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RESEARCH ARTICLE

EFFECT OF INTERNAL HEAT AND VARIABLE ELECTRICAL CONDUCTIVITY ON CONVECTIVE MHD FLOW ALONG A VERTICAL ISOTHERMAL PLATE

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Abstract

The study investigates convective MHD flow of incompressible fluid of low Prandtl number along a vertical non-conducting plate in porous medium at constant temperature where fluid is sucked through a vertical plate with a constant suction velocity under the action of small internal volumetric heat generation and transverse magnetic field applied externally. The electrical conductivity of the fluid is considered to be temperature dependent variable. Incorporating the Boussinesqs approximation for the boundary layer flow and considering the action of local specific dissipation of mechanical energy due to permeability of the medium, numerical values of various flow parameters such as velocity, temperature, skin-friction, heat transfer etc. are calculated numerically, analysed graphically and conclusions are drawn.

Introduction:-

Since last few years convective flow of magnetohydrodynamics(MHD) fluid with variable fluid properties under various geometrical situations in presence of external magnetic field has been a great attention, because of its importance and wide ranging applications in various scientific, engineering and industrial fields. MHD convective flow of variable physical parameters has been investigated to a large extent by many researchers because of its numerous applications in the field of engineering and technology, industrial, extraction of geo-thermal energy, and in many more situations. In the field of nuclear technology MHD convection flow is employed to study the magnetic behaviour of plasmas in fusion reactors, liquid metal cooling of nuclear reactors, electro-magnetic casting etc. By choosing fluids of suitable magnetic field and electrically conductivity, one can control many metallurgical processes involving cooling of continuous strips etc. Variation of fluid properties along with the temperature field can affect the heat generation in MHD flow, (Herwig, et al., 1986). Fluids with small Prandtl numbers are generally free-flowing liquids with higher thermal conductivity, therefore they are naturally a better choice for heat conducting purpose. Liquid metals have small Prandtl number (Pr) of the orders 0.001~0.1 (e.g. Sodium(Na) Pr = 0.01, Mercury(Hg) Pr =0.03) have higher thermal conductivity that is why they are generally used as coolants. Even if there is small temperature difference between the surface and fluid they transport the heat; because of this, they are effectively used for the purpose of disposal of waste heat. This is why liquid metals (e.g. mercury) are used as coolant in nuclear reactor. In all these applications every parameter of the flow affects the flow, and contributes to the heat transfer. Therefore, variations of fluid properties are to be selected and observed properly. The natural convection flow along a vertical isothermal plate with fluids of low Prandtl number in presence of variable electrical conductivity and transverse magnetic field become a point of study for many authors. Flow of such kind of fluids at

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stagnation point has been discussed by (Pai, 1956). (Kay, 1966) reported that thermal conductivity of liquids with low Prandtl number varies linearly with temperature in range of 0^oF to 400^oF. (Arunachalam et al., 1978) studied forced convection in liquid metals with variable thermal conductivity and capacity. (Boracic et al., 2010) studied the unsteady plane MHD boundary layer flow of a fluid of variable electrical conductivity. (Chen, 1998) discussed about Laminar mixed convection flow adjacent to vertical, continuously stretching sheet. (Alam et al., 2011) have studied Heat and Mass Transfer in MHD free convection flow over an inclined plate with hall current. (Chakraborty et al., 2018) has studied about the MHD flow in porous medium with small internal heat generation under variable electrical conductivity.

Motivating with the above works, we have tried to investigate a fully developed convective MHD incompressible flow of low Prandtl number fluid through porous medium where fluid is sucked through a vertical plate with a constant suction velocity under the action of small heat generation and transverse magnetic field. The electrical conductivity of the fluid is considered to be temperature dependent variable. We have considered local specific dissipation of mechanical energy due to permeability of the medium which was seen absence in the paper (Alam et al., 2011). Governing equations are solved numerically using Runga-Kuta method where Shooting method is used in order to get the missing boundary values. The numerical values of various flow parameters such as velocity, temperature, skin-friction, heat transfer etc. are calculated numerically and analysed graphically followed by conclusions.

Formulation Of The Problem

We have considered steady laminar natural convection flow of a viscous incompressible fluid along a vertical non-conducting plate in porous medium at constant temperature T_w. The plate is placed vertically upward along x-axis while y-axis is perpendicular to it . It is considered that the fluid has internal volumetric heat generation Q within the fluid flow and the electrical conductivity of the fluid varies inversely with temperature (Boracic et al., 2010). A uniform magnetic field of intensity B_0 is applied normal to the plate. It is considered that the electrical field due to polarization of charges and Hall effect are negligibly small. Incorporating the Boussinesq approximation for the boundary layer flow and considering the action of local specific permeability of the medium, the equation of continuity, momentum & energy are given as below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{du}{dx}+v\frac{du}{dy}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma_1 B_0^2}{\rho}u-\frac{\vartheta}{K_1}u \qquad (2)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}+Q-\frac{\mu}{K_1}u^2 \qquad (3)$$

Here electrical conductivity σ_1 is variable of temperature as given below

$$\sigma_1=\frac{\sigma}{1+\varepsilon\theta} \qquad (4)$$

The boundary conditions are

Y = 0 : u = 0, v = 0, T= T_w

y → ∞ : $u\to 0,\ T\to T_\infty$ (5)

Solution Of Governing Equations

Introducing the stream function $\varphi(x,y)$ such that

$$u=\frac{\partial\varphi}{\partial y} \text{ and } v=-\frac{\partial\varphi}{\partial x} \qquad (6)$$

$$\text{where } \varphi(x,y)=4\upsilon f(\eta)\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \text{ and } \eta=\frac{y}{x}\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \qquad (7)$$

Following (Crepeau and Clarksean, 1997), the volumetric rate of heat generation is given as

$$Q=S\{k\left(\frac{T_w-T_\infty}{x^2}\right)\left(\frac{Gr}{4}\right)^{\frac{1}{2}}e^{-n}\} \qquad (8)$$

