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RESEARCH ARTICLE

MODELING AND FORECASTING OF THE VOLATILITY OF ALL-SHARE INDEX OF NIGERIAN STOCK EXCHANGE MARKET

Onyishi E.I.[1], C.N. Okoro.[1] and Ibina E.O.[2]

1. Department Of Industrial Mathematics and Applied Statistics Ebonyi State University, Abakaliki, Ebonyi State, Nigeria.
2. Department of Mathematics and Statistics, Akanu Ibiam Federal Polytechnic, Unwana Afikpo. Ebonyi State.

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Abstract

This paper modeled and forecasted the volatility of the Nigerian Stock Exchange Market while incorporating trade volume from January 2022 to December 2024 using both symmetric and asymmetric volatility models such as ARCH, GARCH, GARCH-M, EGARCH, TGARCH, and PGARCH each in Normal, Student's-t and Generalized Error Distributions. The results revealed the following facts: the volatility of the market returns could best be modeled using PGARCH (1, 3) model in normal error distribution, there is evidence of positive leverage effect which reflects that positive shocks induce a larger increase in volatility when compared to the negative shocks of equal magnitude, news about volatility from the previous periods has an explanatory power on current volatility, there is volatility clustering in the market returns, changes in the market returns are explosive, the market risk and returns are inversely related, and the out-sample forecast of the market volatility for the next 2years shows that there will be a low volatility from March to September of the year 2023 and then an increase in volatility up to December 2024. However, for policy and investment decision the paper recommends taking the news on volatility into consideration while forming expectations in the market, investors could invest more within March to September 2023 as the market risk and returns are inversely related, following the forecast estimate that the volatility will be on increase from October 2023 up to December 2024 within which investors are likely to invest less as the market risk and returns are inversely related; monetary authority could contract the money supply so as to control the problem of idle cash if the percentage of the investors in the risk-averse category is greater than that of risk-lovers and risk diversifiers categories.

Introduction:-

A Stock market is a place where stocks, bonds, and other securities are traded; but stock exchange is the body that runs a stock market to help people, firm or companies generate capital. As a primary Market, it provides an avenue for people, firm or companies to sell new shares and bonds to investors. The stock market in Nigeria is run by the Nigerian stock exchange (NSE).According to Humplay (2020), the Nigeria stock exchange (NSE) was incorporated

Corresponding Author:- Onyishi E.I.
Address:**-** Department Of Industrial Mathematics and Applied Statistics Ebonyi State University, Abakaliki, Ebonyi State, Nigeria.

on September 15, 1960 as the Lagos stock exchange and a non-profit making organization, limited by guarantee following the recommendation of the professor R.H. Barbacks committee set up to examine the viability of securities exchange in Nigeria. On June 5th,1961, the exchange opened its door for business. In December 1977, it came to be called the Nigerian stock exchange (NSE). The NSE has a president and Council members, chairman and Board of Directors who are elected at each annual general meeting by members of the exchange. Concisely, the Nigerian stock exchange is the only perfect type of organized market in that the commodities traded on it have little or no cost.

Objectives Of The Nigerian Stock Exchange

The Objectives of the Nigerian Stock Exchange as stated in its memorandum of Association are to create an appropriate mechanism for capital formation and efficient allocation of resources among competing projects, to provide special financing strategies for those projects with long-term gestation period, to maintain discipline in the capital market, to broaden share ownership practice and also maintain fair prices for securities.

Why We Should Invest In The Stock Market

The stock market gives you higher returns than investing in bonds, treasury bills, certificates of deposits (CD's) or putting your money in a saving account. Investment in the stock market helps you stay above the inflation rate. You can invest in the stock market with very little money.

When you buy shares in the stock market, you get to become a shareholder or part owner of a public company with all the privileges entitled to shareholders and do not need to be an expert in shares or even bother yourself about which shares to buy, investment bodies called unit-trusts or mutual funds can manage your investment for you for a small commission.

Financial analyst is the intercessor of data and information as they conduct retrospective analysis towards firm and financial forecast to generate future information. Forecast conducted by financial analyst could help the firm to improve the quality of their financial reporting (Warsono et al.,2019).

