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RESEARCH ARTICLE

SMARANDACHLEY PRODUCT CORDIAL LABELING OF BLOOM TORUS GRAPH (BT m,n)

Dibya Gulab Minj[1], Dr. G.V.V. Jagannadha Rao[1] and Dr. Chiranjilal Kujur[2]

1. Department of Mathematics, Kalinga University, Naya Raipur, Chhattisgarh-492101, India.
2. Department of Mathematics, St. Joseph's College Darjeeling, West Bangal- 734101, India.

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Abstract

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In this paper we work on the Bloom Torus graph $BT_{m,n}$ by satisfying the condition of Smarandachely product cordial labeling.

Smarandachely product cordial labeling on G is such a labeling $f : V(G) \longrightarrow \{0,1\}$ with induced labeling $f(u)\,f(v)$ *on edge* $uv \in E(G)$ *that* $\left|v_f(0) - v_f(1)\right| \geq 2$ *and* $\left|e_f(0) - e_f(1)\right| \geq 2.$

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Introduction:-

The applications of graph labelling are implemented in the development of social network. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Labeling helps to distinguish between any two adjacent vertices or edges. Graph labelling was first introduced in the year 1967 by Rosa [1]. Rosa defined a function as $f : V(G) \rightarrow \{0,1,2,3.........q\}$, f is an injection such that, when each edge xy is assigned the label $\left|f(x) - f(y)\right|$, the resulting edge labels are distinct.

Product cordial labelling of some connected graphs was investigated by Nikhil Parmar and Dr. Dharamvirsinh Parmar [2]. S. K.Vaidya and C. M. Barsara [3] worked and proved the edge product cordial labelling in the context of some Graph Operations and Product Cordial and Total Product Cordial Labelling of P^m_{n+1} by Zhen-Bin Gao, Guang- Yi Sun, Yuan- Ning Zhang, Yu Meng, and Gee- Choon Lau [4].The concept of product cordial labelling of double path union of C_3 related graphs was introduced by Mukund V. Bapat [5]. And Sum divisor cordial labelling for star and ladder related graphs by A. Lourdusamy and F. Patrick [6]. A. Sugumaran and K. Rajesh [7] introduced about the Sum Divisor cordial labelling on some Special Graphs. Samir K. Vaidya and Chirag M. Barasara [8] investigated on Total edge product cordial labelling of graphs and P. Lawrence Rozario Raj and R. Lawrence Joseph Manoharan [9] on Face and Total face product cordial labelling of graphs.. A.Sugumaran and K.Rajesh proved that the plus graph, umbrella graph, path union of odd cycles, kite and complete binary tree are sum divisor cordial graphs. D.Antony Xavier and Deeni C.J. [10] worked on bloom graph. It possesses a unique properly of being both regular and planar. D. Antony Xavier, Deeni C.J introduced about a new interconnection network motivated by the grid, cylinder and torus network. They discussed basic topological properties of Bloom graph. They also proved the hamitonicity. And explored the pancyclicity and broadcasting the proposed graph. Bloom torus: A potential fixed interconnection architecture was introduced by S. Kulandai Therese, D. Antony and AndrewArokiaraj [11]. L (2,

Corresponding Author:- Dibya Gulab Minj
Address:- Department of Mathematics, Kalinga University, Naya Raipur, Chhattisgarh-492101, India.

1)-Labelling for Bloom graph was introduced by Chriranlal Kujur, D.Antony Xavier, Arul Amritha Raja and Francis Xavier

Here especially the researcher will work on the Bloom Torus Graph satisfying the condition of Smarandachely product cordial labelling which is regular and planner graph. We consider graph $\vec{G}=(V(G),E(G))$ having set of vertices $V(G)$ and set of edges $E(G)$ respectively.

Definition- Bloom Torus Graph-

The bloom torus denoted by $BT(m,n); m,n>2$ *consists of vertex set* $V(BT(m,n)=\{(x,y):0\leq x\leq m-1, 0\leq y\leq n-1$, two distinct vertices (x_1,y_1) *and* (x_2,y_2) being adjacent if and only if

1) $x_1=0, x_2=m-1$ *and* $y_2=(y_1-1+\left\lfloor \frac{m}{2}\right\rfloor(\mathrm{mod}\, n))$

2) $x_1=0, x_2=m-1$ *and* $y_2=(y_1+\left\lfloor \frac{m}{2}\right\rfloor(\mathrm{mod}\, n))$

3) $x_2=x_1+1$ *and* $y_1=y_2$

4) $x_2=x_1+1$ *and* $y_1+1\equiv y_2(\mathrm{mod}\ n)$

The first and second conditions describe the wrap-around edges, the third condition describe the vertical edges and fourth condition describes the slant edges.Bloomtorus graph has mn vertices and 2mn edges.