Since equation (6) is satisfied by equation (1), using equation (6), (7), (8) in equation (2) and (3), along with the relation (4), resultant coupled non-linear ordinary differential equations are

$$f^{'''}-2(f^{'})^2+3ff^{''}+\theta-\left[\frac{M}{1+\varepsilon\theta}+\frac{1}{Da}\left(\frac{4}{Gr}\right)^{\frac{1}{2}}\right]f^{'}=0 \qquad (9)$$

$$\theta^{''}+3Prf\theta^{'}+Se^{-n}-16\frac{\Pr E}{Da}\left(\frac{Gr}{4}\right)^{\frac{1}{2}}(f^{'})^2=0 \qquad (10)$$

The boundary conditions are reduced to

$$f(0)=0,\ f^{'}(0)=0,\ \theta(0)=1 \text{ when } \eta\to 0\ ;\ f^{'}(\infty)=0\ ,\ \theta(\infty)=0 \text{ when } \eta\to\infty \qquad (11)$$

Skin-Friction Coefficient at The Boundary Layer (τ)

$$\tau = \frac{(\tau)_{y=0}}{\frac{1}{2}\rho {u_0}^2}$$

$$= 2\left(\frac{Gr}{4}\right)^{\frac{1}{4}} f''(0) \qquad (12)$$

where, $(\tau)_{y=0} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$ shear stress at the plate.

$u_0 = \sqrt{\{g\beta x(T_w - T_\infty)\}}$, convective fluid velocity near the plate.

5. Nusselt Number (Nu)

The Rate of Heat Transfer in terms of Nusselt Number (Nu) at the plate is given as

$$Nu = \frac{qx}{K(T_w - T_\infty)} = -\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \theta'(0) \qquad (13)$$

where, $q = -K(\frac{\partial T}{\partial y})_{y=0}$

Solutions of Equations

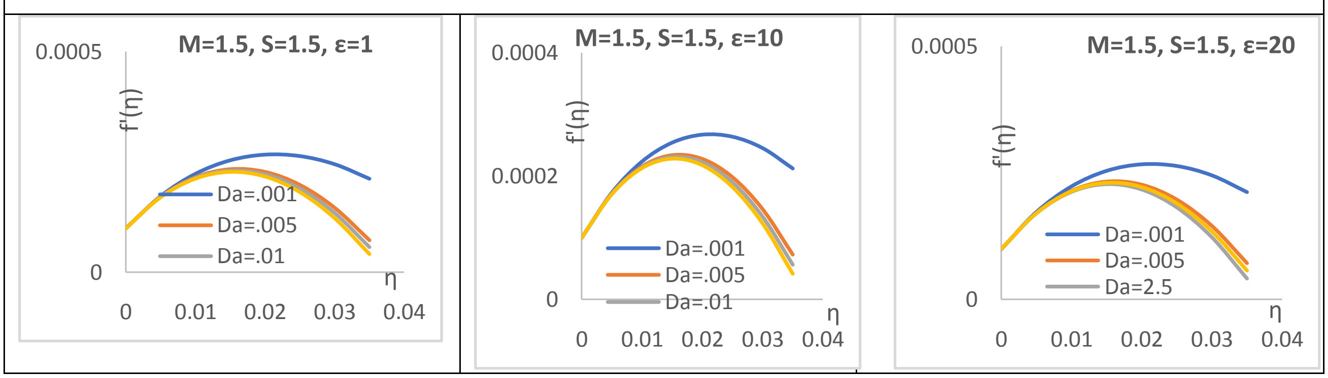
The equations (9 &10) subject to the boundary condition (11) are solved using fourth order Runge-Kutta method along with the Shooting-method (guessing method to find out the missing boundary values) for different values of physical parameters. To calculate the numerical results for physical quantities f, θ, τ and Nu, we have considered, Gr =10 because it relates to the problem of cooling in nuclear reactors; Pr = 0.023 since it is connected to the popular liquids metal mercury at 20^oc ; chosen n =1.0 arbitrarily. The physical parameters that affect fluid flow and energy transfer are varied as Da=0.001 to 2.5 ; ε=1 to 20 ; M=1.5 to 3.5 ; S=1.5 to 3.5. We consider the electrical conductivity of the liquid ε=1.0 as smaller, ε=10.0 as intermediate and ε=20 as larger. The numerical values of non-dimensional parameters for fluid velocity (f ′), temperature (T), skin friction at the plate (τ) and the rate of heat transfer in terms of Nusselt number (Nu) at the plate are obtained and figures are plotted (fig. 1.0 to 8.0) to show and analysis the nature of flow and heat transfer.

Technique For Numerical Solutions

The system of non-linear ordinary differential equation (9 &10) together with the boundary conditions (11) are numerically solved by Nachtsheim-Swigert shooting iteration method along with the fourth order Runge-Kutta initial value solver. (Alam et al., 2011), (Chakraborty et al., 2016) have also used this technique to solve this type of problems.