The widely used symmetric volatility models include autoregressive conditional heteroscedasticity (ARCH) model, generalized autoregressive conditional heteroscedasticity (GARCH) model and GARCH-in-Mean (GARCH-M) model. While the asymmetric models include EGAECH, TGARCH and PGARCH designed to capture the issue of asymmetric effect which the symmetric models are not be able to capture. (Ibrahim, 2017).

Furthermore, the modeling in this paper include the all share index (ASI) which the market returns will be derived from and the lag of lagged trading volume (logVolumet-1) as well as the structural breaks. All are incorporated in the conditional variance equation. The rationale for Lamoureux and Lastrapes (1990) to propose the use of the lag of trade volume instead of the contemporaneous trade volume series is that it may not be strictly exogenous to stock market returns. More so, the logic for not using the absolute lag of the trade volume series but its lagged lag is to obtain efficient estimate (Ekong and Onye, 2017). Nonetheless, the motivation to argument the lagged lag of the trade volume in the conditional variance equation is to solve for the likely problem of simultaneity bias in the conditional variance specification. Moreover, the rational for accommodating structural breaks in the conditional variance equation is to smooth the sudden shifts in the variance due to the fact that it is the specification that contains the persistence parameter which ignoring break in the equation could lead to over estimation of the parameter. However, the structural breaks will be added to the equation as dummy variables that take value 1 as the break occurs in conditional volatility onwards and otherwise it takes value 0.

Kuhe (2018) modeled the volatility persistence and asymmetry with breaks in the Nigerian stock returns using daily closing quotations of stock prices from 3rd July, 1999 to 12th June, 2017. The study used GARCH, EGARCH, and GJR-GARCH models with and without structural breaks. The estimates without breaks provide evidence of high persistence of shocks in the returns series but when incorporating breaks the study found significant reduction in shocks persistence. However, there is evidence of asymmetry in the returns series where positive shocks induce a larger increase in volatility when compared to the negative shocks of equal magnitude and the model that best fits the series is the EGARCH (1, 1) model.

Usman et al. (2018) investigated the volatility of Nigeria stock exchange market returns from January 1996 to December 2015 with particular emphasis to the period of financial boom. The estimation techniques employed include GARCH, GARCH-M, EGARCH, CGARCH and GRJGARCH models. The results obtained revealed that the best and the worst fit models are CGARCH (1, 1) and EGARCH (1, 1) for the training period while for the testing period are ARCH (1) and GARCH (2, 1).

It has been empirically verified that adding the logged lag of trading volume plays a significant role in studying the market volatility. However, the contribution of structural breaks in the analysis has been neglected while incorporating breaks proved to be important in volatility analysis. In addition, none of the studies incorporate structural breaks and trade volume in one model.

Methodology:-

The methodology adopted in this study is autoregressive conditional heteroscedasticity (ARCH) and its generalization (GARCH) models while taking the trade volume and structural breaks into consideration. The methodology has two sections, namely data and variable, and model specifications.

Data and Variable

This paper makes use of the monthly difference change of All Share Index (ASI) of Nigerian Stock Exchange (NSE) market which constitutes monthly equity trading of all listed and quoted companies in the Nigeria Stock Exchange. The paper also makes use of monthly log of lagged trading volume series (logVolumet-1) which is the number of shares or contracts traded in a security or an entire market during a given period of time. For every buyer, there is a seller, and each transaction contributes to the count of total volume. That is, when buyers and sellers agree to make a transaction at a certain price, it is considered one transaction. If only five transactions occur in a day, the volume for the day is five (Hayes, 2019). However, the ASI and trade volume data consist of monthly observations totaling of 404 observations from January 1988 to December 2021 sourced from the Central Bank of Nigeria (CBN) statistical bulletin (2021). Hence, the market returns was calculated as: $r = \log\left(\frac{P_t}{P_{t-1}}\right) = [\log(P_t) - \log(P_{t-1})]$ where, r is the monthly returns for period (t), Log is the natural logarithm, P_t and P_{t-1} denote the closing market index of NSE at the current month and previous month respectively, thus the All Share Indices for months (t) and (t – 1).

