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RESEARCH ARTICLE

WORKING WITH MATHEMATICAL PROBLEMS: AN ANALYSIS OF STUDENTS' MISCONCEPTIONS AND ITS IMPACT ON MATHEMATICS LEARNING

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Abstract

The great misconception about mathematics is the notion that mathematics is about formulas, solving word problems, and doing computations. Hence, it is the impetus for this study to explore why so many students havedifficulty learning mathematics. To achieve this goal, this study focuses on why so many students keep making the same errors over a long period of time. Generally, among the errors committed by the students in solving word problems, it was found out that students usually made encoding errors. These errors were the result of carelessness, rushing through a problem or misreading a problem. These students correctly work out the solution, but cannot express this solution in an acceptable written form.Moreover, this study stresses that one of the foremost problems encountered by the students was their inability to understand the language used in mathematics, which is English. For some students, mathematical disability was a result of problems with the language of mathematics. Students had difficulty in understanding mathematical terminologies which normally were not used outside the mathematics lesson. Furthermore, lack of comprehension of the students in algebraic expressions concepts and operations leads to an error in translating mathematical phrase into mathematical symbol. This was due to insufficient understanding of mathematical expressions and poor skills in mathematical translation. The conclusions drawn from this investigation strongly justify the needs for mathematics teachers to give more emphasis on students' learning in mathematical concepts. They must also need to be empowered in order to help the learners to be conversant in the mathematical language. The study has demonstrated that mathematical language plays a vital role in learners' comprehension of word problems, hence the language that is used in mathematical word problems needs to be taken into cognizance.

Introduction:-

"Diagnose before you dose" is a rule that is applied to medicine. However, this is equally important in teaching and learning mathematics as well. There were several procedures to diagnose student errors in mathematics. Observation of students' work and careful scrutiny of their solution is one way to understand the logic behind their thinking that led to an error. According to Canziz (2011), it is considerably important to identify and correct the mistakes made by

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students about mathematical concepts. One way of trying to find out what makes mathematics difficult is to identify the kinds of errors students commonly commit and then investigate the reasons for these errors. Likewise, if the reasons that students misunderstand mathematical concepts can be well understood, it is helpful to design remedial measures to avoid the misconceptions.

To investigate the reasons behind misunderstandings, we have to inquire deep into students' minds. Some errors are persistent, so that they will occur due to flawed conceptual knowledge (misconceptions) which are amenable to analysis and robust to change, rather than the random errors that merely occur due to human fallibility. Therefore, examining deep into students' thinking and their beliefs is necessary to find reasons for them to make these misconceptions.

Error analysis in mathematics teaching strives to identify the nature of error a learner may commit in dealing with mathematical problems. An analysis of these patterns will provide mathematics educators insights into how students use algorithms to calculate and solve mathematical problems. Checking not only the students' final answers but also the process on how they arrive at these answers help teachers pinpoint students' conceptions as well as misconceptions. Therefore, a systematic analysis and comparison of occurrence of error made by the students is of considerable importance.

A systematic analysis of errors made by the students is of considerable importance. Errors are no longer considered a bad sign in learning (Shavarani, 2012). She further explained that examining students' wrong answers is one way to learn about students' understanding of a concept. In the same vein, students' correct answers may not necessarily indicate a good conceptual understanding of related knowledge because students could have solved the problem correctly by just memorizing procedures or definitions without real understanding (Tan, 2009). Besides, students' correct answers are generally uniform, which does not provide much information for research (Limjap, 2009).

In line with this, in the course of teaching college mathematics, the researcher has observed that college students have some persistent misconceptions that cause them to repeatedly commit the same error in solving mathematical problems. These errors and misconceptions form a pattern and hence become resistant to correction. In this connection, this study is being undertaken to identify patterns of the errors and misconceptions committed by the students in solving word problems. Moreover, these misconceptions are neither inborn nor instantaneous. Rather, students have acquired these misconceptions during their learning process for yet unknown reasons. Whatever the reasons may be, there should be a way to identify and remedy these problems. It is the researcher's belief that there should be a systematic way of studying the problem-solving errors committed by students in order to address them more effectively and adequately.

Methodology:-

The nature of the research problems raised in this investigation lend themselves to a mixed method design which is characterized by the collection and analysis of quantitative data followed by the collection and analysis of qualitative data (Creswell, 2003). Typically, the purpose of a mixed method design is to use qualitative results to assist in explaining and interpreting the findings of a primarily quantitative design. The initial quantitative phase of the study may be used to characterize individuals along certain traits of interest related to the research questions. These quantitative results can then be used to guide the purposeful sampling of participants for a primarily qualitative study. The findings of the quantitative study determine the type of data to be collected in the qualitative phase (Gay, Mills and Airasian, 2006).

