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### RESEARCH ARTICLE

#### GENERALIZATION OF THE POSITION-GENERATING MATHEMATICAL MODEL IN LINEAR WIRELESS SENSOR NETWORKS

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#### Abstract

The Linear Wireless Sensor Networks (LWSN) are a collection of wireless sensors arranged in a linear fashion. In order to allow them a better organization, several topologies have been proposed in the research. One of the most recent of them assumes a topology called redundant k-variant in which the redundancy factor k is a variable. This topology is based on a model generating the Euclidean positions of the nodes in an increasing way, the topology divides the network into two groups of nodes with different redundancy factors. This topology model increases the availability of the network especially in the areas close to the sink, compared to other topological models in which the redundancy factor was fixed for all the nodes of the network. In this paper we propose a generalization of the mathematical model proposed in the redundant k-variant model by generalizing the variation of the redundancy factor for all the nodes of the network and thus eliminating the fixed nature of the redundancy factor within the groups of nodes in the network in the k-variant model. In the approach proposed in this paper, the redundancy factor varies as one moves away from the sink. In this model, we choose, for each node, the best position among all possible ones. We use a system based on an arithmetic sequence to define an equation whose solution will generate the positions of the different nodes. We have also defined a specific resolution method coded in Python to choose the right values from all the solutions. The simulations focused on four series of transmissions based on different topologies. The experiments showed a much higher energy efficiency of the network in favor of the proposed model.

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#### Introduction:-

A wireless sensor network (WSN) is a set of interconnected sensors for a given application. These sensors, after collecting data, send them to a base station for operation [10, 4, 5, 6].

These WSNs, today, are applied in several fields such as environmental (meteorology, acidification of the oceans, dispersion of pollutants, etc.), commercial, medical (implantation of micro-sensors in the human body), military (detection chemical, biological or radiation agents), etc. WSNs are arranged in a given form depending mainly on their application.

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We are now witnessing more and more new applications of WSNs on linear infrastructures such as railway environments, bridges, gas and oil pipelines, borders, waterways etc... These environments are characterized by one or more portions of lines with branches. This has given rise to new types of wireless sensor network topologies called linear wireless sensor network [2, 7, 8, 9]. Linear Wireless Sensor Networks (LWSN) are a special case of (WSN) in which the nodes are arranged linearly.

LWSNs are divided into two categories of topologies: Networks with strictly linear topologies, in which we note the existence of a single linear line starting from the sink, and networks with linear topologies with zones of junctions, in which we note the existence of several linear lines, some of which form crossing points called junctions. In linear wireless sensor networks, nodes are both data sensors and relays, and have at least one neighbor in both linear directions of the network. The number of neighbors is determined by a factor  $k$  as a rule. In practice, the number of neighbors of each node depends on its position in the network, the range of its radio signal with respect to the density of nodes.

In [3], the authors propose a topology based on a  $k$ -redundant architecture in which each node of the network has  $k$  neighbors in the direction of the sink and in the opposite direction. If it is a strictly linear network, each node has at most  $2*k$  neighbors and at least  $k$  neighbors. In this type of topologies, the availability of the WSN strongly depends on the value of  $k$ . The larger this value, the better the network in terms of availability.

Given the linear form of the network, the data are constrained to go through a single path for the case of strictly linear wireless sensor networks. This particularity leads to faster energy depletion for nodes that are closer to the sink compared to other nodes in the network further away. Indeed, given the linear shape of the network, the closer you get to the sink, the more the activity of the node concerned increases because the node, in addition to its own data, is obliged to relay the data of its neighbors further away from the sink.

In [1] the authors propose a new kind of linear topology called redundant  $k$ -variant in which the redundancy factor  $k$  is not fixed for the whole network. They set up a mathematical model making it possible to generate the Euclidean positions of the nodes in an increasing way, thus making it possible to vary the redundancy factor  $k$ . The topology resulting from their model forms two groups of nodes with, in each, a different redundancy factor. Such a topology increases the availability of the network especially in the zones close to the sink by reducing the effort (energy) of transmission at the level of the nodes as one approaches the sink.