Model Specifications

The first step in the model specifications involve specifying a model for the mean returns series known as conditional mean equation and this can be specified as:

$r_t = \mu + e_t$ (1) However, the second step in the model specifications is based on the following symmetric and asymmetric conditional heteroskedasticity models where both the structural breaks and trade volume are inclusively modeled in.

Symmetric Volatility Models

Symmetric volatility models are heteroscedasticity models where the conditional variance depends only on the magnitude of the return of an asset and not on its sign. The widely used symmetric volatility models include autoregressive conditional heteroscedasticity (ARCH) model, generalized autoregressive conditional heteroscedasticity (GARCH) model and GARCH-in-Mean (GARCH-M) model. However, these models discussed as follows:

ARCH Model:

An autoregressive conditional heteroskedastic (ARCH) pioneered by Engel (1976). It is a model where the variance of the error term at time t regresses against the p lags of the squared error terms at lag order p. For example, if the variance is a function of only last period volatility then the ARCH process is known as ARCH (1) and with dummy variables and trade volume in the conditional variance equation it can be specified as: ARCH (1):

$\delta_t^2 = \propto_0 + \sum_{i=1}^{n} D_i + \propto_1 \mu_{t-1}^2 + \theta \log \text{volume}$ (2), where δ_t^2 is the variance of the error term / conditional variance at time t, $\propto_0$ is constant, n represents total number of breaks, D_i represents dummy variables taking value 1 as the break comes out in conditional volatility and zero for else, $\propto_1$ is the coefficient of the ARCH term, μ_{t-1}^2 is lag one of the squared error terms, and θ is the coefficient of the trading volume.

However, the process may depend on any number of lagged volatilities i.e. ARCH (p) which can be is specified as: ARCH (p): $\delta_t^2 = \propto_0 + \sum_{i=1}^{n} D_i + \propto_1 \mu_{t-1}^2 + \ldots + \propto_p \mu_{t-p}^2 + \theta \log \text{volume}_{t-1} = \sum_{i=1}^{n} D_i + \propto_1 \mu_{t-1}^2 + \theta \log \text{volume}_{t-1}$ (3)

where $\propto_1 > 0$; $\propto_j \geq 0$; and θ <, >0. However, the rationale for θ <, >0 is that the use of trading volume has no imprecise role in financial research yet, however, Lamoureux and Lastrapes (1990: 4) and Tiang and Guo (2006:12) assumed $\theta > 0$.

GARCH Model:

Bollerslev (1986) and Taylor (1986) consider that the process of ARCH may cause a large number of parameters to be estimated without any precision and therefore, they generalized the process as generalized autoregressive conditional heteroscedastic (GARCH). Hence, a GARCH model is an extension of ARCH model employed to avoid having large number of parameters to be estimated and therefore, making it less likely to violate the non-negativity constraint (Campbell et al., 1998; Pindyck and Rubinfield, 1998). A GARCH model is a model in which the variance of the error term at time t regresses against the p lag of the squared error terms and the q lag of the variance of the error term. Therefore, the model allows the variance of the error terms / conditional variance to be dependent on its lags and the lags of the squared error terms and this is the advantage of GARCH model over ARCH model. As an extension of ARCH process, a simple GARCH (1, 1) model can be represented as follows: $\delta_t^2 = \propto_0 + \sum_{i=1}^{n} D_i + \propto_1 \mu_{t-1}^2 + \ldots + \beta_1 \delta_{t-1}^2 + \theta \log \text{volume}_{t-1}$ (4) where δ_t^2 is the variance of the error term / conditional variance at time t, $\propto_0$ is constant, n represents total number of breaks, D represents dummy variables taking value 1 as the break comes out in conditional volatility and zero for else, $\propto_1$ is the coefficient of the ARCH term, μ_{t-1}^2 is the term of the last period variance of the error term / conditional variance, β_1 is the coefficient of the GARCH term, and δ_{t-1}^2 is the last period variance of the error term / last period's forecast variance. However, the process may depend on any number of lagged volatilities i.e. GARCH (p, q) which can be specified as: GARCH (p, q): $\delta_t^2 = \propto_0 + \sum_{t=1}^{n} D_t + \propto_1 \mu_{t-1}^2 + \cdots + \propto_p \mu_{p-1}^2 + \beta_1 \delta_{t-1}^2 + \beta_p \delta_{t-p}^2 = \propto_0 + \sum_{t=1}^{n} D_t + \sum_{j=1}^{p} \alpha_j \mu_{t-1}^2 + \sum_{j=1}^{q} \beta_k \delta_{t-k}^2 + \theta \log \text{volume}_{t-1}$ (5) where the p and q are the respective orders of the GARCH processes, However, for the requirements of stationarity in basic GARCH model: $\alpha_j + \beta_k < 1$, $\alpha_j \geq 0$, and $\beta_k \geq 0$.