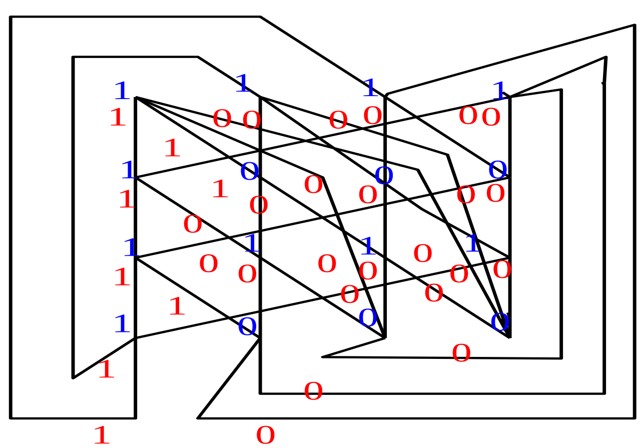
Theorem 1 – The $m\times n$ dimensional Bloom Torus graph $BT_{m,n}$ admits Smarandachely Product Cordial labeling $m,n\geq 3$.

Case1-When m, n both are even –

Label all the vertices of all odd rows with (1,1,1…..) and first five vertices of all the even rows are labelled as (1,0, 0,0,1,0) respectively and rest of the vertices as (0,1,0,1,0,1……..) alternatively.

Figure 1.1:-

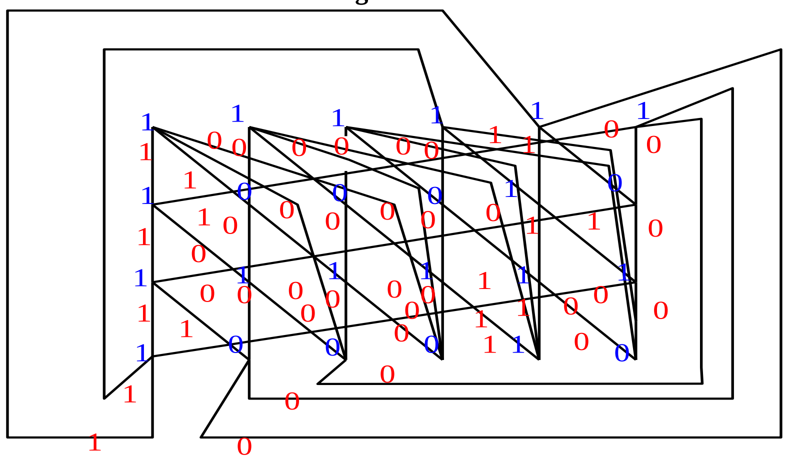
$BT_{4,4}$



$$v(1)=10, v(0)=6$$
$$\left|\sum v(0)-\sum v(1)\right|\geq 2$$

$$|6-10| \geq 2$$
$$4 \geq 2$$
$$e(1)=8, e(0)=24$$
$$\left|\sum e(0)-\sum e(1)\right| \geq 2$$
$$|24-8| \geq 2$$
$$16 \geq 2$$

Figure1.2:-

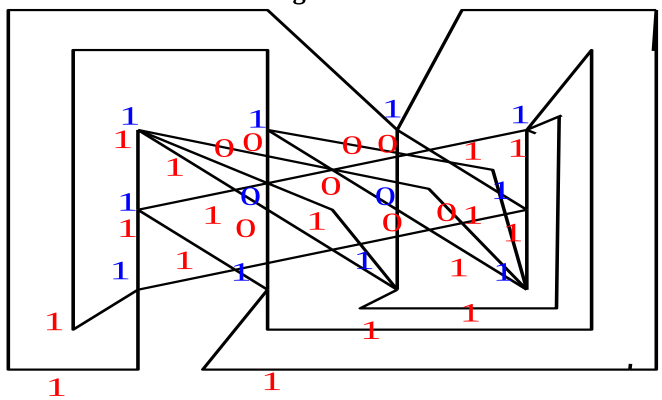


$$BT_{4,6}$$
$$v(1)=16, v(0)=8$$
$$\left|\sum v(0)-\sum v(1)\right| \geq 2$$
$$|8-16| \geq 2$$
$$8 \geq 2$$
$$e(1)=16, e(0)=32$$
$$\left|\sum e(0)-\sum e(1)\right| \geq 2$$
$$|32-16| \geq 2$$
$$16 \geq 2$$

Case 2- when m is odd, n is even-

Label first four vertices of the odd rows with (1, 1, 1…...) respectively and first two vertices of even rows are 707abeled with (1, 0) and rest of the vertices is labeled as (0, 1, 0, 1…..) alternatively.