In this paper we propose a generalization of the mathematical model proposed in [1] in which the redundancy factor is generalized for all the nodes of the network. Indeed, in [1], even if the redundancy factor is no longer fixed throughout the network, it still remains fixed within the groups. Consequently, the energy problem that had led to the reflection of this model still remains when it is applied individually in the groups of nodes generated by the model.

In the approach proposed in this paper, the redundancy factor varies as one moves away from the sink, thus eliminating the notion of group of nodes. In this model, we choose, for each node, the best position among all possible ones. The model uses a system based on arithmetic sequences to define an equation that will be used to generate the positions. We also defined a specific method of resolution coded under python to choose the good values in the whole of the solutions.

The organization of the paper will be as follows: In the first part, we will present in detail the mathematical model generating positions proposed in [1] to which this paper brings an extension. In part 2 we will present our model as well as the simulation results obtained. We will end with the conclusion and some perspectives.

#### **Related work:**

In [1] the authors propose a mathematical model generating positions in strictly linear wireless sensor networks. Their model generates a topology they call redundant  $k$ -variant made up of two groups of nodes, each with its own redundancy factor.

As a basic postulate the authors consider a homogeneous network of strictly linear wireless sensors [3]. The authors note  $d$  the distance from the linear axis of the network,  $C$  the cardinality of the network (the number of nodes that make up the network) and  $p$  the maximum transmission range of a sensor.

**Convergence condition:**

The authors set a convergence condition for the network, i.e. what should be the values of p, Cr and d so that the network covers the entire linear distance of the network with availability in all places. This condition is given by equation (1)

$$p \times C_r \geq d \Rightarrow p \geq \frac{d}{C_r} (1)$$

A consequence of this condition is that for any LWSN satisfying the convergence condition, there exists a non-zero real  $k_f$  such that the network is  $k_f$ -redundant (2).

$$k_f \geq \frac{p \times C_r}{d} (2)$$

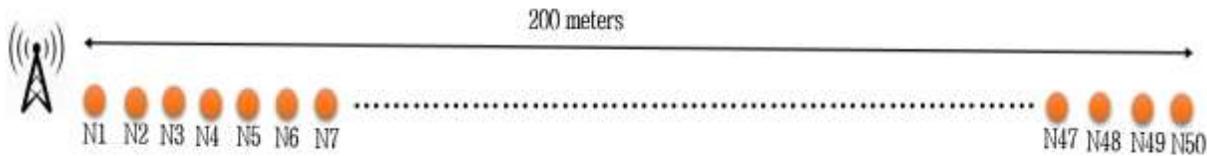
The position-generating mathematical model proposed by the authors is based on a set map given by equation (3)

$$\begin{aligned}
 & \text{If } p = \frac{d}{C_r}: f(C_r, d, p) = \frac{p \times C_r}{d} = k = 1 \text{ then the network is 1 redundant} \\
 & \text{If } p > \frac{d}{C_r}: f: N^* \times N^* \times N^* \rightarrow M_{C_r, 2}(N^*): \forall (C_r, d, p) \in N^{*3} \text{ we associate } f(C_r, d, p) | \\
 f(C_r, d, p) = & \begin{cases} G_1 \left(\frac{p}{k}\right) + G_2 \left(\frac{p}{k-1}\right) = d \text{ with } (G_1, G_2) \in [2, C_r - 2]^2 \\ G_1 + G_2 = C_r, k \in N^* \setminus \{1\} \end{cases} \quad (3)
 \end{aligned}$$

where :

- k is the maximum network redundancy factor.
- G1 and G2 two groups of nodes of respective redundancy factor k and k-1. The nodes of G1 will be distanced from each other by  $\frac{p}{k}$  those of G2 will be distanced from  $\frac{p}{k-1}$ .
- $M_{C_r, 2}(N^*)$  is the position matrix containing the positions of all nodes in the network. The generated matrix is defined, on each line, by the couple (Id<sub>node</sub>, D<sub>sup</sub>) with Id<sub>node</sub>, the identifier of the node and D<sub>sup</sub> the distance between the node and its neighbor greater than one jump (in the direction of the sink).