GARCH-in-Mean (GARCH-M):

The theory of positive risk premium states that stocks with high volatility have higher probability of high return. To model such assertion one might consider GARCH-M model developed by Engle et al. (1987)where "M" stands for GARCH in the mean where the conditional mean of return series depends on its conditional variance (Maqsood et al. 2017).

A GARCH-M model is defined by two equations, namely the conditional mean equation and the variance equation, and this can be depicted using a simple GARCH (1, 1) as follows:

Conditional mean equation: $r_t = \mu + \lambda \delta_t^2 + e_t$ (6)

Variance equation: $\delta_t^2 = \propto_0 + \sum_{t=1}^{n} D_t + \sum_{j=1}^{p} \alpha_j \mu_{t-1}^2 + \sum_{j=1}^{q} \beta_k \delta_{t-k}^2 + \theta \log volume_{t-1}$ (7) where μ constant, λ is also constant but is called the risk premium parameter and a positive λ indicates that the return is positively related to its volatility and therefore, as volatility increases; the returns correspondingly increase while a negative λ implies that the returns and volatility are inversely related.

Asymmetric Volatility Models

Symmetric volatility models are based on the assumption that the impact of positive shocks on the market returns has no significant difference when compared to the negative shocks of equal magnitude, and therefore, the same. In such models, the conditional variance cannot respond asymmetrically to rises and falls in the error term μ_t while such effects are important in the behaviour of stock returns (Ahmed and Suliman, 2011). However, the tendency for volatility to decline when returns rise (good news / positive shocks) and to rise when returns fall (bad news / negative shocks) is often called leverage effect and this cannot be captured using symmetric volatility models (Enders, 2004). Therefore, symmetric volatility models may not be appropriate in analyzing the volatility behavior of the returns of financial assets. Consequently, new models called asymmetric models were introduced to deal with this shortcoming. These models include EGAECH, TGARCH and PGARCH. However, the models have their distinguished methods of capturing asymmetric effect, but their uniform objective is to capture the asymmetric effect (Ibrahim, 2017).

Exponential GARCH (EGARCH) Model:

Exponential GARCH is an asymmetric heteroscedasticity model introduced by Nelson (1991) to provide for the shortcoming of GARCH model on nonnegative constraints on the parameters, and thus, to allow for asymmetric effects between positive and negative asset returns by logging the conditional variance so that there are no restrictions on these parameters (Mhmoud and Dawalbait, 2015; Kuhe and Chiawa, 2017). The EGARCH model can be stated as:

$$\text{Log}(\delta_t^2) = \propto_0 + \sum_{i=1}^{n} D_i + \sum_{j=1}^{p} \alpha_j \left\{\left|\frac{\mu_{t-j}}{\delta_{t-j}}\right|\right\} + \sum_{k=1}^{r} \beta_k \log\left(\delta_{t-j}^2\right) + \sum_{l=1}^{r} r_l \left[\frac{\mu_{t-1}}{\delta_{t-1}}\right] + \theta \text{ log voume}_{t-1} \quad (8)$$

where the difference between α_j and r_l is the shocks impact on the conditional volatility, β denotes the logged GARCH term, r represents the asymmetric or leverage effect parameter and it is sign determines the nature of the asymmetric volatility. If $r = 0$ then there is no asymmetric volatility but if $r \neq 0$ then there is asymmetric effects of shocks on volatility where a positive value signifies that positive shocks increase the volatility more than negative shocks while negative value indicates that negative shocks increase the volatility more than positive shocks(Brooks, 2014).

