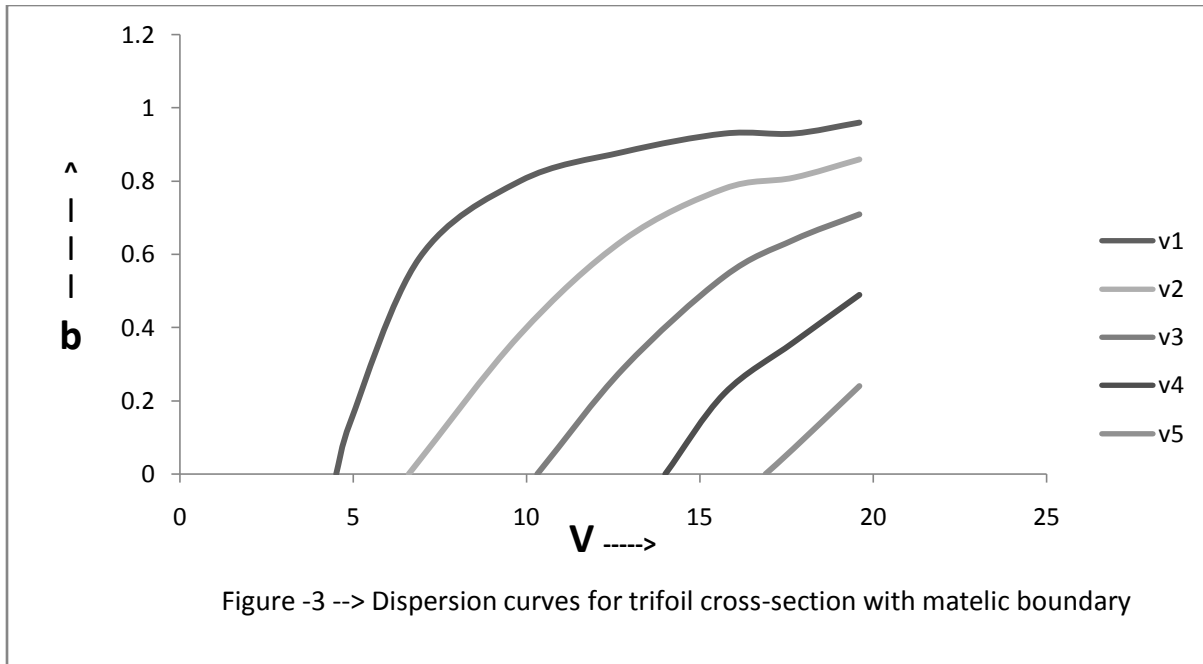**Fig. 1:-** An optical fiber with a core having a trefoil cross-section.

Fig.2:- The curve $r^3 = a^3 \sin 3\theta$ with the orthogonal curve $r^3 = b^3 \cos 3\theta$

Figure -3 --> Dispersion curves for trifoil cross-section with matelic boundary

Refrence:-

1. C.D. Hussey and G. Pask, Electron Lett., 17 p. 644, (1981)
2. I.C. Goyal, J.IETE, 32, p. 196, (1986)
3. M.P.S Rao, B. Prasad, P. khastgir and S.P. Ojha, Microwave and Optical Tech. Lett., 14, p. 177, (1997).
4. Shibukawa and M. Kobayashi, Apple. Opt., 20, p. 2444, (1981)
5. A.K. Shahi, V.Singh, S.P. Ojha, Progress in Electromagnetic Research, PIER 75, 51-62, 2007. Sharma, Optik, 122, p.1535-1537, 2011.
6. Kh.S Singh, P.K. Choudhury, V. Misra., P. Khastgir and S.P. ojha, J.phy. Soc. Japan, 62, p. 3778, (1993)
7. S. Ganguly. Kh.S Singh, S.P. Ojha and P.Khastgir, Microwave Opt. Technol. Lett., 8, p. 18, (1995).
8. R.B. Dyott, Electron. Lett., 26, p. 1721, (1990).
9. K.M. Flood and D.L. Jaggard, IEEEJ. Quantum Electron, p. 339, (1994).