In [1] the authors apply the model to a topology defined by the following characteristics: Cr = 50 nodes, p= 10 m, d= 200 m (figure 1)



**Figure 1:-** Example of linear topology.

The convergence condition is verified because  $10 > \frac{200}{50}$

$$f(50, 200, 10) = \begin{cases} G_1 \left(\frac{10}{k}\right) + G_2 \left(\frac{10}{k-1}\right) = 200 \\ G_1 + G_2 = 50 \end{cases} \text{ with } (G_1, G_2) \in [2; 48]^2 \quad (4) \\ k \in N^* \setminus \{1\}$$

Once the expression of the map **f** has been established, the authors seek a real for **k** belonging to [2; p] for which solutions G1 and G2 exist for **f**.

For **k=3**

$$f(50, 200, 10) = \begin{cases} G_1 \left(\frac{10}{3}\right) + G_2 \left(\frac{10}{2}\right) = 200 \\ G_1 + G_2 = 50 \end{cases} \text{ with } (G_1, G_2) \in [2; 48]^2 \\ k \in N^* \setminus \{1\}$$

So, G1=30 and G2=20. Concretely, this results in a network composed of two groups: a group of 30 3-redundant sensors and a group of 20 2-redundant sensors. In the G1 group the distance at the level of the sensors is  $\frac{p}{k} = \frac{10}{3}$  meters, in the G2 group the distance will be  $\frac{p}{k-1} = \frac{10}{2} = 5$  meters (figure 2) [1]

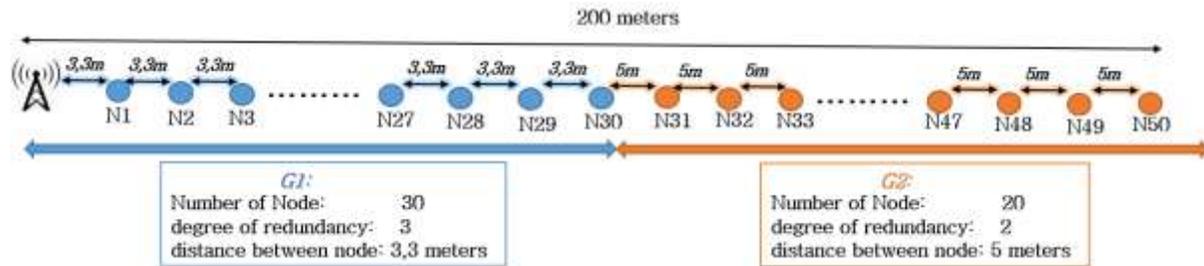


Figure 2:- Topology-generated by model.

The matrix of positions obtained is:

$$M_{50,2} = \begin{pmatrix} N1 & 3.3m \\ N2 & 3.3m \\ N3 & 3.3m \\ \vdots & \vdots \\ N29 & 3.3m \\ N30 & 3.3m \\ N31 & 5m \\ N32 & 5m \\ N33 & 5m \\ \vdots & \vdots \\ N48 & 5m \\ N49 & 5m \end{pmatrix}$$

Figure 3:- Matrix of positions.

**Discussion:-**

In [1] the authors proposed a completely new kind of topology which they named kvariant-redundant. The mathematical model they use generates a topology composed of two groups of nodes, each with its own distance value. The lower redundancy factor, *k*, at the level of the group closest to the sink makes their topology more optimal in terms of energy because the nodes, being in the group closest to the sink, will expend a lower transmission power than compared to nodes in the other group which makes their model more energy efficient than other existing k-redundant topology models in which the redundancy factor is fixed, which puts all nodes at the same level of effort.

However, their model suffers from some incompleteness. Indeed, even if the redundancy factor is no longer fixed for the entire network, it still remains fixed within each group. Consequently, the problem of the fixed redundancy factor remains, partially, always present within each group.

In the following we will present our model in which this problem has been completely solved.