Figrue2.1:-



$BT_{3,4}$

$v(1)=10, v(0)=2$

$\left|\sum v(0)-\sum v(1)\right| \geq 2$

$|2-10| \geq 2$

$8 \geq 2$

$e(1)=16, e(0)=18$

$\left|\sum e(0)-\sum e(1)\right| \geq 2$

$|18-16| \geq 2$

$2 \geq 2$

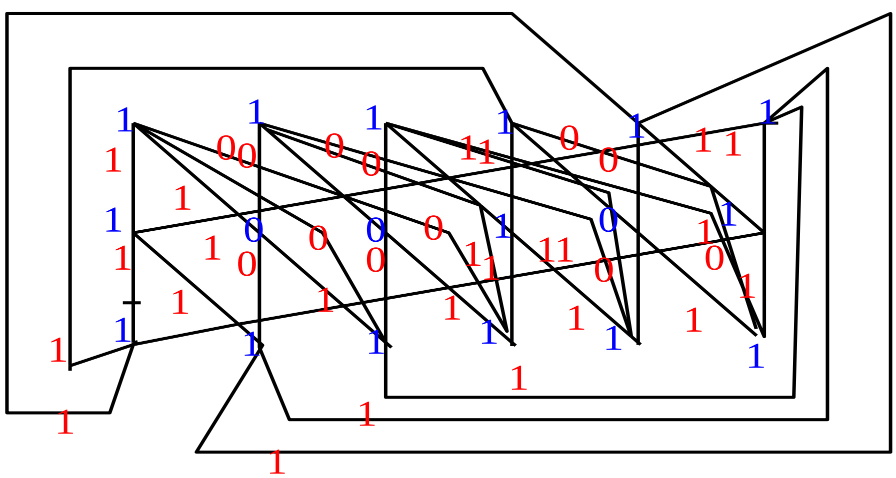


Figure2.2:-

$BT_{3,6}$

$v(1)=15, v(0)=3$

$\left|\sum v(0)-\sum v(1)\right| \geq 2$

$|3-15| \geq 2$

$12 \geq 2$

$e(1)=24, e(0)=12$

$\left|\sum e(0)-\sum e(1)\right| \geq 2$

$|12-24| \geq 2$

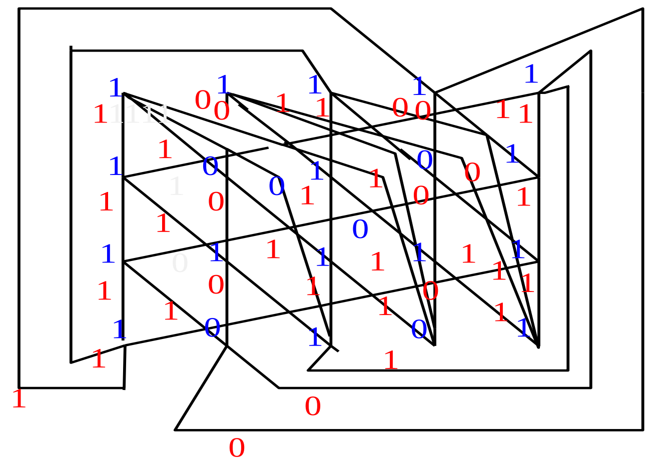
$12 \geq 2$

Case -3- When m is even, n in odd –

Label all vertices of the odd rows with (1, 1, 1.....). And the vertices of all even rows are labelled as (1, 0, 1, 0.......) alternatively.

Figure3.1:-

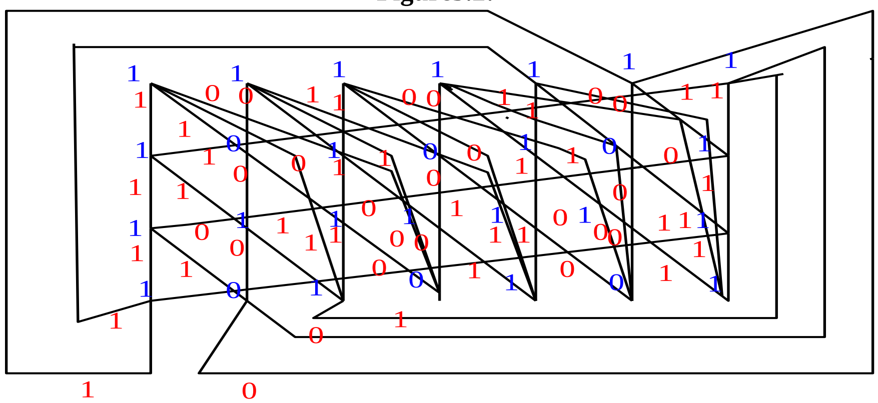
$BT_{4,5}$



$$v(1) = 16, v(0) = 4$$
$$\left|\sum v(0) - \sum v(1)\right| \geq 2$$
$$|4 - 16| \geq 2$$
$$12 \geq 2$$
$$e(1) = 24, e(0) = 16$$
$$\left|\sum e(0) - \sum e(1)\right| \geq 2$$
$$|16 - 24| \geq 2$$
$$8 \geq 2$$

