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RESEARCH ARTICLE

CASCADE RELIABILITY ESTIMATION FOR TWO DISTRIBUTIONS

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Abstract

The present paper describes to obtain the system reliability of n-cascade system. For this estimation Lindley stress and exponential strength have been considered. Under this assumption the reliability expression of n-cascade system Rn is given. Here stress-strength are considered as random variables with the given density function. Some numerical values of reliabilities are given in tabular form for some selected values of the parameters.

Introduction:-

Many authors such as Hanagal [3], Kapur and Lamberson [5], Bhowal [1], have studied different models in reliability without taking time into consideration. They have studied only single impact systems. Cascade system were first developed and studied by Pandit and Sriwastav [6]. A Cascade system is a type of n-standby system [6] where the stresses on subsequent components are attenuated by a factor 'k', called attenuation factor. This factor is generally assumed to be a constant for all the components or parameter having different fixed values for different components. But an attenuation factor may be a random variable also [2]. Most of the discussions of interference models assume that the parameters of stress and strength distributions are constants. But in many cases this assumption may not be true and the parameters may be assumed themselves (parameters) to be random variables. For example, solutions corrosive action may be highly influenced by variation in its temperature [4] and hence the distribution of stress (corrosive action) may have different parametric values which vary randomly with temperature or in other words, the stress parameter may be taken as a random variable.

Let $X_1, X_2, ..., X_n$ be the strengths of n-components in the order of activation and let $Y_1, Y_2, ..., Y_n$ are the stresses. In Cascade system every after failure the stress is modified by a factor k which is also called attenuation factor such that

$$Y_2 = kY_1, Y_3 = kY_2 = k^2 Y_1 ..., Y_i = k^{i-1} Y_1 \text{ etc.}$$

In stress strength model the reliability, R of a component (or system) is defined asthe probability that its strength X, is not less than the stress Y working on it, where X and Y are random variables.
i.e. R = Pr(X ≥ Y)

In this paper n-cascade system have been considered with this model. The basic aim of this paper is to obtain the system reliability R_n with this model where the strength is considered as one-parameter exponential distribution and

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stress is Lindley distribution. The paper is organized as follows. In section 2 the general model is developed for n-cascade system. In section 3 the reliability expressions for n-cascade system is calculated when the stress-strength of the components follow particular distributions. In section 3.1 the expressions of R_n, is obtained when strength is considered as one-parameter exponential distribution and stress is Lindley distribution. The reliabilities R(1), R(2) and R_2 are tabulated with some numerical values for each cases in section 4. Results and Discussions are discussed at the end of the paper.

Mathematical Formulation

Let us consider n-cascade system where n components are numbered from 1 to n in their order of activation. Let X_i be the strength of the i^{th} component, in the order of activation, and when activated faces the stress Y_i, i=1,2,…, n. In case of cascade system with attenuation factor 'K' (constant).

$$Y_i = K^{i-1}Y_1, \quad i = 1,2,...,n \tag{2.1}$$

The system reliability is given by

$$R_n = R(1) + R(2) + ... + R(n) \tag{2.2}$$

Now the marginal reliability $R(1)$, $R(2)$, $R(3)$,…, $R(n)$ may be obtained as

$$R(1) = \int_{-\infty}^{\infty} \overline{F}(y_1) g(y_1)\, dy_1 \tag{2.3}$$

$$R(2) = \int_{-\infty}^{\infty} F(y_1)\overline{F}(ky_1)\, g(y_1)\, dy_1 \tag{2.4}$$

$$R(3) = \int_{-\infty}^{\infty} F(y_1)\, F(ky_1)\overline{F}(k^2 y_1)\, g(y_1)\, dy_1 \tag{2.5}$$

Similarly,

$$R(n) = \int_{-\infty}^{\infty} F(y_1)F(ky_1)F(k^2 y_1)...\overline{F}(k^{n-1}y_1)\; g(y_1)\, dy_1 \tag{2.6}$$

where r^{th} component marginal reliability is given by

$$R(r) = P[X_1 < Y_1, X_2 < kY_1,..., X_{r-1} < k^{r-2}Y_1, X_r \geq k^{r-1}Y_1] \tag{2.7}$$

Stress-Strength follows Specific Distributions

When Stress-Strength follows particular distributions the expression (2.6) can be evaluated and then the system reliability is obtained. In the following sub-section different particular distributions have been considered for all the Stress-Strength involved and obtain expressions of system reliability.