**Position-generating mathematical model with generalization of the redundancy variance factor:**

In this part we present a generalization of the mathematical model detailed previously [1] with a generalized variation of the redundancy factor.

**Postulate and Notations:**

We consider a homogeneous network of strictly linear wireless sensors [3] with the same topological characteristics used for the k-variant model [1].

1. We note **d** the distance from the linear axis of the network ;
2. We note **C** the cardinality of the network (the number of nodes in the network);
3. We note **p** the maximum range of a node: this value is a constant, given the homogeneity of the network;
4. We note **N** the set of nodes of the wireless sensor network;
5. We note **U<sub>n</sub>** the arithmetic sequence giving the Euclidean distance separating a given node  $n \in N$  from the sink;
6. We note **x** the Euclidean distance separating the node closest to the sink (**U<sub>1</sub>**) to the sink;

$$U_1 = x \text{ with } x \in ]0 ; P[ \quad (5)$$

7. We note **k** the variance factor of the redundancy of the LWSN;
8. For full coverage, the condition described by equation (6) must hold

$$P \geq x + k(C - 1) \quad (6)$$

**Convergence condition:**

As exactly in [1], the convergence condition remains the same: The number of nodes with their range must cover the entire linear distance of the network.

$$p \times C \geq d \Rightarrow p \geq \frac{d}{c} \quad (7)$$

**Model:**

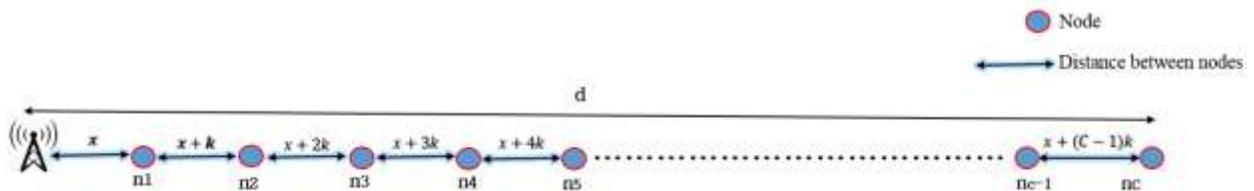
1. If  $d = P \times C \Rightarrow$  the network is 1-redundant defined by  $U_n$

$$\forall n \in N, U_n = U_{n+1} - P, \text{ with } U_1 = P \quad (8)$$

2. If  $P > \frac{d}{c}$ :

$$\exists k \in ]0 ; \frac{d}{\sum_{i=0}^{c-1} i} [ \quad | \quad \boxed{U_{n+1} = U_n + x + (n \times k)} \quad (9)$$

Figure (3) describes the topology resulting from the model:



**Figure 4:-** Topology generated by the model.

Once the expression of  $U_n$  established we can determine the expression of x

We have:

$$\begin{aligned}
 U_1 &= x ; \\
 U_2 &= U_1 + x + k = 2x + k ; \\
 U_3 &= U_2 + x + 2k = 3x + 3k ; \\
 U_4 &= U_3 + x + 3k = 4x + 6k ; \\
 U_5 &= U_4 + x + 4k = 5x + 10k ; \\
 U_6 &= U_5 + x + 5k = 6x + 15k ; \\
 &\dots\dots\dots \\
 U_c &= U_{c-1} + x + (c - 1)k = 2x + k ;
 \end{aligned}$$

$$\begin{aligned}
 U_n &= (n \times x) + \left( \sum_{i=0}^{n-1} i \right) k \\
 d &= (C \times x) + \left( \sum_{i=0}^{C-1} i \right) k \\
 d &= (C \times x) + \left[ \frac{C(C-1)}{2} \right] k
 \end{aligned}$$

$$\boxed{x = \frac{d - \left[ \frac{C(C-1)}{2} \right] k}{C}} \quad (10)$$

Then, we seek among all the possible  $k$  the one which minimizes  $x$ .

In the following, we will present the methodology for solving (10) adopted for the search for the efficient  $k$ .