Denscomb (2007) defines mixed methods as using data collecting methods that collect both qualitative and quantitative data. Collins et al. (2006), point out that using mixed methods is advantageous because mixed methods improve the accuracy of data. It also produces a more complete picture by combining information from complementary kinds of data or sources. This assists because biases intrinsic to single-method approaches are avoided, as a way of compensating specific strengths and weaknesses associated with particular methods. The quantitative dimension brings to the research numerical data usually in the form of frequencies and the qualitative dimension would add textual description of the phenomenon.

Consistent with mixed-method design adopted in this study, a combination of quantitative and qualitative measures was used to collect data to answer the research questions raised in the preceding chapter. In the quantitative part of this study, the researcher used a test instrument to identify and classify student errors. A total of 10-item word problems solving test were used in the study as part of the subject course requirements in Math 4: Solid Mensuration. Interviews were then being conducted to unravel students' reasoning and misconceptions that resulted in such errors in the qualitative part of the study. In this study, an interview technique guided by Newman's Error Analysis was used to further clarify the errors made by the students and the reasons for such errors.

Results and Discussion:-

Students' Errors in Solving Routine Problem

Majority of the twelve (12) common errors committed by students were due to inability to use the correct process in routine problems. Some students who were able to understand and translate the problem were impeded in their progress by their inability to use the correct mathematics to solve the problem. They failed to solve the problem using a correct solution path as well as the needed working equation. Students failed to perform basic computation due to misunderstanding of mathematical concepts. These errors were due to poor mastery of mathematical skills.

In addition, students usually made errors in solving problems involving algebraic expressions. These students add and subtract the expression by combining terms even if they were dissimilar. They could not see how variables were related to each other and failed to see the meaning among the algebraic numerical expressions that could serve as their basis for structuring their solution. The study confirms with a previous observation that most of the students could not hurdle the demands of algebraic manipulations and analysis of the variables, especially in written word problems (Denly, 2009).

Some students identified an appropriate operation or sequence of operations but did not know the necessary procedures to carry out these operations accurately. Most of them attempted calculation but were not able to provide a logical solution. They were able to guess and check their answers with the problem but found it hard to create a working solution to be able to arrive at their theorized answers. Similar to what Ashlock (2006) reported, students can jump into the answers without any working solution. They had their solution in their head but cannot write their solutions, that is, they can give the answers right away but when asked of their solutions, fail to present any.

Failure to write the final answer in an acceptable written form is considered by Ragma (2014) as a common error caused by carelessness or impulsiveness rather than due to misunderstanding of the mathematical concepts. This type of error was also committed by students, though to a lesser degree when compared to process error as shown in Table 1a. In some case, students' final answer is not in its acceptable simplified form and does not indicate the unit.

There were errors that occurred due to lack of strategy knowledge and inability to translate the problem in mathematical form. Some students who had no difficulty comprehending the problem were impeded in their progress as they appeared to have no knowledge of ways in which a routine problem might be approached and their inability to translate the problem into a mathematical form. In some cases, students who were able to fully understand the thrust of the problem used incorrect formula in solving it. Moreover, students encountered difficulties in translating word problem into mathematical sentences. In the phrase *the length is to be three times its width*, the measures of *length* and *width* are interchanged and *length* is written as *l=w+3* instead of *3w*.

Further analysis shows that there is a significant difference between the number of errors committed by the high achieving and low achieving groups. Confirming expectations, students in the high achieving group committed fewer errors compared with those who are in the low achieving group.

Table 1a:- Common Errors Committed by Students in Solving Routine Problems.

