Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI:10.21474/IJAR01/14678
DOI URL: http://dx.doi.org/10.21474/IJAR01/14678

RESEARCH ARTICLE

CASSI - AN OPTIMAL ZEROS ASSIGNMENT METHOD FOR SOLVING ASSIGNMENT PROBLEMS

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Manuscript Info

Manuscript History
Received: 05 March 2022
Final Accepted: 08 April 2022
Published: May 2022

Key words:-
Assignment Problems, Hungarian Method, TERM Method, CASSI Method

Abstract

An assignment problem (AP) is a meticulous case ofa transportation problem, in which the goal is to allocate a number of facilities to an equal number of activities at an overall maximum profit (or minimum cost, distance, time). It occupies a verysignificantrole in the real physical world. The well-known method applied to solve the APs is the Hungarian method, which generates optimal solution to a given AP. A little bit difficulty in the Hungarian method is to cover all the zero entries of a reduced cost matrix using minimum number of horizontal and vertical lines. However, this task has been made easy if one applies the 'ME Rules'presented in the 'Mantra' technique. In this research article, we make an attempt to bring in a new technique named asCASSIfor obtaining an optimal solutionto anygiven AP using an optimality testing and improving technique. The added advantage of this method is that for any AP, the solution obtained by applying any method based on zeros assignment approach can be tested for optimality and can also be improved towards optimal, if it's not optimal.

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Introduction:-

The assignment problem (AP) is one of the mostvital applications in the realworld and it is a particular class of linear programming in which our goal is to assign n number of jobs to n number of persons at minimum cost/ maximum profit on the whole. There are different ways to solve the AP. A most popular algorithm for solving AP wasintroduced by Kuhn [5] in 1995, named as Hungarian method (HM). In the modern years, quite a lot of methods have been available for obtaining the optimal solution to APs. Amongst them a few methods have been introduced such as New Method, TVAM Method, NS-TVSNM, and MAP Method and it is said that these methods achieve the optimal solutiondirectly to the APs. However,in factthe solutionsproduced by them are not optimal ones for some problems. A concisenarration about the above supposed direct methods are given below:

A. Ahmed and Afaq Ahmad [1], in 2014introduced a new method for obtaining an optimal solution of a broadchoice of assignment problems straight away. It consumes less time and is very simple to be acquainted with and apply. In reality this algorithm resembles the algorithm of the ASM Method due to Abdul Quddoos and Shakeel Javaid and M.M. Khalid [4] for generating an optimal solution for transportation problems.

In 2015, A. Thiruppathi and D. Iranian [3] proposed an innovative method named by TVAM to determine an optimal solution directly in less number of iterations for any AP. This algorithm applies systematic process and is

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very easy to understand. Actually, in this algorithm the authors make use of the Vogel's Approximation Method (VAM) procedure on the reduced cost matrix resulting from the given assignment cost matrix.

In April 2016, N. Sujatha and A.V.S.N. Murthy [7] proposed an advanced method named NS-AVSNM for obtaining optimal solution of assignment problemsin a straight way. The unbalanced problems can also be solved straightwith no conversion into minimization problems. It is aneasy and efficient method compared with Hungarian method. Really this algorithm has generated an optimal solution with the overall minimum cost of $24 for a 10 × 4 unbalanced minimization assignment problem against the solutiongenerated by the famous Hungarian method with the overall cost of $43.

In December 2016, A. Seethalakshmy and N. Srinivasan [2] introduced a new straight method for solving a maximization assignment problem (MAP). However, it can be applied in all kinds of assignment problems. This method generates an optimal solution straight in a small number of steps for the maximization APs.

In October 2020, R. Murugesan and T. Esakkiammal [9] introduced a new method namely TERM for solving a broad choice of APs with leastendeavor of mathematical calculations. The TERM method is based on the principle of reducing the given cost matrix to a matrix of opportunity costs (MOC) having at least one zero in every row and every column and creating assignments to the selected 0-entry cells of MOC which ensures best solution for a known AP.

In March 2021, R. Murugesan and T. Esakkiammal [8] introduced a very simple technique, known as MANTRA, which finds optimal solution directly to a given unbalanced AP without converting it into a balanced AP. This technique takes less time to solve an unbalanced AP in comparison to the Hungarian method.

In April 2021, T. Esakkiammal and R. Murugesan [10] introduced a new ones assignment method namely MASS (Modified Assignment) for obtaining optimal solutions to a broad choice of APs. This method obtains the optimal solution to a given AP in two phases. In the first phase, a solution is created using the ones assignment technique. Optimality testing and optimizing of the obtained solution is passed out in the second phase.

In this article, we have projected a new technique entitled CASSI, which performs the optimality testing and optimizing of the solution obtained through the TERM method. The performance of the CASSItechnique has been experienced over the recognized 30 benchmark problems from the literature and the outcomes are compared and discussed.