#### Solving method:

The goal is to find among all the possible solutions for (10), the one that minimizes  $x$ .

Indeed, if  $P > \frac{d}{C}$  : there exists at least one solution of (10) for which  $P \geq x + k(C - 1)$  is verified.

To do this, we will proceed as follows:

- ✓ We define a minimum value  $k_\phi$  which will constitute the step for the iteration. The iteration interval is  $]0 ; \frac{d}{\sum_{i=0}^{C-1} i} [$ ;
- ✓  $\forall i \in [0 ; \frac{d}{\sum_{i=0}^{C-1} i} [ ; i = i + k_\phi$
- ✓ For each value of  $i$  we check if  $P \geq x + i(C - 1)$  is true
  - If yes, the iteration continues
  - Else,  $i = i - k_\phi$ , we take the last value of  $i$  before stopping the iteration

The algorithm was coded in python, the pseudocode is presented below. Please note that we have not given preset values to the variables  $P, C$  and  $d$  (replaced by '?' in the script). Therefore, the tester will have to replace them with their own values.

```

1 def sigma(first, last): # -----function performing the sigma from first to last
2     sum = 0
3     for i in range(first, last + 1):
4         sum = sum + i
5     return sum
6
7 #----- resolution method-----
8 d=?
9 C=?
10 P=?
11 if P > d/C:
12     k = 0
13     x=0
14     a=1
15     while P > x + (C-1)*k and a>0: # The last two nodes must be distanced by a value less than or equal to P
16         k = k + 0.01
17         a = (d-(sigma(1,C-1)*k))/C
18         if a > 0:
19             x = a
20         k = k - 0.01
21         x = (d-(sigma(1,C-1)*k))/C
22     print ("Solution of the equation (x,k) = (",x,",",k,")")
23
24     print ("for each node we give its distance from its lower neighbor, at one hop\n")
25     for i in range(1,C +1):
26         print("N",i,"--->",x + (i-1)*k)
27
28     print ("For each node we give its distance from the Sink\n")
29     for i in range(1,C +1):
30         print("N",i,"--->",(i*x + sigma(0,i-1)*k))
31 elif d == P*C:
32     print("the kvariant-redundant arrangement is impossible the topology is 1-redundant with a distance of ",P)
33 else:
34     print("arrangement is impossible")

```

[Figure 4:- resolution algorithm.]

**Application**

In order to test our model we compared it with the k-variant model. We carried out five series of transmission, for each one, we calculate the lifetime of the network, i.e. the time elapsing until the extension of the last neighbor of the sink (figure 4). The script is executed for each topology, x represents the distance separating the sink from its closest neighbor,  $k_{max}$  represents the maximum degree of redundancy which corresponds to the number of neighbors of the sink,  $k_{min}$  represents the minimum degree of redundancy corresponding to the number of neighbors of the node furthest from the sink.

The simulations were carried out under Castalia. All nodes initially have an energy reserve equal to 18720 joules. In the following, we give the results obtained for each round of transmission:

**Round 1**

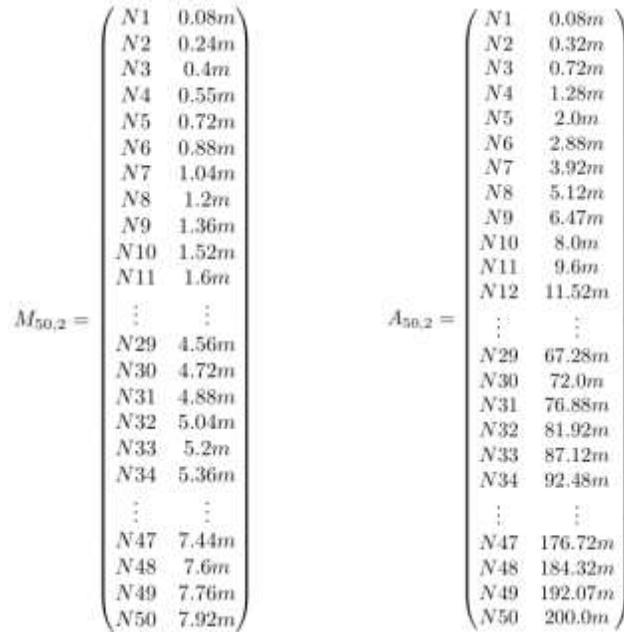
The transmissions in this round are applied to a topology with the same characteristics as those used in [1]: C = 50 nodes, p= 10 m, d= 200 m. The results obtained are presented in Table 1