Results And Discussions:-

Fig.1 (i – ix); Velocity distribution, f '(η) vs η, at Gr =10, Pr =.023, n=1.0



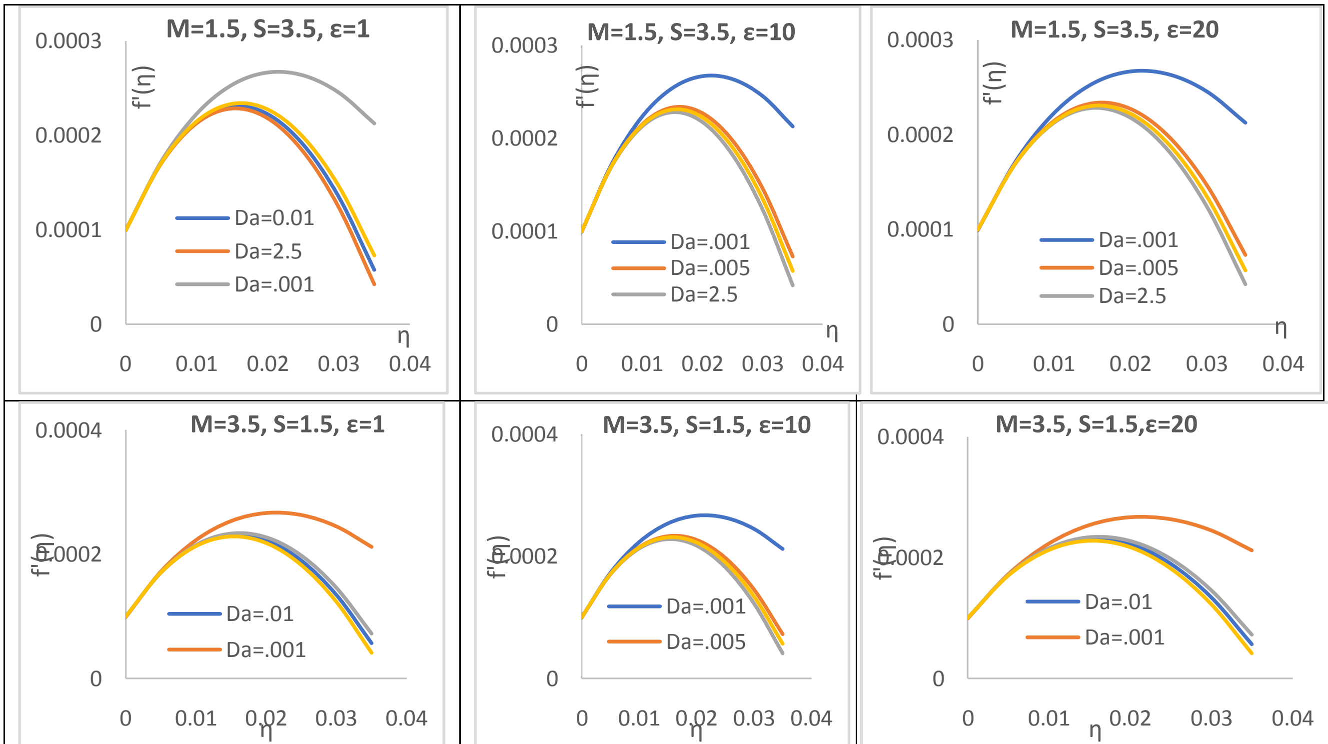


Fig.2 (i-ix); Velocity variation with ε at η=0.015, Gr =10, Pr =0.023, n=1

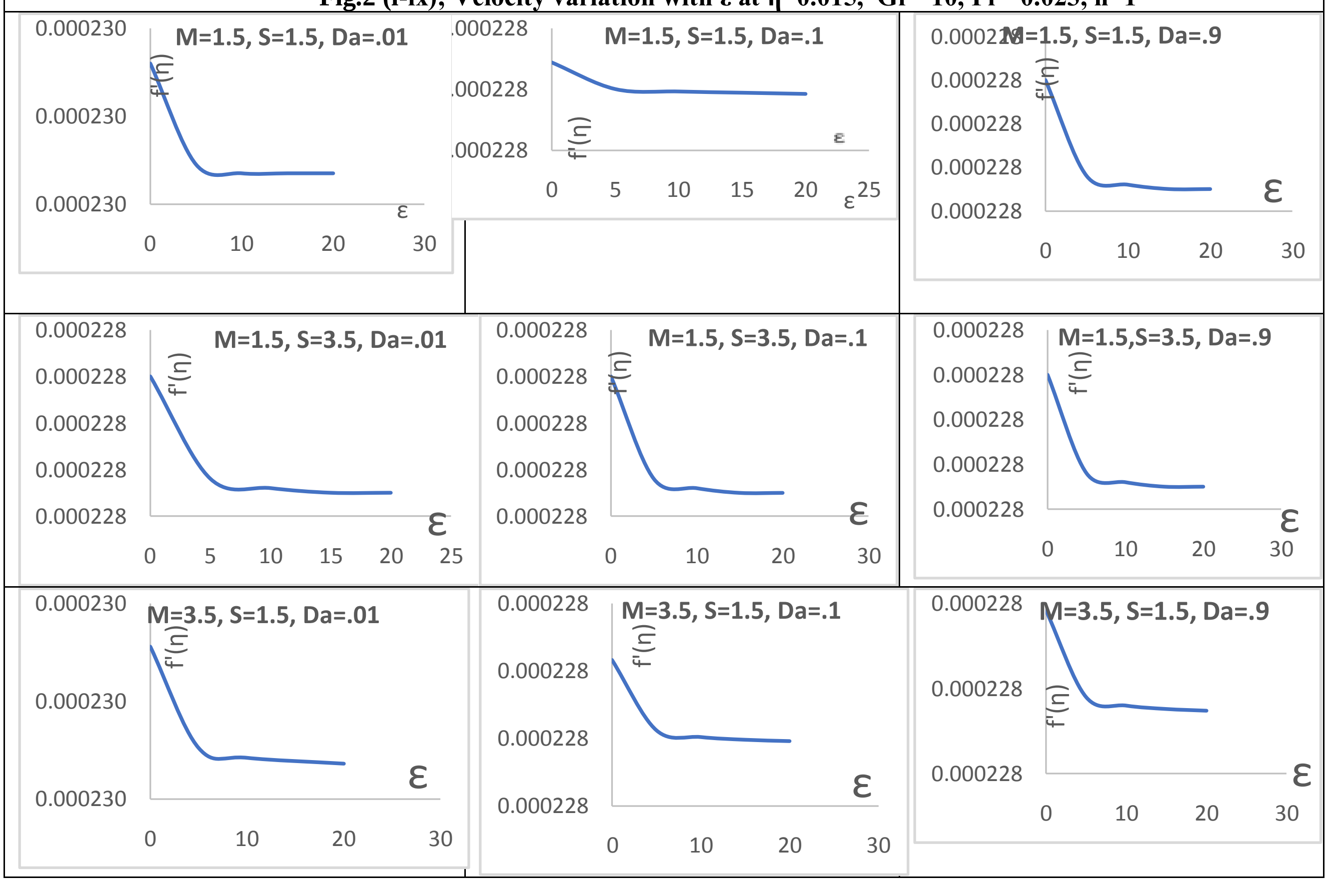


Fig.3 (i-ix); Velocity variation with Da; (f'(η) vs Da) at η=0.015, Gr=10, Pr=.023, n=1