Descriptions of Errors	Example	Group	*Mean*	*Grand Mean*	*T-Value*	*P-Value*
1. Mathematical translation is incorrect.	*The length is to be three times its width* $l = 3x$ $w = x$	HG	5.4	6.3	4.0249*	0.0038*
		LG	7.2			
2. The formula is not correctly indicated.	$A\Delta = bh$ $b^2 = c^2 + a^2$	HG	5.6	6.3	2.7456*	0.0252*
		LG	7			
3. Unfinished answer	$b^2 = 13^2 - 5^2$ $240\ ft^2 = 3x^2$	HG	5.4	6.1	3.1305*	0.0140*
		LG	6.8			

	$\frac{240\ ft^2}{3} = \frac{3x^2}{3}$					
4. Error in calculation	$1125 = 25h$ $49 = h$	HG LG	5.4 7.2	6.3	4.0249*	0.0038*
5. Attempted calculation but did not know the solution made.	$a^2 + a^2 = (25\sqrt{2})^2$ $a = 25$	HG LG	5.4 6.8	6.1	3.1305*	0.0140*
6. Error in solving Algebraic Expression	$240 = 3x^2$ $\frac{240}{3x} = \frac{3x^2}{3x}$ $80 = x$	HG LG	5.2 7.0	6.1	3.6742*	0.0063*
7. The equation is solved incorrectly.	$A = lw$ $1125 = (25)(w)$ $1125 - 25 = w$ $1100 = w$	HG LG	5.6 6.8	6.2	2.6833*	0.027*
8. Mathematical operations on radicals is solved incorrectly.	$240 = 3x^2$ $\sqrt{240} = \sqrt{3x^2}$ $4\sqrt{15} = 3x$	HG LG	5.8 7.6	6.7	2.5456*	0.0344*
9. Incorrect operation	$13^2 = 5^2 + a^2$ $169 = 25 + a^2$ $169 + 25 = a^2$	HG LG	5.4 6.8	6.1	3.1305*	0.0140*
10. The answer is not simplified.	$x^2 = 80$ $x = \sqrt{80}$	HG LG	5.4 7.6	6.5	3.0509*	0.0158*
11. The unit is not written in the final answer.	$P\Delta(Triangle)$ $= 5km + 12\ km$ $+ 13\ km\ ;\ P\Delta = 30$	HG LG	6 7.2	6.6	2.4495*	0.0400*
12. The unit is not correctly written.	$A\blacksquare = (10\ ft)^2$ $A\blacksquare = 100\ ft$	HG LG	5.6 7	6.3	2.7456*	0.0252*

**Significant at 0.05 level of significance*

Students' Errors in Solving Non-Routine Problems

The data summarized in table 1b reveals that most of the common errors committed by students in solving non-routine problems were along process. The students incorrectly used distributive property of multiplication over addition in simplifying the expression *2(100+2x).* Instead of multiplying each of the terms in the other factor, the first term is multiplied by 2. Some students also committed errors on problems involving algebraic expressions mathematical concepts. These errors occur due to misunderstanding combining dissimilar terms.

In mathematical operations, most of the students incorrectly answered problems involving special products and factoring. These students committed errors in expanding the term $(x+6)^2$. Instead of using the pattern $x^2+2xy+y^2$, they just squared the first term and the second term, respectively.

These findings are consistent with the study of Egodawatte (2011) who reported that most students committed processing errors along word problems involving algebraic, polynomial, and rational expressions. The problems were too symbolic and students who find difficulty with the required mental operation often abandon solving them.

Errors along misunderstanding the thrust of the problem may be due to the students' lack of critical ability to deduce major concepts from a given problem. To White (2007), most problems involving situations were misunderstood by students because of their insufficient exposure to such problems and poor mastery.

An analysis of the students' solutions further show errors along transformation. These students failed to write the complete and correct representation of the phrase *"The length of the entire garden is 6 feet more than the length of the planted area".* This was traced to insufficient understanding of mathematical expressions and poor skills along mathematical translation. Though there were students who went beyond the processing level, these students failed to write their final answer in an acceptable written form. Some of the students whose calculations were correct still made mistakes by failing to indicate the correct unit in their final answer or express simplified answers in simplest terms. The one-way t-test for difference of means indicated that a significant difference in the number of errors committed by high and low achieving groups with the pattern of errors in solving non-routine problems.

Table 1b:- Common Errors Committed by Students in Solving Non-Routine Problems .