Thisresearch article is prepared as follows: In Section 1, shortintroduction is given. The algorithm of the projected CASSItechniqueis presented in Section 2. In Section 3, one benchmark problem from the literature has been illustrated by the projected technique.The recognized 30 benchmark APs of balanced and unbalanced types are listed in Section 4. Section 5 demonstrates the assessment of the outcomes and discussion on the CASSI and Hungarian methods. Last of all, conclusions are drawn in Section 6.

The CASSI Algorithm

The term CASSI is coined from the two words '**C**ustomized' and '**Assi**gnment'. The CASSItechnique consists of two phases. In Phase #1, a complete assignment plan, known as a solution, is determined using the TERM method and in Phase #2 optimality testing and optimizing of the obtained solution are carried out based on the computed improvement indices of the unused cells. The algorithm is as follows:

Phase #1
(Finding a solution of the Assignment Problem)

For the given AP, a solution is found by applying the TERM method due to Esakkiammal T and Murugesan R [9]. The readers may refer Appendix-A for the TERM algorithm.

Phase #2
(Optimality testing and optimizing the obtained solution)

Step 1: **Construct the final MOC**

Consider the final MOCmarked with assignments, obtained through the TERM method. The cells with assignments marked are called *used cells* and the remaining cells are called *unused cells*. An unused cell may have 0-entry or 1-entry or 2-entry and others.

Step 2: **Identify and include the corner cells of various loops**

Each row and each column of an assignment table of size n will have only one used cell (or corner cell) and hence the assignment table will have n corner cells in total. For forming a loop from an unused cell, the assignment table should have at least 2n-1 corner cells. In this step the possible corner cells are included as follows:

(a) 0-entries inclusion

In the original assignment cost matrix, first include all the 0-entries (used as well as unused) from the final MOC as the corner cells.

(b) Row-wise inclusion

In the first row of the matrix, include the cell having least entry with just > 0as a corner cell. If tie occurs in including such a cell, include the cell which has the least original assignment cost figure in the row. Again, if tie occurs among the original assignment cost figures, include any one cell arbitrarily.In the same way, include the corner cells for the remaining rows.

(c) Column-wise inclusion

In the first column of the matrix, include the least entry cell having entry with just > 0 as a corner cell. If tie occurs in including such a least entry cell, include the cell which has the least original assignment cost figure in the column. Again, if tie occurs among the original assignment cost figures, include any one cell arbitrarily. Likewise, include the corner cells for the remaining columns.The cells having assignments are called *used corner cells* and the other corner cells are called *unused corner cells*. It is noted that each row and each column will have exactly one used corner cell and at least oneunused corner cell.

NOTE:

1. If a column has more than one identical least entry just > 0, say u and with original cost figures C_1, C_2, C_3, … such that $C_1 < C_2 < C_3$ ...and suppose that the least entry cell with cost C_2 or above has been included during the row-wise inclusion, then the least entry u with original least cost figure C_1 must be included during the column-wise inclusion.
2. If a column has two successive least entries just > 0, say u and v such that u > v and if the cell with entry u has already been included during the row-wise inclusion, then the cell with entry v must be included during the column-wise inclusion.

Step 3: **Compute the improvement index for every unused cell**

In the assignment table with the original assignment cost figures along with the identified and included corner cells, trace a loop starting from an unused cell. An unused cell may be a corner cell or not. There may be one loop or more than one loop from an unused cell. Mark (+) and (-) sign alternatively at each corner of a loop, starting from the unused cell. Compute the effect on cost for the unused cell, by adding together the original unit costfigures found in each corner cell containing a plus sign and then subtracting the original unit cost figures found in each corner cell containing a minus sign. This effect on costis called the *net cost change(NCC) value* for the unused cell associated with the chosen loop. If the unused cell has more than one loop, then compute the NCC value for the cell associated with each possible loop. The maximum among the computed NCC values is called the *improvement index*of the unused cell. In the same way, compute the improvement index for every unused cell in the assignment matrix row-wise. In a square assignment matrix of order n, there will be exactly $n^2 – n$ unused cells and hence exactly $n^2 – n$ improvement indices.

NOTE: The improvement index for an unused cell may be negative or zero or positive. If we make a new assignment only in the unused cell with the most negative improvement index, then the overall assignment cost may improve (decrease).

Step 4: **Test the optimality condition**

If theimprovement indexcomputed for each of the unused cells is non-negative, then definitely the current solution is an optimal one for the given AP. If negativeimprovement indices occur for certain unused cells, then the current solutionis not optimal and it has to be optimized further.

Step 5: **Optimize the current solution**

(i) Select an appropriate unused cell for new assignment

Select the unused cell with the most negative improvement index to include in the new solution. If tie occurs among the unused cells with identical most negative improvement index, then select each such cell for the new assignment as a separate case. Such a state may generate different optimal solution to anyknown AP. The most negative improvement index of an unused cell indicates the overall assignment cost improvement (decrease)that can be achieved by making an assignment in that cell.