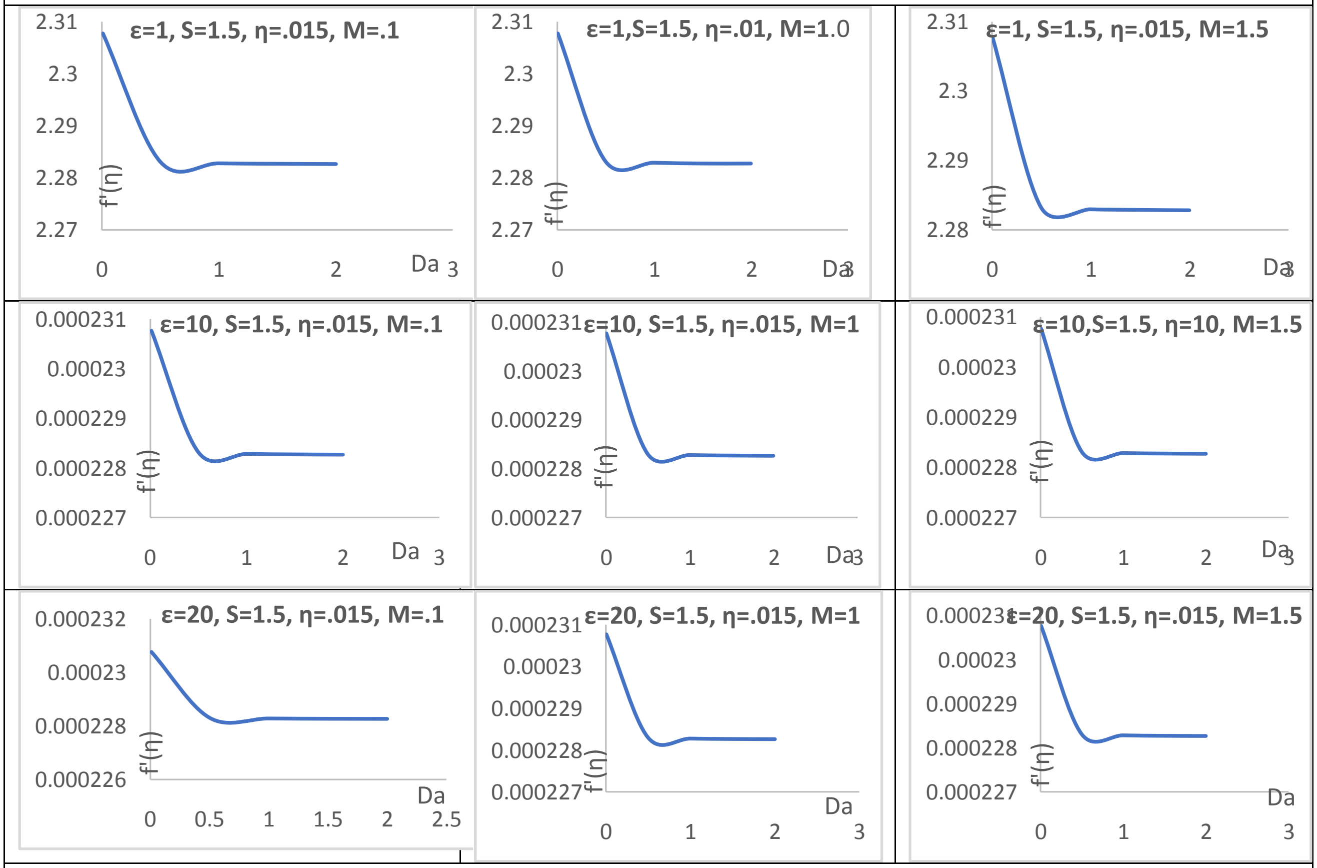
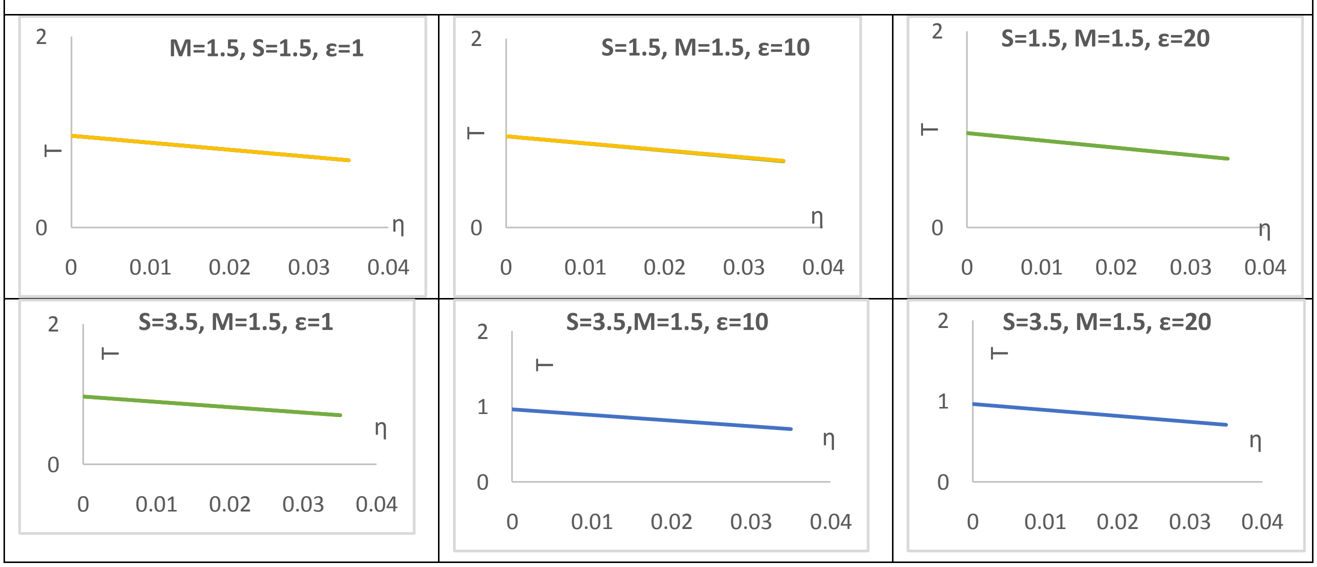