| Model     | x      | $k_{max}$ | $k_{min}$ |
|-----------|--------|-----------|-----------|
| k-variant | 3.3 m  | 3         | 2         |
| Our model | 0.08 m | 11        | 1         |

Tab 1:- The results of round 1.

The generated positions are represented in two matrices  $M_{50,2}$  and  $A_{50,2}$ :

1. In  $M_{50,2}$ , each line indicates, for each node of the network, the distance which separates it from its neighbor less than one hop.
2. In  $A_{50,2}$ , each line indicates, for each node of the network, the distance which separates it from the sink.



[Figure 5:- Matrix of positions.]

The matrix generated by the model presented in [1] is presented in the related work section. We can notice that the first node is located at 0.08 meters from the sink in comparison with [1] where the first node is located at 3.3 meters from the sink, this difference in favor of our continuous model until the twenty-second node which is located 3.44 meters from its lower neighbor in comparison with [1] where its equivalent in [1] is located 3.3 meters from its lower neighbor.

We can also notice that the sink has 11 neighbors while for the k-variant model the sink has 3 neighbors. We measure the time that elapses before the extinction of the last neighbor of the sink from several series of transmissions. K-variant performs 17018.18 seconds before the last neighbor of the sink goes out while our model performs 234000 seconds. The lifetime of the k-variant network is thus multiplied by 13.7 compared to the lifetime of the network for k-variant. However, we notice a greater energy expenditure at the level of the sensors furthest from the network.

**Round 2:**

The transmissions in round two (2) are applied to a topology with the following characteristics: C = 100 nodes, p= 15 m, d = 500 m. The results obtained are presented in Table 2.

The sink has 17 neighbors for our model while for the k-variant model the sink has 3 neighbors.

K-variant performs 7800 seconds before the extinction of its last neighbors while our model performs 374400 seconds, i.e. a multiplication of 48 of the lifetime of the network. However, like round 1, we notice a greater energy expenditure at the level of the sensors furthest from the network.

| Model     | x      | k <sub>max</sub> | k <sub>min</sub> |
|-----------|--------|------------------|------------------|
| k-variant | 4.68 m | 3.2              | 2.2              |
| Our model | 0.05 m | 17               | 1                |

Tab 2:- The results of round 2.

**Round 3:**

The transmissions in round three (3) are applied to a topology with the following characteristics: C = 150 nodes, p= 20 m, d = 1000 m. The results obtained are presented in Table 3.

The sink has 15 neighbors for our model against 3 for the k-variant model.

K-variant performs 9360 seconds before the extinction of its last neighbors while our model performs 26742.85 seconds, i.e. a multiplication of 2.8 of the lifetime of the network.

In this round, too, a higher energy expenditure at the level of the sensors furthest from the network was recorded for our model.

| Model     | x      | $k_{max}$ | $k_{min}$ |
|-----------|--------|-----------|-----------|
| k-variant | 6.45 m | 3.1       | 2.1       |
| Our model | 0.7 m  | 15        | 1         |

Tab 3:- The results of round 3.

**Round 4:**

The transmissions in round four (4) are applied to a topology with the following characteristics: C = 200 nodes, p= 30 m, d = 2000 m. The results obtained are presented in Table 4.

The sink has 24 neighbors for our model against 3 for the k-variant model.

K-variant performs 6038.7 seconds before the extinction of its last neighbors while our model performs 374400 seconds, i.e. a multiplication of 62 of the lifetime of the network.