(ii) Make a new assignment in the selected cell

If the cell (i, k) has the most negative improvement index in the i^{th} rowand the cell (i, j) is the currently assigned cell in the i^{th} row, move the assignment from the cell (i, j) to the new cell (i, k). Equivalently, the assignment in the j^{th} column is first moved to the k^{th} column. Due to the unique assignment property in a row and column, this move will induce the current assignment in the k^{th} column, say (m, k) to move to another appropriate column. So, move the allocation from the cell (m. k) to the cell (m, n) having most negative improvement indexor next to the most negative improvement index among the improvement indices of the cells in the m^{th} row. Move the current assignments in this way until to get a new assignment in the j^{th} column from which we have started our first move.

(iii) Write the modified assignment plan as the new solution

Write the corresponding modified assignment plan, which is a new solution, and compute the associated overall cost.

Step 6: R**epeat the process**

Consider the final MOC marked with the new assignments and repeatSteps 3 to 5until there is no negative improvement index for all unused cells. That is, the current solution is an optimal one. Write the optimal solution and compute the associated overall minimum cost of assignment.

Unique / Alternative solution:

In an optimal assignment table, if one or more unused cell has improvement index as zero, it indicates that the given AP will have an alternative optimal solution. Also, if the improvement indices for all the unused cells are strictly > 0, then the given AP has a unique optimal solution only.

3. Illustrative Problem

Aptdescriptive solution makes the readers to realize the projected CASSItechniquesystematically. Keeping in mind, one assignment problem from the literature has been illustrated.

Illustration:

Consider the following cost minimizing APhaving5 jobs and 5 typists, referred from J. K. Sharma [6], which is exposed in Table 1.

Table 1:- The known Minimization AP.

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	85	75	65	125	75
2	90	78	66	132	78
3	75	66	57	114	69
4	80	72	60	120	72
5	76	64	56	112	68

Solution by the CASSItechnique

Phase #1: (Finding a solution)

By applying the steps of Phase-I in the CASSItechnique, one can get the final MOChaving at least one 0-entry in every row and in every column, as shown in Table 2, with a complete assignment plan.The cells with starred 0s denote the assigned cells.

Table 2:- Matrix of opportunity costs with assignments.

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	2	2	0	4	**0***
2	6	0	**0***	10	2
3	**0***	1	0	1	2
4	2	**0***	0	4	2
5	2	0	0	**0***	2

The corresponding assignments and the overall assignment cost are made known in the following Table 3.

Table 3:- The solution table.

Job	Typist	Cost in $
1	5	75
2	3	66
3	1	75
4	2	72
5	4	112
Overall cost		400

Now, the obtained solution is tested for optimality by Phase-II of the proposed CASSI algorithm.

First iteration
Step 1: **Construct the final MOC**
The final MOC along with the assignments is shown in Table 2.

Step 2: **Identify and include the corner cells of various loops**
In order toperform the optimality testing and optimizing of the obtained solution, we identify and include certain cells as corner cells.The assignment table with the original cost figures along with the included corner cells is shown in Table 4. Note that, in the 1st row the least entry just > 0 is 2, which occurs at the cells (1, 1) and (1, 2) with original cost figures 85 and 75 respectively. The cell (1, 2) with the least cost is taken as the additional corner cell in the 1st row. Also, in the 3rd row the least entry just > 0 is 1, which occurs at the cells (3, 2) and (3, 4) with original cost figures 66 and 114. Observe that, the cell (3, 2) is included as an additional corner cell in the 3rd row during the row-wise inclusion and the cell (3, 4) is included in the 4th column during the column-wise inclusion.Similar situation occurs for the cells (5, 5) and (5, 1) in the 5th row.The cell (5, 5) is included as an additional corner cell in the 5th row during the row-wise inclusion and the cell (5, 1) is included in the 1st column during the column-wise inclusion.The additional corner cells included in the 2nd and 4th rows, and 2nd and 4th columns are obvious. Note that, there are 5 used corner cells and 13 unused additional corner cells.

Table 4:- The assignment table with the original cost figures and corner cells.

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	85	2 75	0 65	125	**0**
2	90	0 78	**0**	132	2 78
3	**0**	1 66	0 57	1 114	69
4	80	**0**	0 60	120	2 72
5	2 76	0 64	0 56	**0**	2 68

Step 3: **Compute the improvement index for every unused cell**
Next, for each of the unused cells (corner cells as well as other cells) we compute the NCC values by forming possible number of loops using the corner cells. A loop should have not more than two corner cells, including the starting cell, in succession. The computed NCC values and thereby the improvement index for each of the unused

cells are shown in Table 5. The improvement index of an unused cell is the maximum among the computed NCC values for the cell using possible loops formulation from that cell.

Table 5:- Computation of the improvement indices for the unused cells in Table 4.