Threshold GARCH (TGARCH) Model:

This is an asymmetric GARCH model which allows the conditional variance to have a different response to past negative and positive innovations, thereby accounting for the possible asymmetries. The TGARCH model can be expressed as:

$$\delta_t^2 = \propto_0 + \sum_{i=1}^{n} D_i + \sum_{j=1}^{p} \alpha_j\, \mu_{t-j}^2 + \sum_{k=1}^{r} \beta_k \delta_{t-j}^2 + \sum_{l=1}^{r} r_l\, \mu_{t-1}^2 d_{t-1} + \theta \text{ log voume}_{t-1} \quad (9)$$

where d_{t-1} is a dummy variable and can be specified as:

$$d_{t-1} = \begin{cases} 1 \text{ if } \mu_{t-1} < 0, \text{ bad news} \\ 0 \text{ if } \mu_{t-1} \geq 0, \text{ good news} \end{cases} \quad (10)$$

The coefficient r is called leverage term. In the model, effect of good news shows their impact by α, while bad news shows their impact by α + r. In addition, if r ≠ 0 then the news impact is asymmetric, and thus leverage effect, otherwise the modal collapses to symmetric GARCH**.** The sign of the leverage effect is the opposite compared to the E GARCH, and if r > 0, then negative shocks will increase the volatility more than positive shocks, while if r < 0, then positive shocks will increase the volatility more than negative shocks (Dutta,2014).

Power GARCH (PGARCH) Model:

Power GARCH (PGARCH) is an asymmetric GARCH model developed by Ding et al. (1993). PGARCH is different from other GARCH families in the sense that it models the standard deviation rather than the variance as in most of the GARCH families. In PGARH the power transformation is endogenized rather than fixed arbitrarily. Among its different advantages, the PGARCH structure is flexible enough to nest both the conditional variance (Bollerslev) and the conditional standard deviation (Taylor) models as particular cases. It thus provides an encompassing framework for model analysis and selection (Ann-Sing and Mark, 2005). The model also offers the opportunity to estimate the power parameter δ instead of imposing it on the model (Ocran and Biekets, 2007). The PGARCH (p, q) can be specified in the following form:

$$\delta_t^2 = \propto_0 + \sum_{i=1}^{n} D_i + \sum_{j=1}^{p} \alpha_j (/\, \mu_{t-j} / - r_l\ \mu_{t-1})^{\delta} + \sum_{k=1}^{r} \beta_k \delta_{t-j}^{\delta} + \theta \text{ log voume}_{t-1} \quad (10)$$

where α_j and β_k are the parameters of ARCH and GARCH respectively, r_l is the leverage effect parameter where a positive value of r_l means that positive shocks are associated with higher volatility than negative shocks and vice versa, and δ is the parameter for the power term; δ> 0,
/r / ≤ 1 for j = 1, 2,….., r, y = 0 for all j<r and r ≤ p. However, when δ = 2, the model simply becomes a standard GARCH model that allows for leverage effect while for δ = 1, the conditional standard deviation will be estimated (Ahmed and Suliman, 2011; Maqsood et al., 2017).