In this round, too, a higher energy expenditure at the level of the sensors furthest from the network was recorded for our model.

| Model     | x      | $k_{max}$ | $k_{min}$ |
|-----------|--------|-----------|-----------|
| k-variant | 7.6 m  | 3.9       | 2.9       |
| Our model | 0.05 m | 24        | 1         |

Tab 4:- The results of round 4.

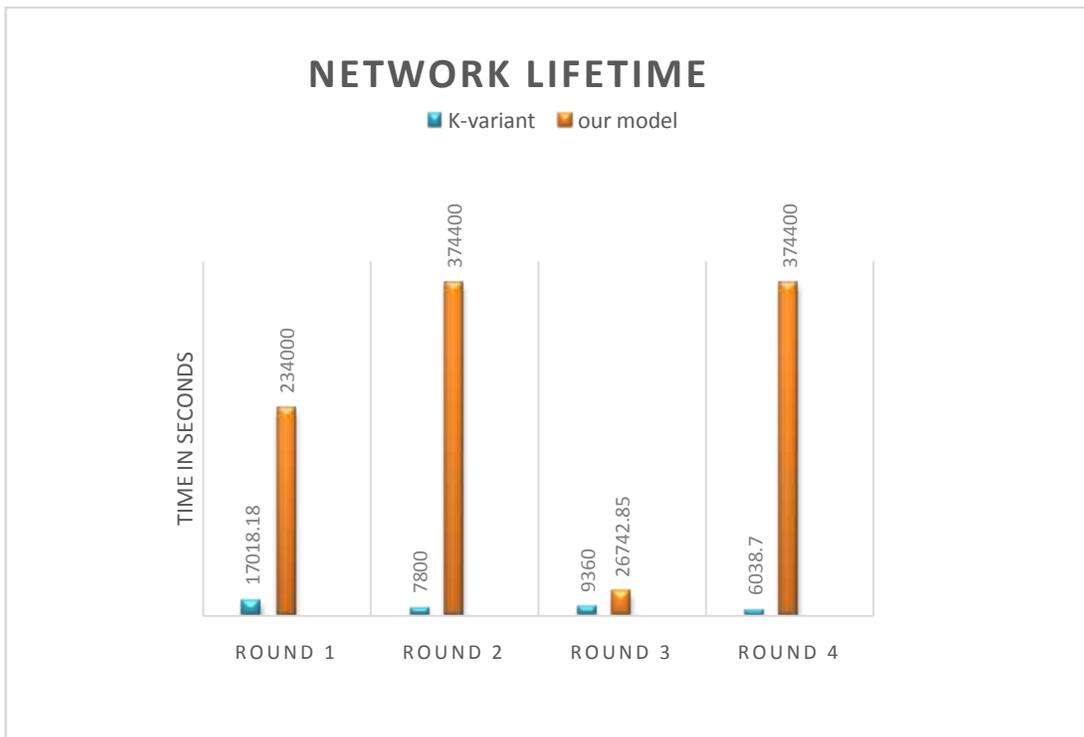


Figure 6:- Network lifetime per transmission series.

The simulations focused on four series of transmissions based on different topologies. The experiments showed a much higher energy efficiency of the network for our model. However, we have observed a greater energy expenditure at the level of the sensors farthest from the sink.

### Conclusion:-

In this paper we have presented an extension of the position-generating mathematical model presented in [1]. We worked on the generalization of the latter in which the problem of the fixed redundancy factor still existed within the two groups of nodes generated by the model. In the approach proposed in this paper, the redundancy factor varies as one moves away from the sink, thus eliminating the notion of group of nodes. In this model, we choose for each node the best position among all those possible. The model uses a system based on arithmetic sequences to define an equation that will be used to generate the positions. We have also defined a specific resolution method coded in python to choose the right values from all the solutions. In the approach that we have proposed in this paper, the redundancy factor varies as one moves away from the sink and this, regardless of where one is in the network. We thus choose, for each node, the best position among all those possible.

The simulations involved four series of transmissions based on different topologies. The experiments showed much higher network energy efficiency in favor of the proposed model.

In future perspectives we plan to apply the model in linear wireless sensor networks with junction areas.

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