Unused Cell	NCC values due to the possible loops traced	Improvement Index
(1, 1)	85-75+78-66+57-75 = -6 85-75+66-78+66-65 = -1 85-75+66-72+60-66+78-75 = 1 85-76+112-114+57-66+78-75 = 1 85-75+68-64+72-60+57-75 = 8 85-75+68-112+114-75 = 5 85-75+72-72+66-75 = 1	8
(1, 2)	75-75+72-72 = 0 75-75+78-66+60-72 = 0 75-75+72-60+66-78 = 0 75-75+68-112+114-57+66-78 = 1 75-75+68-64+72-60++66-78 = 4 75-75+68-76+75-57+66-78 = -2 75-75+68-76+75-66 = 1	4
(1, 3)	65-75+78-66 = 2 65-75+72-72+78-66 = 2	2
(1, 4)	125-75+68-112+114 = 6 125-75+72-60+56-112 = 6	6
(2, 1)	90-66+57-75 = 5 90-66+60-72+66-75 = 3	5
(2, 2)	78-66+60-72 = 0 78-66+65-75+72-72 = 2 78-72+60-56+112-114+57-66 = -1	2
(2, 4)	132-112+56-66 = 10	10
(2, 5)	78-75+65-66 = 2 78-75+75-66+57-66 = 3 78-68+112-114+57-66 = -1	3
(3, 2)	66-75+76-56+66-78 = -1 66-75+76-112+114-57+66-78 = 0 66-75+76-64+72-60+66-78 = 3 66-57+66-78+75-75= -3	3
(3, 3)	57-66+78-68+112-114 = -1 57-75+66-64+72-60 = 6 57-66+78-72+72-66 = 3	6
(3, 4)	114-112+76-75 = 3 114-57+66-78+72-60+56-112 = 1 114-112+68-78+66-57 = 1 114-112+68-75+75-78+66-57 = 1	3
(3, 5)	69-114+112-64+72-72 = 3 69-75+75-72+60-56+112-114 = -1 69-78+66-56+112-114 = -1 69-57+66-78+75-75+72-72 = 0	3
(4, 1)	80-75+66-72 = -1 80-75+114-112+64-72 = -1	-1
(4, 3)	60-66+78-72 = 0 60-66+78-75+75-72 = 0 60-56+112-114+66-72 = -4 60-57+75-76+64-72 = -6 60-65+75-68+64-72 = -6	0
(4, 4)	120-112+68-75+75-72 = 4	4

	120-112+56-65+75-72 = 2 120-112+76-75+57-65+75-72 = 2 120-112+64-72 = 0	
(4, 5)	72-75+65-60+78-72 = 2 72-72+75-75 = 0	2
(5, 1)	76-112+114-75 = 3 76-75+66-72+60-57+114-112 = 0	3
(5, 2)	64-112+114-57+66-78 = -3 64-72+60-65+75-68 = -6 64-72+72-78+66-57 +75-76= -6 64-78+75-65+66-78 = -16	**-3**
(5, 3)	56-112+114667+72-60 = 4 56-68+75-65+66-78+72-60 = -2	4
(5, 5)	68-78+66-57+114-112 = 1 68-112+114-57+66-78+72-60+65-75=3	3

Step 4: **Test the optimality condition**
From Table 6, the improvement indices for certain unused cells are negative. Therefore, the current solution is not an optimal one. Now, we move to the next step for optimizing the current solution.

Step 5: **Optimize the current solution**
a) Select an appropriate unused cell for new assignment
Now, we try to bring the cell corresponding to the most negative improvement index into the assignment for further improvement. The cell (5, 2) has the most negative improvement index as -3 and so bring it into assignment.
b) Make a new assignment in the selected cell (5, 2)
The effect of bringing this cell (5, 2) into assignment is shown in Table6.

Table 6:- Shifting of certain currently assigned cells to new cells.

Currently assigned cell	Newly assigned cell	Improvement Index
(5, 4)	(5, 2)	-3
(4, 2)	(4, 1)	-1
(3, 1)	(3, 4)	3
Effective improvement index		-1

In Table 6, we make a new assignment in the cell (5, 2) from the currently assigned cell (5, 4). Due to this assignment and the unique assignment property of each row and column, we are forced to make the assignment from the already assigned cell (4, 2) in the 4th row to the new cell (4, 1) having the least improvement index -1 among the 4th row unused cells (Refer Table 5). Due to this new assignment and the unique assignment property in each row and column, we are again forced to make the assignment from the already assigned cell (3, 1) in the 3rd row to the new cell (3, 4) having the next possible least improvement index 3 among the 3rd row unused cells (Refer Table 5). Note that, it is not possible to make the new assignment at the cell (3, 2) in the 2nd column because we have already made a new assignment in the cell (5, 2), which is in the 2nd column. Also, note that first we have started shifting from the 4th column [that is, the cell (5, 4)] and finally ended at the same 4th column [that is, the cell (3, 4)]. Therefore, no further shifting of assignments is not possible.

c) Write the modified assignment plan as the new solution
The new solution resulted due to the shifting of certain currently assigned cells into new assigned cells is shown in Table 7.

Table 7:- The new solution (optimal solution) table.

Job	Typist	Cost in $
1	5	75
2	3	66
3	4	112

4	1	80
5	2	64
Overall cost		399

It is observed that, the overall cost of assignment is reduced by \$1 from \$400 to \$399.

Testing the optimality of the new solution
Now, this new solution is tested for optimality by using Phase-II of the CASSI algorithm.

Second iteration
Step 1: **Construct the final MOC**
The final MOC along with the new assignments is shown in Table 8.