Fig.4 (i-ix); Temperature distribution, T Vs η at Gr=10, Pr=0.023, n=1



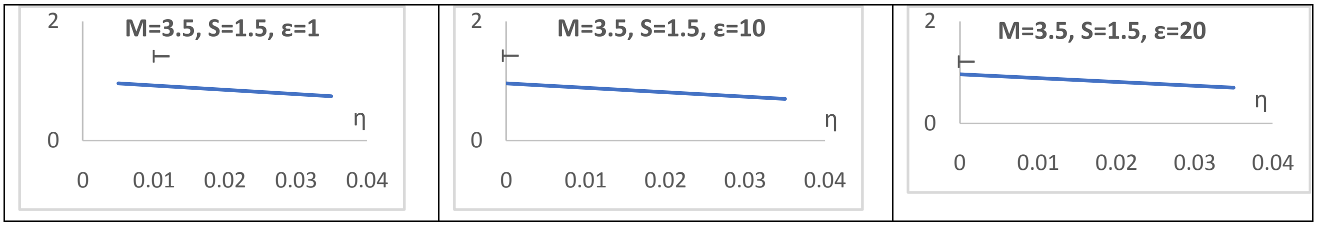
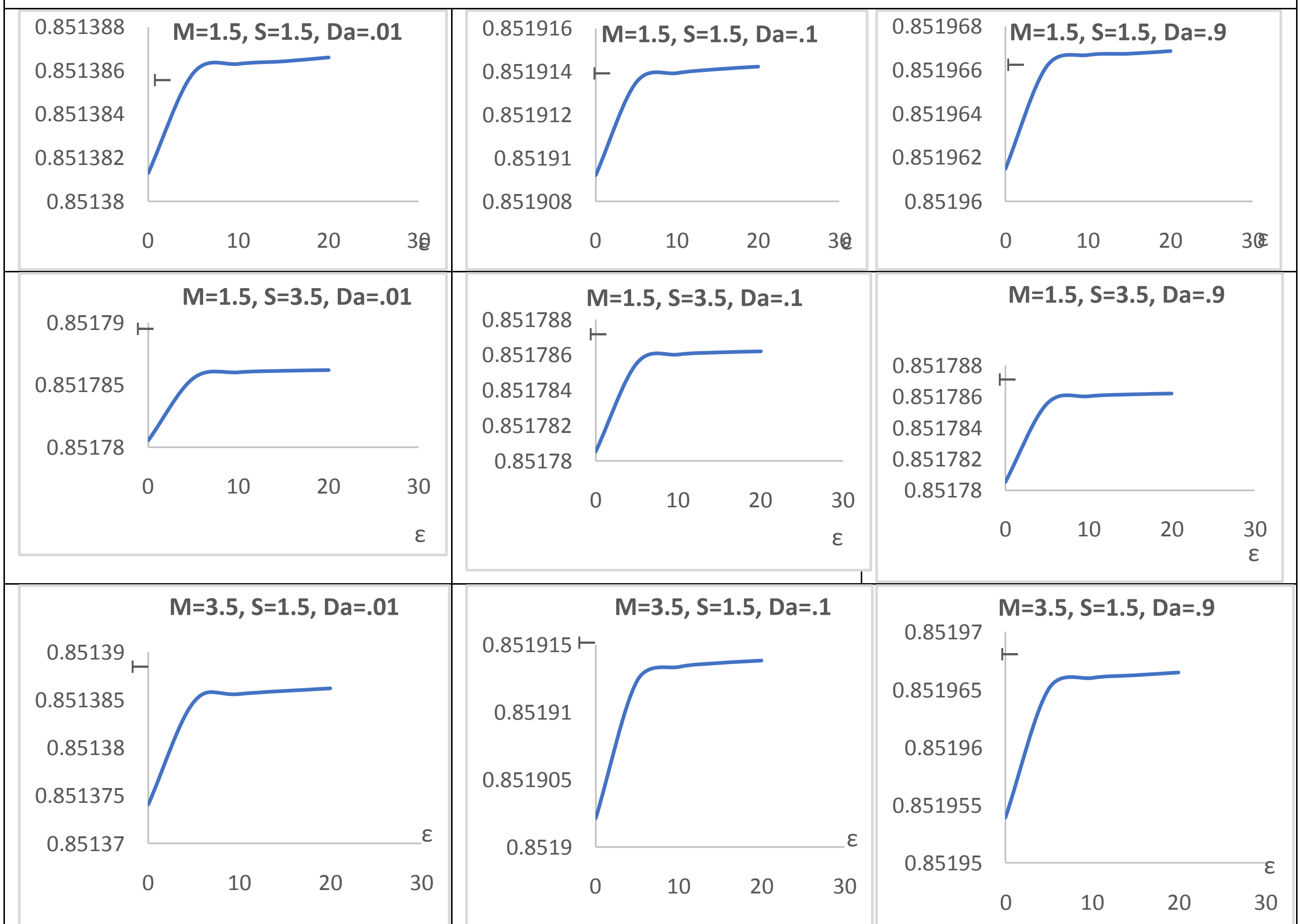
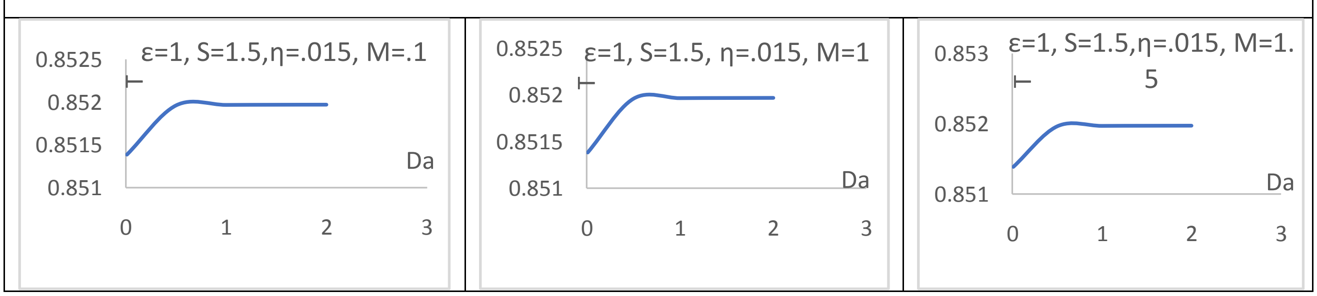