Estimation Techniques:

The analysis started by estimating the descriptive statistics in order to examine the descriptive properties of the market returns. Next is the unit root tests of the market returns in order to verify whether the returns are stationary and to achieve that, following Onwukwe and Isaac (2011), Banumathy and Azhagaiah (2015), and Maqsood et al. (2017), the augmented Dickey-Fuller (ADF) (1979) and Philips-Perron (PP) (1988) unit root tests were used in addition to Ng-Perron (Ng and Perron, 2001) unit root test because the ADF and PP unit root tests were reported to suffer from severe size distortions and low power problems depending on the sample size. The Ng-Perron test accumulates intellectual heritage of number of previous tests e.g. Elliot et al. (1996), etc. More so, the test emphasizes on adjusting unit root tests for specification of lag length, and thus, a test optimal in choosing lag length where a set of four test statistics proposed for testing unit root, namely the modified detrended Za transformation of the standardized estimator(MZa), the modified detrended Zt transformation of the conventional regression t statistic(MZt), the modified Bhargava R1statistic(MSB) and the ERS modified detrended point optimal statistic(MPT). Afterward, is the Bai-Perron (2003) multiple breakpoints test. Subsequently, is the test for heteroscedasticity using Lagrange Multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) and after the evidence of heteroscedasticity was found; the paper proceed with the estimation of both the symmetric and asymmetric volatility models. Last, is forecasting, which starts with the forecasting performance between the best two models selected by the information criteria under symmetric and asymmetric volatility models using three statistical error functions, namely the root mean square error (RMSE), mean absolute error (MAE), and mean absolute percent error (MAPE). Then, in an attempt to test the forecasting performance of the best selected model, a graphical comparison was made between the original market returns series and the conditional variance of the estimated selected model. Moreover, in an attempt to again test the forecasting performance of the best selected model, full in-sample forecast was conducted and then landed with the out-sample forecast for two and half years ahead.

Results and Analysis:-

This part presents the empirical results and the discussion of the results. It starts with the analysis of the descriptive statistics, unit root tests, heteroscedasticity test, estimation of the volatility models, and lastly, is the forecasting.

Descriptive Statistics

The descriptive statistics of the series was computed in order to obtain the statistical characteristics of the series such as mean, maximum, minimum, standard deviation, skewness, kurtosis, Jarque-Bera, and the number of observations of the series.

Table 1:- Descriptive Statistics of the All-Share Index of the Nigerian Exchange Market.

Mean	**Median**	**Maximum**	**Minimum**	**Std. Dev**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**Probability**	**Sum**	**Sum Sq. Dev.**	**Observations**
30496	**24518**	**65652**	**7415**	**34672**	**-0.73**	**19.65**	**749.3912**	**0.0000**	**3659560**	**1.37221 E+ 11**	**120**

Source: Author’s Computation

Table 1 represents the descriptive statistics of the market returns. The average monthly share index is 30496. The range of the monthly share index is 7415 to 65652 and this suggests high variability of the share index. The monthly standard deviation is 34672 and this is greater than the mean value, hence, there is high level of volatility in the share index. The skewness is –0.73 and this means that the share index has a long left tail and thus, non-normal. Kurtosis is 19.65 and this means that the share index is peaked and thus, not normal. Under the null hypothesis, the Jarque-Bera’s is 0 and in this case, the Jarque-Bera is 749.3912. Hence, the share index deviated from normal distribution. In summary, the descriptive results show that the share index is low and volatile, and the possibilities of receiving negative return are higher than that of positive return in the market. The unit root tests, namely the ADF, PP, and Ng-Perron were employed to test for the stochastic properties of the series and therefore, the stationarity or otherwise of the share index series.

Table 2:- Result of ADF and PP Unit Root Tests.

Value	**ADF**	**PP**
t-statistic	-6.895622	-18.22780
p-values	0.0000	0.0000
Critical Value		
1%	-3.446402	-3.446241
5%	-2.868511	-2.868440
10%	-2.570549	-2.570511

Source: Author’s Computation

Table 2 shows the unit root tests using ADF and PP. The p-values of ADF and PP are < 0.01, thus, reject the null hypothesis that the series has a unit root at 1% level and this leads to conclude that the share index series is stationary.

Table 3:- Results of Ng-Perron Unit Root test.

		MZa	MZt	MSB	MPT
Ng-Perron test statistics		-73.5313	-6.06303	0.08246	0.33416
Asymptotic critical values*	1%	-13.8000	-2.58000	0.17400	1.78000
	5%	-8.10000	-1.98000	0.23300	3.17000
	10%	-5.70000	-1.62000	0.27500	4.45000

Source: Author’s Computation

Table 3 shows the unit root test using Ng-Perron. All the four tests, namely MZa, MZt, MSB and MPT reject the null of unit root at 1% level. This leads to conclude that the NSE market share index series is stationary.