Table 8:- Matrix of opportunity costs with the new assignments.

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	2	2	0	4	**0***
2	6	0	**0***	10	2
3	0	1	0	**1***	2
4	**2***	0	0	4	2
5	2	**0***	0	0	2

It is noted that, the unused 1-entry and 2-entry cells(3, 4) and (4, 1) have got new assignments.

Step 2: **Identify and include the corner cells of various loops**
In order toperform the optimality testing and optimizing of the new solution, we make use of the already included corner cells. The assignment table with the original cost figures along with the new assignments and the already included corner cells is revealed in Table 9.

Table 9:- The assignment table by means of the original cost figures and the corner cells.

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	85	2 75	0 65	125	**0**
2	90	0 78	**0**	132	2 78
3	0 75	1 66	0 57	**1** 114	69
4	**2**	0 72	0 60	120	2 72
5	2 76	**0**	0 56	112	2 68

Step 3: **Compute the improvement index for every unused cell**
The computed NCC values and thus the improvement index for each of the unused cells are shown in Table 10. The improvement index of an unused cell is the maximum among the computed NCC values for the cell using possible loops formulation.

Table 10:- Computation of the improvement indices for the unused cells in Table 9.

Unused cell	Computation of NCC values	Improvement index
(1, 1)	85-80+60-66+78-75 = 2 85-80+72-78+66-65 = 0 85-75+114-112+64-75 = 1 85-76+64-78+66-65 = -4	2
(1, 2)	75-66+114-112+68-75 = 4	4

	75-64+56-66+78-75 = 4	
(1, 3)	65-75+68-112+114-57 = 3 65-66+78-75 = 2	3
(1, 4)	125-114+66-78+66-65 = 0 125-75+68-64+66-114 = 6	6
(2, 1)	90-80+60-66 = 4 90-76+64-66+57-66= 3	4
(2, 2)	78-64+56-66 = 4 78-72+80-76+56-66 = 0	4
(2, 4)	132-114+54-66 = 9	9
(2, 5)	78-75+65-66 = 2	2
(3, 1)	75-80+72-64+112-114 = 1 75-57+66-78+75-75+68-64+72-80 = 2 75-57+66-78+75-75+72-80 = -2	2
(3, 2)	66-64+112-114 = 0 66-57+66-78+68-64 = 1 66-114+112-68+75-75 = -4	1
(3, 3)	57-66+78-64+112-114 = 3 57-66+78-68+112-114 = -1 57-65+75-68+112-114 = -3	3
(3, 5)	69-78+66-56+112-114= -1 69-75+80-72+64-68 = -2 69-75+75-78+66-57 =0	**0**
(4, 2)	72-64++76-80 = 4 72-64+112-114+75-80 = 1	4
(4, 3)	60-80+76-64+78-66 = 4 60-72+75-75+78-66 = 0	4
(4, 4)	120-114+75-80 = 1 120-112+64-66+75-80 = 1	1
(4, 5)	72-78+66-57+75-80 = -2 72-75+75-78+66-57+75-80 = -2 72-60+66-78+75-75 = 0	**0**
(5, 1)	76-80+72-64 = 4	4
(5, 3)	56-64+78-66 = 4	4
(5, 4)	112-114+66-64 = 0 112-114+57-66+78-64 = 3	3
(5, 5)	68-75+65-66+78-64 = 6	6

Step 4: **Test the optimality condition**

From Table 10, we see that the improvement indices for all the unused cells are ≥ 0. This indicates that, there will be no further reduction in the overall assignment cost of $399 and hence the current solution is optimal only. The optimal solution is shown in Table 8 and/ Table 9.

Alternative optimal solution

Observe that, at the optimal level the unused cells (3, 5) and (4, 5) have improvement indices as zero. This indicates that the given AP has an alternate optimal solution. By shifting the assignment from the currently assigned cell (3, 4) into the cell (3, 2), we can get an alternate optimal solution to the given AP. The shifting of certain currently assigned cells to the new cells and the resulting alternate optimal solution are shown in Table 11 and in Table 12 respectively.

Table 11:- Shifting of certain currently assigned cells to new cells.

Currently assigned cell	Newly assigned cell	improvement index
(3, 4)	(3, 2)	1
(5, 2)	(5, 4)	3

Effective improvement index	4

Table 12:- Alternate optimal solution.

Job	Typist	Cost in $
1	5	75
2	3	66
3	2	76
4	1	80
5	4	112
Overall cost		399

Therefore, the given AP has two optimal solutions with the minimum overall assignment cost of $399.

4. Benchmark Problems

To validate the effectiveness of the projected CASSItechnique, we have solved 30 benchmark APs in dissimilar sizes, from a range of literatures, which are all included in Appendix B.

5. Assessment of Outcomes

To determine the efficiency of the projected CASSItechnique, 30 benchmark problems, listed in Appendix B, have been experienced and the outcomes are compared with the outcomes of the existing Hungarian method (HM). The assessment of outcomes is exposed in Table 13.

Table 13:- Comparison of outcomes obtained by the CASSI and HM.