Descriptions of Errors	Example	Group	*Mean*	*Grand Mean*	*T-Value*	*P-Value*
1. Misunderstanding in the concept of semicircle.	*"A running track with straight sides and semi-circular ends". -the shape of the track is associated with oval*	HG LG	5.6 6.6	6.1	2.8868*	0.0203*
2. Mathematical translation is incorrect.	*The perimeter of the poster is 3/2 times the perimeter of the printed area. P. Printed area=3/2(P. Poster)*	HG LG	5.8 7	6.4	3.2071*	0.0125*
3. Mathematical Model is incorrect.	*Width =100+2x* *Length =140+2x* *Perimeter of poster=2l+2w* *2(100)+2x +2(140)+2x*	HG LG	5.4 8.4	6.9	5.3033*	0.0007*
4. Incorrect expansion	*Perimeter of rectangle* *2(100+2x)+2(140+2x) = 200+2x+280+2x* *Area of square* $(x+6)^2 = x^2+36$	HG LG	5.6 7.4	6.5	3.8376*	0.0050*
5. Unlike terms are combined	*480+8x=720* *488x=720*	HG LG	5.4 7	6.1	4.000*	0.0033*
6. Simplifying incorrectly algebraic equation	$(x+3)^2=225$ $\sqrt{(x+3)^2}=225$ $x+3=225$ $x=222$	HG LG	5.8 7.8	6.8	4.7140*	0.0015*
7. Transposition errors	$x-18=10$ $x=10-18$ $x=-8$	HG LG	5.6 7.4	6.5	3.1820*	0.0130*
8. Unfinished answer	$\left(\frac{52-x}{4}\right)^2+\left(\frac{x}{4}\right)^2 = 109$	HG LG	5.6 7.4	6.5	3.1820*	0.0130*
9. The answer is not simplified.	$225=(x-3)^2$ $\sqrt{225}=\sqrt{(x-3)^2}$ $\sqrt{225}=x-3$ $\sqrt{225}+3=x$	HG LG	5.4 7	6.2	4.000*	0.0039*
10. The unit is not written in the final answer.	*Width =100+2x* *Length =140+2x* *x=10* *Width=100+2(10)* ***=120***	HG LG	5.6 7.4	6.5	3.1820*	0.0130*

11. The unit is not correctly written.	*The radius of the semi-circular parts is **12 square feet.***	HG	5.6	6	2.3094*	0.0497*
		LG	6.4			

Significant at 0.05 level of significance

Students' Misconceptions in Mathematics

Investigating the reasons behind misunderstandings require inquiring deep into students' minds. Persistent errors occur due to flawed conceptual knowledge (misconceptions) which are amenable to analysis and robust to change, rather than being random errors that merely occur due to human fallibility. Examining deep into students' thinking and their beliefs is therefore necessary to find reasons for such misconceptions.

This investigation examines a range of significant and common mathematical mistakes committed by the students, discussing the nature and origin of the misconceptions that may explain them.

Misconception 1:

Adding Two Dissimilar Terms by Adding their Numerical Coefficients

$$\mathbf{2\pi + 3 = 5\pi}$$

This error was committed by students in adding two terms that are not similar. In evaluating $\mathbf{2\pi + 3,}$ most of the students answered $\mathbf{5\pi}$as the sum of the given expression. One misconception that could explain this error is in the context of simplifying expressions by collecting like/similar terms. Students would add the expressions $\mathbf{2\pi}$and $\mathbf{3}$as $\mathbf{5\pi}$ without understanding the role of the constant$\boldsymbol{\pi}$in the first expression. Students have concept in their minds that $\mathbf{2\pi}$and $\mathbf{3}$are similar terms and thereby can be combined.

This error was also committed when students simplify the expression$\mathbf{2x + x + 5}$. Most of them wrote $\mathbf{8x}$ instead of$\mathbf{3x + 5}$. Again, these students had in their mind that they can add all the terms in the given expression by adding each terms numerical coefficient regardless of whether these terms are similar or not.

The findings of this study corroborate with the analysis of Allen (2007) revealing that students had such misconceptions in dealing with Algebra. He further added that students had misunderstandings of the concepts of similar and dissimilar terms. That is, they have a tendency to combine terms which are unlike in solving algebraic expressions.

Misconception 2:

$$\boldsymbol{AddingDisimilarFractionswithoutrewritingthemas}$$

Equivalent Fractions

$$\mathbf{\frac{1}{2} + \frac{2}{3} = \frac{3}{5}}$$

This error occurred when students encountered problems that involve fractions having different denominators. Students committed errors in adding dissimilar fractions by simply adding the numerators and adding the denominator. These students assumed that they can apply any given operation to a pair of fractions simply by adding that operation to the numerators and denominators taken separately. Unfortunately, students failed to express the given fractions having different denominators in to an equivalent fraction before using the rules in addition of fractions.

This finding corroborates with Hart (1981) divulging that in solving problems involving fraction, a very common errors made by the student was to use the rule "add tops (numerator), add bottoms (denominator)". Initially, these errors were made by students who had misunderstandings that the rule mentioned is only applied for equivalent fractions and thus, the picture may be different in adding fractions having different denominators.