Fig.5 (i-ix); Temperature variation with ε at η=.015, Gr=10, Pr=0.023, n=1



ig.6 (i-ix); Temperature variation with Da, T Vs Da at η=.015, Gr=10, Pr=0.023, n=1



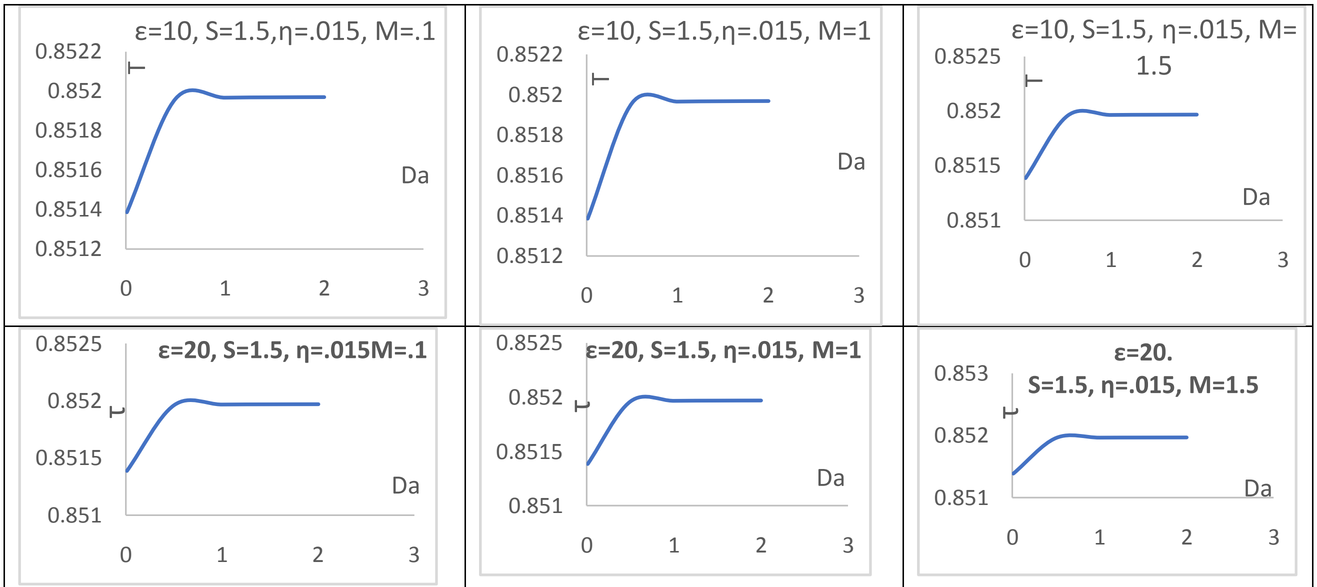


Fig.7 (i-ix); Skin friction variation(τ) with ε at η=0, Gr=10, Pr=0.023, n=1

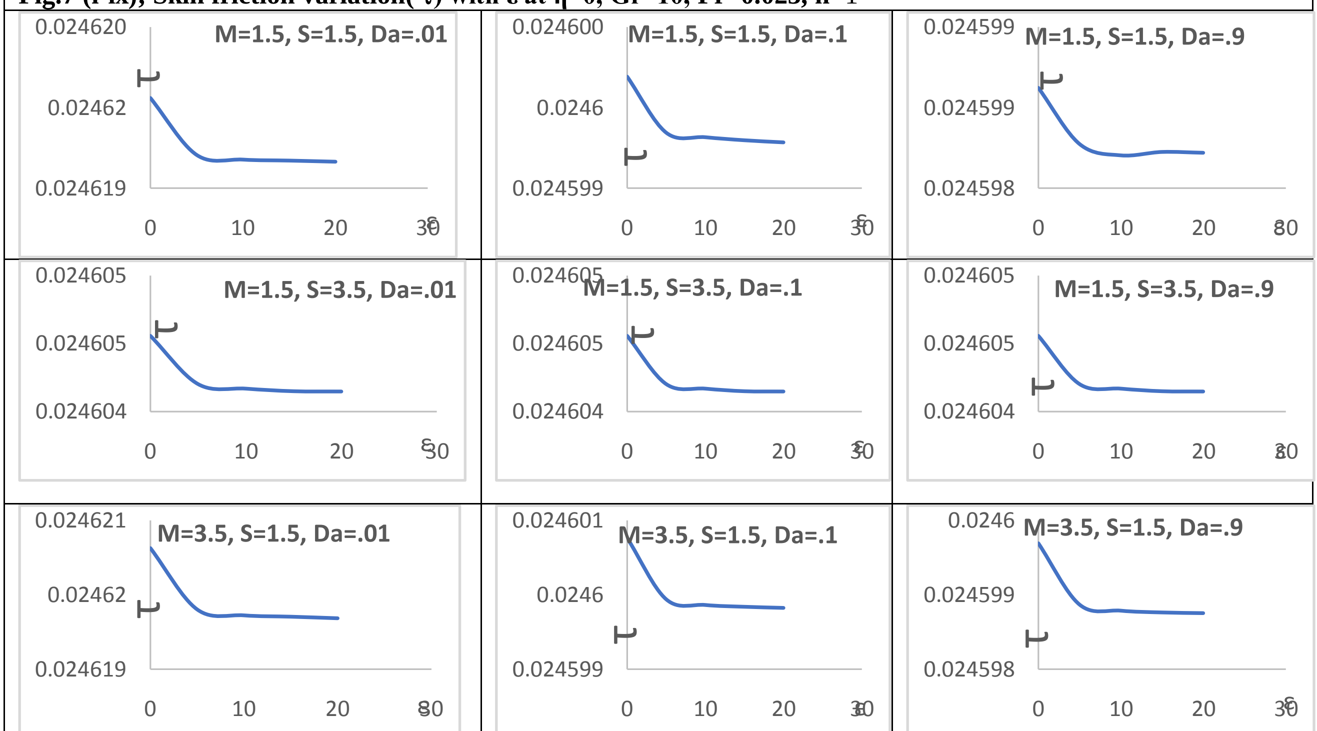
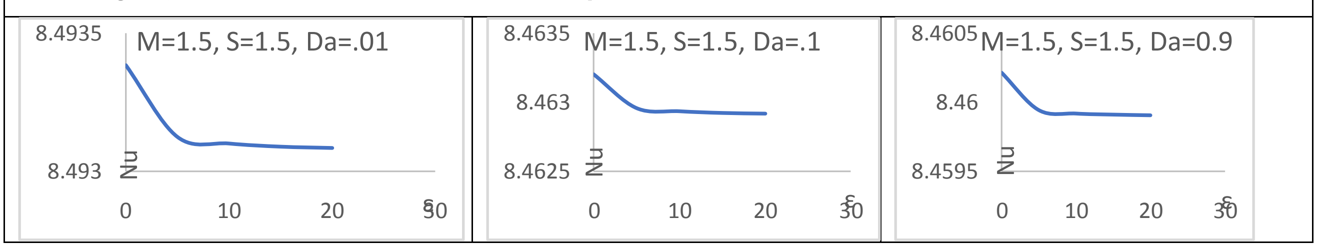
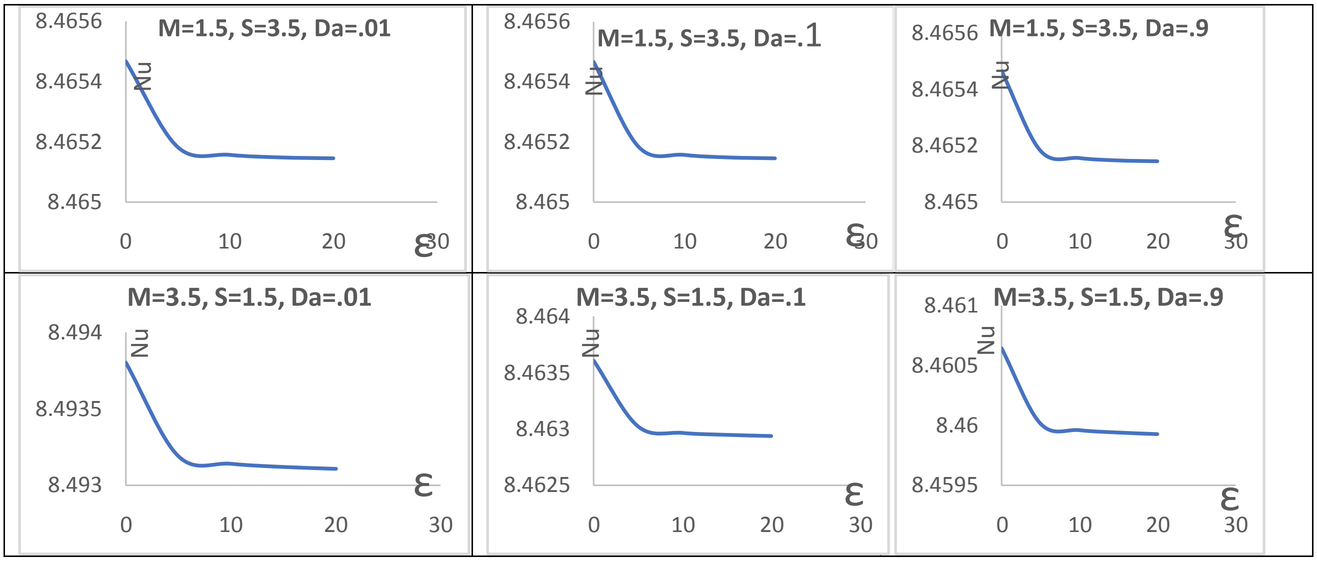


Fig. 8 (i-ix); Nusselt Number(Nu) with ε at η=0, Gr=10, Pr=0.023, n=1.0





In fig.1 (i-ix), velocity distribution f ′(η = 0 to η → ∞), the magnitude of f ′(η) is higher when Da is smaller [e.g. for η=0.015, S=1.5, M=1.5; at Da=0.001, f'(η)=0.000253890 ; while at Da=0.005, f'(η)=0.000233302]**;** f ′(η) increases slowly with the rise of ε [e.g. for η=.015, at ε=1.0, f ′ (η)=0.00022830 while at ε =10.0, f ′(η) = 0.000230767]; in fig.1 (i & iv), (ii & v) and (iii & vi), f ′ (η) increases with the increase of S [e.g. for η=0.015, at S=1.5, f ′(η) = 0.00022830 while at S=3.5, f ′ (η)=0.000230949]; in fig.1(i & vii), (ii & viii) and (iii & ix), f ′ (η) increases as M increases [e.g. for η =0.015 at M=1.5, f ′(η) = 0.00022830 while at M=3.5 , f ′(η) = 230797].