Prob. No.#	**CASSI**	**HM**	**Prob. No.#**	**CASSI**	**HM**	**Prob. No.#**	**CASSI**	**HM**
1.	48	48	**11.**	13	13	**21.**	50	50
2.	14	14	**12.**	900	900	**22.**	214	214
3.	59	59	**13.**	81	81	**23.**	54	54
4.	71	71	**14.**	399	399	**24.**	15	15
5.	09	09	**15.**	67	67	**25.**	54	54
6.	14	14	**16.**	392	392	**26.**	08	08
7.	29	29	**17.**	114	114	**27.**	15	15
8.	21	21	**18.**	99	99	**28.**	73	73
9.	24	24	**19.**	248	248	**29.**	870	870
10.	27	27	**20.**	191	191	**30.**	24	24

According to Table 13, we find out that out of 30 benchmark problems experienced, for 22 problems the projected CASSItechnique has created optimal solution straightaway in Phase #1 itself. For the problems numbered with 13 – 14, 22 and 25, and 27 – 30 only, we have to go to Phase #2 in order to improve the solution obtained through Phase #1. Further, it is observed that for the problems numbered with 28, 29 and 30, the CASSItechnique has produced optimal solutions, whereas the HM has not produced optimal solutions by few authors.However, for these APs (and any AP) one can achieve easily the optimal solutions directly by the Hungarian method, if one applies the ME Rules presented in the 'Mantra' technique due to Murugesan R. and Esakkiammal T. [8], to cover all the 0-entries using minimum number of lines. Really, Hungarian method is an efficient one to solve APs since 1955. Next, we give the novelty found in the projected CASSI technique.

The proposed CASSI Technique in terms of Novelty

In the literature, for solving Transportation Problems only, methods such as MODI method and Stepping Stone method areavailable for testing the optimality of an obtained solution and optimizing it, if not optimal. But, in the literature, so far no such methods are available for testing and optimizing the solutions of APs. CASSI method is the first of its kind for testing the optimality of an obtained solution using zeros assignment technique, and optimizing the solution, if it's not optimal.

Conclusion:-
In this article, we have projected a new technique named CASSI to get an optimal solution to assignment problems. This method finds the optimal solution to a given AP in two phases. In Phase #1, a solution is found out using the TERM method.Optimality testing and optimizing of the obtained solution is carried out in Phase #2 based on the improvement indices computed for the unused cells. The projected method is tested for 30 classical benchmark APs from the literature. The obtained results substantiate that the projectedCASSItechnique is the most competentone, which generates optimal solution to all 30problems. Hence, it is guaranteed that by applying the CASSItechnique one can get an optimal solution to a given AP. The added advantage of this technique is that for any AP, the solution obtained by applying any method based on zeros assignment approach can be tested for optimality and can also be improved towards optimal, if it's not optimal.

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Appendix A

(Algorithm for the ExistingTERM Method)

The TERM method is an efficient method of finding the best solution of an assignment problem without making a direct comparison of every solution. This method also works on the principle of reducing the given cost matrix to a matrix of opportunity costs (MOC) and making assignments to the selected zero-entry cells of the MOC in a way different from that of by Hungarian method. In this section, the algorithm for the TERM method (minimization case) for determining the best solution of APs has been proposed. The following are the sequence of steps involved in it:

(1) Checking the Balanced Condition. Check whether the given AP is balanced or not. If the AP is balanced, go to Step 3; otherwise, go to Step 2.

(2) Conversion to Balanced AP. If the AP is not balanced, then anyone of the following two cases may arise:

a) If the number of rows exceeds the number of columns, introduce required number of additional dummy columns to the assignment table to equalize with the rows. The unit assignment cost for the cells in these dummy column(s) is set to 'M', where $M > 0$ is a very large but finite positive quantity. Go to Step 3.

or

b) If the number of columns exceeds the number of rows, introduce required number of additional dummy rows to the assignment table to equalize with the columns. The unit assignment cost for the cells in these dummy row(s) is set to 'M', where $M > 0$ is a very large but finite positive quantity. Go to Step 4.

(3) Constructing the Matrix of Opportunity Costs (MOC).

a) Perform the Row Minimum Subtraction (RMS) Operation.

Subtract the minimum cost from each of the costs of every row of the balanced AP. This will result in a resultant matrix.

Perform the Column Minimum Subtraction (CMS) Operation.
Subtract the minimum cost from each of the costs of every column of the resultant matrix obtained in Step 3(a). Go to Step 5.

(4) Constructing the Matrix of Opportunity Costs (MOC).
a) Perform the Column Minimum Subtraction (CMS) Operation.
Subtract the minimum cost from each of the costs of every column of the balanced AP. This will result in a resultant matrix.

b) Perform the Row Minimum Subtraction (RMS) Operation.
Subtract the minimum cost from each of the costs of every row of the resultant matrix obtained in Step 4(a). Go to Step 5.