Misconception 3:

$$\boldsymbol{SubtractionisCommutative}$$

$$\mathbf{x - 2 = 2 - x}$$

Generally, most of the students think that the expressions $x - 2$and $2 - x$ are equal. They tend to apply the commutative property not only to the operation of addition but also to subtraction. Furthermore, these students had conception that the two expressions are equal due to commutative property of equality regardless of what operation use in the expression. In addition, their misconception was strengthened by the concept that even the variable $x$and the constant 2changed their positions, still the picture may be the same since the operation $(-)$did not change.

The analysis of the study relates with Elis (2013) showing that high percentage of students who have thought that$x - y = y - x.$ He further explained that this error was caused by such incorrect understanding that even the two terms interchanged with each other, still the expressions will be the same.

Misconception 4:

$$\boldsymbol{Monomialfactorismultipliedtoonlythefirstterm}$$
$$\mathbf{2(100 + 2x) = 200 + 2x}$$

Most of the students failed to simplify the expression$2(100 + 2x).$ Instead of writing$200 + 4x$, students failed to distribute the constant 2 correctly. Though some of them used distributive property of multiplication over addition but an error was made due to misunderstandings in the rules applied in using distributive property. These students thought that in simplifying the expression $a(b + c)$using distributive property, this results to $ab + c.$ That is, students were unable to apply the correct concept of this property due to some misconceptions that hinder their problem solving skill.

The finding of the study harmonized with the study of Li (2007) stating that students had difficulty in dealing with equations. He explained that the students did not master the mathematical principles behind simplification of such concept. This error points out to the fact that the mastery was not attained.

In addition, the findings also relate with the study of Pamani (2009) showing that the students had some misunderstandings in dealing with equations. It was stressed that students failed to understand the rudiments of these algebraic concepts.

Misconception 5:

$$\boldsymbol{Thesquareofabinomialisthesumofthe}$$
$$\boldsymbol{squaresofthefirstandsecondterms}$$
$$\boldsymbol{(x + 6)^2 = x^2 + 36}$$

Students made errors in squaring the binomial$x + 6$. Instead of writing $x^2 + 12x + 36$, most of them wrote $x^2 + 36$ as their answer. These students thought that they can just distribute the exponent to the two terms involved. That is, they have in their minds that in the expression$(x + y)^2$, the correct simplified form is $x^2 + y^2$ which is incorrect. Though there were students used the concept of FOIL method but these students could not productively use the method in getting the product of two binomials. The findings of the study adhere to the study of Wood (2003) emphasizing that students have some misconceptions in Mathematics especially algebra topics such as special product and factoring patterns.

Further, the study also jibes with the study of Bucsit (2009) stating that the students had poor performance in dealing with special products due to some imperfect understandings. She mentioned that this very dismal performance pointed out to the fact that the students had not very well understood the concepts and processes involved in special products.

Conclusion and Recommendation:-

One of the goals of this investigation was to identify and compare errors committed by students in solving word problems. This investigation is classified as mixed-methods design which tried to capture the strengths of both quantitative and qualitative research. Consistent with this end, students were bifurcated into high and low achieving group. As discussed in the preceding part of this study, participants belonging to high group were those whose general weighted average belonged to the first quadrant, whereas members of low group were those whose grades belong to the fourth quadrant.

The data were obtained by means of two sets of word problems which were administered in one-time point. Error analysis was done by adopting Newman's Error Analysis Guide. Statistical calculations such as frequencies, percentages, means, standard deviations, and test concerning two means were used to analyze the data gathered. Two weeks after the test, students were interviewed to identify their misconceptions and their reasoning. In the interview process, students were asked to explain their thinking while they were doing the same problems again. Some prompting questions were asked to facilitate this process and to clarify more about students' claims.

The analyses suggest the errors usually made by the students in solving routine and non-routine problems varied, hence it can be deduced that students have difficulty in dealing with non-routine problems which involve unexpected and unfamiliar solutions and requires higher-order thinking in the process of understanding, analysis, exploration, and application of mathematical concepts. Therefore, mathematics teachers should give more emphasis on students' learning in mathematical concepts such as algebraic expressions and factoring polynomials to reduce committed errors since it was found out that students encountered difficulties in working with these topics because it is too symbolic. In addition, teachers must also need to be empowered in order to help the learners to be conversant in the mathematical language. The study has demonstrated that mathematical language plays a vital role in learners' comprehension of word problems, hence the language that is used in mathematical word problems needs to be taken into cognizance.

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