Fig.2 (i-ix); velocity variation with ε , within a smaller range of ε (~ 0.001 – 0.7), f ′ (η) rapidly decreases with rise of ε, whereas f ′(η) slowly decreases as ε ≥ 0.7 for the all values of S and M ; when Da is increased, f ′(η) decreases slowly [e.g. in fig.2 (i & ii) at ε=5; for Da=0.01, f ′ (η) = 0.000230769 while for Da=0.1, f ′(η)=0.000228500]; in fig.2(iv – vi) and (vii – ix) when S and M are increased f ′ (η) increases [e.g. at ε=10; for S=1.5, f ′ (η) = 0.000230767 while S=3.5, f ′ (η) =0.000230938; at ε=10; for M=1.5, f'(η)=0.000230767 while M=3.5, f'(η)=0.000230771].

Fig. 3(i-ix), for smaller range of Da(~ 0.001 to 0.5), f ′ (η) decreases with rise of Da thereafter f ′ (η) increases slowly with the increase of Da ; f ′ (η) increases when M is increased from 0.1 to 1.5 [e.g. Da=0.5, S=1.5 for M=0.1, f ′ (η) =0.000228310 while for M=1.0, f' (η) = 0.000228318] ; from fig.3 (i, iv, vi); f ′ (η) decreases with increase of ε (= 1 to 20) increases[(e.g. at Da=0.5, S=1.5, M=0.1 for ε=1, f ′(η)=0.000228310 while for ε=10 f ′ (η) =0.000228309].