/* The resultant matrix obtained in Step 3(b) or Step 4(b) is known as the matrix of opportunity costs (MOC). It is noted that there will be at least one zero entry in each row and in each column of an MOC. The cells having only zero entries in an MOC are called zero-entry cells. */

(5)Making assignments in the MOC by applying the Tie Breaking Techniques.
(i) List all the zero-entry cells (row-wise) from the obtained MOC.
(ii) For each such cell, count the total number of zeros (excluding the selected one) in its row and column. Now choose a zero-entry cell for which the number of zeros counted is the *minimum* and make an assignment to that cell.
(iii) Again, if tie occurs in case of (ii), then make the assignment to that cell for which the total sum of all the elements in the corresponding row and column (of MOC) is the *maximum*.
(iv) Over again, if tie occurs in case of (iii), then make the assignment to that cell for which $i < k$ and / or $j < l$ where (i, j) and (k, l) are the competing cells for assignment.
(v) All over again, if tie occurs in case of (iv) such that

(a) The competing cells form a square:If (i, j), (i, l), ((k, j) and (k, l) are the competing cells for the allocation, which form a square, then select the cell at random for assignment. If we select the cell (i, j) for allocation, it will induce, in turn, to make an assignment in the cell (k, l) and vice versa. Similarly, if we select the cell (k, j) for allocation, it will induce, in turn, to make an assignment in the cell (i, l) and vice versa. This situation determines alternative solutions to the given AP.
(b) The competing cells do not form a square:If (i, j), (i, l), ((m, n) and (p, n) are the competing cells for the allocation, which do not form a square, then select the cell at random for assignment. If we select the cell (i, j) for allocation, it will induce, in turn, to make an assignment in the cell (m, n) and vice versa. Similarly, if we select the cell (i, l) for allocation, it will induce, in turn, to make an assignment in the cell (k, n) and vice versa. This situation determines alternative solutions to the given AP.

(6)Reducing the MOC. After performing Step 5, delete the row as well as the column of the cell for which assignment is made for further calculation as they will not be taken into account for making any more assignments..

(7)Developing the new revised MOC.
Check whether the resultant matrix obtained in Step 6 possesses at least one zero in each row and in each column. If so, go to Step 5 for making the next assignment; otherwise, go to Step 3 or Step 4 accordingly, for constructing a new revised MOC.

(8) Repeat Steps.
Repeat Steps 3 to 7 until and unless all the assignments have been made.

(9)Writing the assignments.
Write the assignments one by one row-wise.

(10)Computing the Total Cost.
Finally, compute the total cost corresponding to the assignments [ignoring the assignments in the dummy row(s) or column(s)] obtained in Step 9 using the original cost matrix.

Appendix B:-
(List of classical benchmark APs)