Strength follows Exponential Distribution and Stress follows Lindley Distribution

Let us consider the strengths of n components be i.i.d. with p.d.f. $f(x)$ which follows one parameter exponential distribution with mean $1/\lambda$ and the p.d.f. of Y_1 be Lindley density with parameter θ i.e

$$f(x,\theta) = \lambda e^{-\lambda x}; \qquad x \geq 0,\ \lambda \geq 0$$

$$g(y_1,\theta) = \frac{\theta^2}{(1+\theta)}(1+y_1)e^{-\theta y_1}; \qquad y_1 > 0,\ \theta > 0$$

then from (2.3) to (2.6) we get

$$R(1)=\frac{\theta^2}{1+\theta}\left[\frac{1}{\theta+\lambda}+\frac{1}{(\theta+\lambda)^2}\right]$$

$$R(2)=\frac{\theta^2}{1+\theta}\left[\left\{\frac{1}{\lambda K+\theta}+\frac{1}{(\lambda K+\theta)^2}\right\}-\left\{\frac{1}{\theta+\lambda K+\lambda}+\frac{1}{(\theta+\lambda K+\lambda)^2}\right\}\right]$$

$$R(3)=\frac{\theta^2}{1+\theta}\left[\begin{array}{l}\left\{\frac{1}{\lambda K^2+\theta}+\frac{1}{(\lambda K^2+\theta)^2}\right\}-\left\{\frac{1}{\lambda+\lambda K^2+\theta}+\frac{1}{(\lambda+\lambda K^2+\theta)^2}\right\}\\ -\left\{\frac{1}{\lambda K+\lambda K^2+\theta}+\frac{1}{(\lambda K+\lambda K^2+\theta)^2}\right\}+\\ \left\{\frac{1}{\lambda K+\lambda K^2+\theta+\lambda}+\frac{1}{(\lambda K+\lambda K^2+\theta+\lambda)^2}\right\}\end{array}\right]$$

Similarly,

$$R(n)=\frac{\theta^2}{1+\theta}\left[\begin{array}{l}\left\{\frac{1}{\lambda k^{n-1}+\theta}+\frac{1}{(\lambda k^{n-1}+\theta)^2}\right\}-\left\{\frac{1}{\theta+\lambda k^{n-1}+\lambda}+\frac{1}{(\theta+\lambda k^{n-1}+\lambda)^2}\right\}\\ -\ldots+(-1)^{n+1}\left\{\frac{1}{\lambda+\lambda k+\ldots+\lambda k^{n-1}+\theta}+\frac{1}{(\lambda+\lambda k+\ldots+\lambda k^{n-1}+\theta)^2}\right\}\end{array}\right]$$

The numerical values of $R(1), R(2)\, and\, R_2$ are tabulated in Table 1 for different values of the parameters.

Numerical Evaluation

The marginal reliabilities R(1), R(2) and system reliability R_2 are calculated for some particular values of the parameters which are given in the following table.

Table 1:- Values of R(1), R(2) and R_2 when stress strength are Lindley and Exponential variates.

λ	θ	K	R(1)	R(2)	R_2
1	1	1	.4550	.1729	.6279
1	2	2	.6726	.0986	.7712
1	3	3	.8731	.0612	.9333
2	1	1	.2312	.1342	.3654
2	2	2	.4197	.0818	.5015
2	3	3	.5434	.0246	.5680
3	1	1	.1863	.0846	.2709
3	2	2	.3600	.0559	.4159
3	3	3	.4725	.0321	.5046

Results and Discussions:-

The marginal reliabilities $R(1), R(2)$ and system reliability R_2 have been calculated for some specific values of the parameters from their expressions obtained in Sub-Section 3.1.From the **Table 1**, it is seen that if the stress parameter θ increases then the system reliability R_2 increase. When the strength parameter λ remain constant then $R(1)$ increases but $R(2)$ decreases. For instance, if θ=1, $R(1)=0.4550$ and if $\theta=2,\ R(1)=0.2312$. In general we see that when θ, k increases and for fix value λ then $R(1)$ and R_2 will increases i.e. but $R(2)$ decrease.

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