Fig. 4(i-ix), T decreases with increase of η = 0 to η → ∞ . In fig4(i-iii) T increases with rise of ε [e.g. for η=0.015, Da=0.1; at ε=10 T=0.851913929 while at ε=20 T=0.851914227]. When M increases (1.5 to 3.5) , T decreases slowly (e.g. in fig (i & vii); at η=0.015 for M=1.5 T=0.851911783 while for M=3.5, T=0.851908207). and T decreases slowly with rise of S (1.5 to 3.5) [e.g. in fig (i & iv); at η=0.015 when S=1.5 then T=0.851911783 while for S=3.5 T=0.851731300].

Fig.5(i-iii), T increases with rise of ε [e.g. at η=0.015, Da=0.1; for ε=10, T=0.851913929 while for ε=20, T=0.851914227] ; in fig.5 (i-ix), within smaller range of ε (~ 0.1 - 5.0) T increases sharply with ε thereafter T increases slowly with rise of ε ; T decreases when S increases from 1.5 to 3.5 [e.g. in fig.5(i&iii); for S=1.5 T=0.851385891 while for S=3.5 T=0.851205289]; in fig.5 (i & vii)), (ii & viii) and (iii & ix) T decreases when M increases from 1.5 to 3.5 [e.g. in fig.5(i & vii); for M=1.5, T=0.851385891 while for M=3.5 T=0.851384759].

Fig 6; in fig (i-ix) T increases rapidly with Da within smaller range of it (~ 0.1 to 0.6) and thereafter T increases slowly. T decreases when M increases from 0.1 to 1.5 [e.g. From fig.6(i&ii); at Da=0.5, for M=0.1 T=0.851961613 while for M=1.0, T=0.851959705]. T increases with the rise of ε from 1.0 to 20 [e.g. in fig.6(i&iv); at Da=0.5, for ε=1.0, T=0.851961613 while for ε=10.0, T=0.851961792]. T decreases slowly with rise of S from 1.5 to 3.5 [e.g. in fig6.(i & iv); at η=0.01 for S=1.5, T=0.851911783 while for S=3.5 T=0.851731300].

Fig.7 (i-ix) τ decreases rapidly within a smaller range of ε(~ 0.1-0.5), thereafter decreases slowly; τ decreases when Da is increased from 0.10 to 0.9 [e.g. in fig.7(i&ii); at ε=5.0, for Da=0.01, τ=0.024619706 while for Da=0.1, τ=0.024599845]. τ increases when S is increased from 1.5 to 3.5 [e.g. in fig.7(i&iv) ; at ε=5.0, for S=1.5, τ=0.024619706 while for S=3.5, τ=0.024625642] ; τ increases when M is increased from 1.5 to 3.5 [e.g. in fig.7(i&vii); at ε=5.0, for M=1.5, τ = 0.024619706 while M=3.5, τ = 0.024619805]

Fig.8 (i-ix) the rate of heat transfer in terms of Nusselt Number(Nu) falls rapidly within smaller range of ε (~0.1-0.5), thereafter decreases slowly; fig.8 (i-iii) Nu decreases slowly when Da is increased from 0.01 to 0.9 [e.g. in fig.8(i&iii) at ε=5; for Da=0.01, Nu=8.493123055 while for Da=0.9, Nu=8.459942818]; Nu increases with increase of S from 1.5 to 3.5 [e.g. fig.8(i&iv) at ε=5; for S=1.5 Nu=8.493123055 while for S=3.5 Nu=8.498356819] ; Nu increases when M is increased from 1.5 to 3.5 [e.g. in fig.8(i& vii) at ε=5.0; for M=1.5, Nu=8.493123055 while for M=3.5, Nu=8.493187904].

Conclusions:-

1. Under the action of internal heat, applied magnetic field, fluid velocity decreases rapidly within the smaller range of permeability thereafter it increases gradually. Permeability has an effect on fluid velocity that decreases with the increase of electrical conductivity. For higher magnetic field, the fluid velocity increases for all values of electrical conductivity and permeability. Fluid temperature increases with the increase of conductivity, the rate of increase is higher within the smaller value of conductivity.
2. Permeability has an effect on variation of fluid temperature with electrical conductivity; temperature increases with the increase of permeability. Temperature is decreases when the magnetic field is increased.
3. Skin-friction at the plate decreases with the increase of electrical conductivity; the rate of decreases is higher within the smaller value of it. Skin-friction decreases for higher medium permeability. Skin-friction increases when magnetic field is increased.
4. The rate of heat transfer at the plate decreases with the increase of electrical conductivity, the rate of decrease is higher within the smaller value of conductivity. Rate of heat transfer decreases when medium permeability is higher. When the magnetic field is increased the rate of heat transfer increases.

Appendices	
f: dimensionless stream function g: Acceleration due to gravity B_0: magnetic field intensity x, y: cartesian coordinates u, v: velocity components along x- and y-directions, respectively S: heat generation parameter Nu: Nusselt number C_f: Skin-friction coefficient K: Permeability of medium T: Temperature of the fluid T_∞: Temperature of fluid far away from plate β: coefficient of thermal expansion υ: kinematic viscosity $(=\frac{\mu}{\rho})$ Pr: Prandtl number $(=\frac{\mu C_p}{K})$ Gr: Grashof number $\{=(\frac{g\beta(T_w-T_\infty)x^3}{\upsilon^2})\}$	Q: volumetric rate of heat generation $\{=K(\frac{T_w-T_\infty}{x^2})(\frac{Gr}{4})^{\frac{1}{2}}e^{-n}\}$ Da: Darcy number $(=\frac{K_1}{x^2})$ C_p: Specific heat at constant pressure Θ: dimensionless temperature $(=\frac{T-T_\infty}{T_w-T_\infty})$ ρ: density of fluid υ: kinematic viscosity $(=\frac{\mu}{\rho})$ ε: electrical conductivity parameter μ: coefficient of viscosity β: coefficient of thermal expansion Κ: coefficient of thermal conductivity σ: electrical conductivity η: similarity variable Ψ: stream function σ_1: variable electrical conductivity $'$: differentiation with respect to η

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