Balanced Minimization AP	**Balanced Maximization AP/ Unbalanced Minimization AP**
Problem No.,(Author(s), Year,)	**Problem No.,(Author(s), Year)**
Problem 1 (Anwar Nsaif Jasim, 2017) $[C_{ij}]$ 4×4= [8 20 15 17; 15 16 12 10; 22 19 16 30; 25 15 12 9]	**Problem 16 (R. S. Porchelvi, et al., 2018)** $[P_{ij}]$ 4×4= [140 112 98 154; 90 72 63 99; 110 88 77 121; 80 64 56 88]
Problem 2 (M.D.H. Gamal, 2014) $[C_{ij}]$ 4 ×4= [4 5 2 5; 3 1 1 4; 13 1 7 4; 12 6 5 9]	**Problem 17 (K.P. Ghadle et al., 2013))** $[P_{ij}]$ 4×4= [8 26 17 11; 13 28 4 26; 38 19 18 15; 19 26 24 10]
Problem 3 (K. Ghadle, Y. Muley, 2015) $[C_{ij}]$ 4 ×4= [18 26 17 11; 13 28 14 26; 38 19 18 15; 19 26 24 10]	**Problem 18 (A. Seethalakshmi et al., 2016))** $[P_{ij}]$ 4×4= [42 35 28 21; 30 25 20 15; 30 25 20 15; 24 20 16 12]
Problem 4 (Rajendra B. Patel, 2018) $[C_{ij}]$ 4×4= [10 24 30 15; 16 22 28 12; 12 20 32 10; 9 26 34 16]	**Problem 19 (Aderinto Y.O. et al., 2015)** $[P_{ij}]$ 5×5= [8 26 34 22 16; 13 52 13 52 26; 38 19 36 30 76; 19 26 48 20 38; 46 30 46 22 44]
Problem 5 (H.D. Afroz, M.A. et al., 2017) $[C_{ij}]$ 5×5= [8 4 2 6 1; 0 9 5 5 4; 3 8 9 2 6; 4 3 1 0 3; 9 5 8 9 5]	**Problem 20 (J.K. Sharma, 2017)** $[P_{ij}]$ 5×5= [32 38 40 28 40; 40 24 28 21 36; 41 27 33 30 37; 22 38 41 36 36; 29 33 40 35 39]
Problem 6 (M.D.H. Gamal, 2014) $[C_{ij}]$ 5×5= [12 8 7 0 4; 7 9 1 14 10; 9 0 12 6 7; 7 6 14 6 10; 9 6 12 10 6]	**Problem 21 (Hadi Basirzadeh, 2012)** $[P_{ij}]$ 5×5= [5 11 10 12 4; 2 4 6 3 5; 3 12 5 14 6;; 6 14 4 11 7; 7 9 8 12 5]
Problem 7 (A. Ahamed et al., 2014) $[C_{ij}]$ 5×5= [5 5 7 4 8; 6 5 8 3 7; 6 8 9 5 10; 7 6 6 3 6; 6 7 10 6 11]	**Problem 22 (A. Seethalakshmi, et al., 2017)** $[P_{ij}]$ 5×5= [30 37 40 28 40; 40 24 27 21 36; 40 32 33 30 35; 25 38 40 36 36; 29 62 41 44 39]
Problem 8 (M.D.H. Gamal, 2014) $[C_{ij}]$ 5×5= [7 8 4 15 12; 7 9 1 14 10; 9 1 1 6 7; 7 6 14 6 10; 1 6 12 10 6]	**Problem 23 (N. Sujatha, AVSN Murthy, 2015)** $[P_{ij}]$ 4×3= [11 8 8; 4 33 5; 10 33 5; 1 25 10]
Problem 9 (K.P. Ghadle, et al., 2013) $[C_{ij}]$ 5×5= [12 8 7 15 4; 7 9 1 14 10; 9 6 12 6 7; 7 6 14 6 10; 9 6 12 10 6]	**Problem 24 (N. Sujatha, AVSN Murthy, 2015)** $[C_{ij}]$ 4×5= [5 7 11 6 5; 8 5 5 6 5; 6 7 10 7 3; 10 4 8 2 4]
Problem 10 (Anuj Khandelwal, 2014) $[C_{ij}]$ 5×5= [4 6 7 5 11; 7 3 6 9 5; 8 5 4 6 9; 9 12 7 11 10; 7 5 9 8 11]	**Problem 25 (J.K. Sharma, 2017)** $[C_{ij}]$ 5×4= [9 14 19 15; 7 17 20 19; 9 18 21 18; 10 12 18 19; 10 15 21 16]
Problem 11 (Anuj Khandelwal, 2014) $[C_{ij}]$ 5×5= [2 9 2 7 1; 6 8 7 6 1; 4 6 5 3 1; 4 2 7 3 1; 5 3 5 9 1]	**Problem 26 (Abdur Rashid, 2017)** $[C_{ij}]$ 6×4= [3 6 2 6; 7 1 4 4; 3 8 5 8; 6 4 3 7; 5 2 4 4; 5 7 6 2]
Problem 12 (Aderinto Y.O., Oke M.O., Raji R.A, 2015) $[C_{ij}]$ 5×5= [280 220 310 340 360; 230 200 220 280 310; 240 200 270 300 310 ; 60 130 60 130 180; 70 100 30 120 170]	**Problem 27 (Anju Khandelwal1, 2018)** $[C_{ij}]$ 6×4= [6 5 1 6; 2 5 3 7; 3 7 2 8; 7 7 5 9; 12 8 8 6; 6 9 5 10]
Problem 13 (A. Thirupathy, et al., 2015) $[C_{ij}]$ 5×5= [20 30 25 15 35; 25 10 40 12 28; 15 18 22 32 24; 29 8 43 10 40; 35 23 17 26 45]	**Problem 28 (J.G. Kotwal,T.S. Dhope, 2015)**$[C_{ij}]$ 8×4= [53 62 42 89; 18 35 39 55; 93 80 91 83; 79 23 96 56; 43 16 12 20; 87 70 87 31; 35 79 25 59; 27 16 12 20]
Problem 14 (J.K. Sharma, 2017) $[C_{ij}]$ 5×5= [85 75 65 125 75; 90 78 66 132 78; 75 66 57 114 69; 80 72 60 120 72; 76 64 56 112 68]	**Problem 29 (V. Yadaiah, et al., 2016)** $[C_{ij}]$ 8×5= [300 290 280 290 210; 250 310 290 300 200; 180 190 300 190 180; 320 180 190 240 170; 270 210 190 250 160; 190 200 220 190 140; 220 300 230 180 160; 260 190 260 210 180]

Problem 15 (M. Khalid, M. Sultana, F. Zaidi, 2014)	**Problem 30 (J.G.Kotwal, T.S. Dhope, 2015)**
[C_{ij}] 6×6= [20 23 18 10 16 20; 50 20 17 16 15 11; 60 30 40 55 8 7; 6 7 10 20 25 9; 18 19 28 17 60 70; 9 10 20 30 40 55]	[C_{ij}] 10×4= [11 8 9 8; 4 5 29 33; 10 5 29 33; 1 18 25 31; 23 22 33 30; 3 9 13 19; 6 8 27 32; 32 30 39 38; 36 35 31 21; 15 11 10 28]

Note: Problems 1 – 15 are balanced minimization case, 16 – 22 are balanced maximization case, 23 is unbalanced maximization case and 24 – 30 are unbalanced minimization case.