$BT_{4,7}$

Figure3.2:-



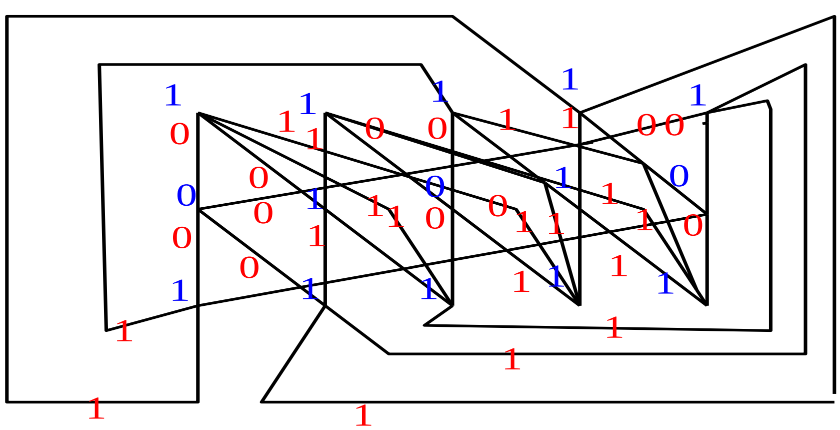
$$v(1) = 22,\ v(0) = 6$$
$$\left|\sum v(0) - \sum v(1)\right| \geq 2$$
$$|6 - 22| \geq 2$$
$$16 \geq 2$$
$$e(1) = 32,\ e(0) = 24$$
$$\left|\sum e(0) - \sum e(1)\right| \geq 2$$
$$|24 - 32| \geq 2$$
$$8 \geq 2$$

Case 4 – When m, n both are odd –

Label all the vertices of all odd rows with (1, 1, 1…) alternatively. And all the vertices of even rows are labelled as (0, 1, 0, 1......).

Figure 4.1:-

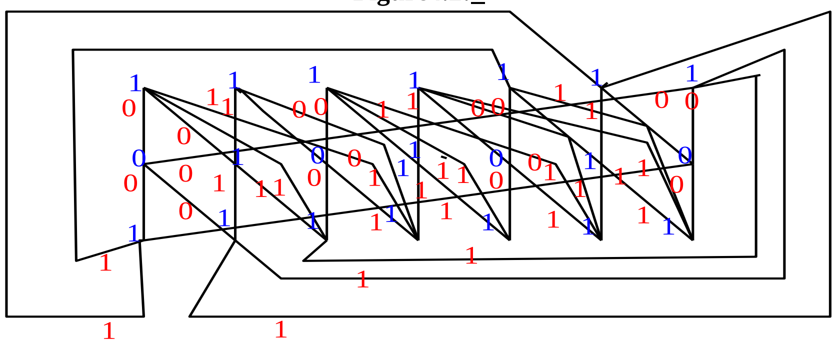
$BT_{3,5}$



$$v(1) = 12,\ v(0) = 3$$
$$\left|\sum v(0) - \sum v(1)\right| \geq 2$$
$$|3 - 12| \geq 2$$
$$9 \geq 2$$

$$e(1)=18, e(0)=12$$
$$\left|\sum e(0)-\sum e(1)\right|\geq 2$$
$$|18-12|\geq 2$$
$$6\geq 2$$

Figure4.2:_



$$BT_{3,7}$$

$$v(1)=17, v(0)=4$$
$$\left|\sum v(0)-\sum v(1)\right|\geq 2$$
$$|4-17|\geq 2$$
$$13\geq 2$$

$$e(1)=26, e(0)=16$$
$$\left|\sum e(0)-\sum e(1)\right|\geq 2$$
$$|16-26|\geq 2$$
$$10\geq 2$$

Conclusion:-

Here the researcher labels the Bloom Torus Graph, which satisfies the SmaranDachley Product Cordial Labeling that is a labeling $f:V(G)\rightarrow\{0,1\}$ with induced labeling $f(u)f(v)$ *on edge* $uv\in E(G)$ *that* $\left|v_f(0)-v_f(1)\right|\geq 2$ *and* $\left|e_f(0)-e_f(1)\right|\geq 2.$

Many research is done in this area and still work is being carried out for this particular Graph.

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