Table 4:- Bai-Perron Test for Multiple Breaks.

This test was conducted to examine the break dates in the market returns series. The test presents different break dates in the series.

All-Share Index of NSE	Break points	Time periods
Share Indices	3	March, 2019
		May, 2014
		June, 2021

Source: Author’s Computation

Table 4 presents 3 breaks in the series occurred in May 2014, June 2021, and March 2019. This structural breaks might be a result of change of government as well as the global financial crises around the year 2014 and 2019. There are other factors such as registration of securities, prospectus, allotment, unit trusts, reconstructions, mergers and takeover as well as insider trading.

Table 5: Heteroscedasticity Test:

To model the volatility of a time series variable, it is mandatory to test for the presence of heteroscedasticity in the residuals of the series before allowing the series to depend on the history of its errors (ARCH) or both its errors and the variance of the errors (GARCH). To test for the presence of ARCH effect, the paper runs an ordinary least squares (OLS) regression of the returns series by trying the autoregressive (AR) process, moving average process (MA), and a combination of the two process called ARMA process with the aim to select the regression that best fit the data. However, autoregressive order one i.e. AR (1) was selected as the regression that best fit the series and therefore, to be used as the conditional mean equation.

Table 5:- Result of Heteroscedasticity Test: (ARCH-LM Test for Residuals).

ARCH-LM (Test Statistics (TR^2)	60.47710
Prob. Chi-square (I)	0.0000

Source: Author’s Computation

Table 5 presents the result of heteroscadasticity test for ARCH effect. However, it shows that there is evidence of heteroscedasticity in the series at 1% level of significance, thus, rejecting the null hypothesis of there is no ARCH effect and this gives credence to employ the volatility models to estimate and forecast the volatility of the share indices.

Estimation of the Volatility Models

To evaluate the contribution of alternative error distributions for vigorous modelling of the share indices, all the models (GARCH, GARCH-M, EGARCH, TGARCH and PGARCH) were estimated using the Maximum Likelihood Estimators (MLE) based on the Broyden, Flectcher, Goldfarb and Shanno (BFGS) iterative algorithm to search for optimal parameters and solving unconstrained non-linear problems under the assumption of five alternative error distributions, namely normal distribution (ND), student’s-t distribution (STD), generalized error distributions (GED), skewed (SSTD), and skewed GED (SGED). Various information criteria which include the Log likelihood (LogL), Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan Quinn

Criterion (HQC) where the highest LogL and minimum information criteria were used to choose the order of each model for error distribution.

Table 6:- Model Order Selection.

S/N	Model	Error Distribution	AIC	SIC	HQC	LogL
1.	**GARCH (1,1)**	**ND**	**3.121619**	**3.032802**	**3.085463**	**-613.2964**
2.	GARCH (1,1)	STD	3.309991	3.211312	3.310436	-646.0482
3.	GARCH (4,1)	GED	3.306316	3.197408	3.135591	-641.3412
4.	GARCH (2,1)	SSTD	3.281205	3.188117	3.214137	-618.3341
5.	GARCH (1,3)	SGED	3.141421	3.147331	3.324116	-616.1183
1.	**GARCH-M(1,1)**	**ND**	**3.141003**	**3.044191**	**3.012216**	**-621.0142**
2.	GARCH-M(1,2)	STD	3.392001	3.249113	3.417138	-692.4971
3.	GARCH-M(1,1)	GED	3.371139	3.211418	3.307332	-681.4431
4.	GARCH-M(4,1)	SSTD	3.365441	3.201189	3.281215	-679.2162
5.	GARCH-M(4,1)	SGED	3.341622	3.199321	3.251229	-669.3211
1.	**EGARCH (1,5)**	**ND**	**3.161127**	**3.016342**	**3.151147**	**-663.8120**
2.	EGARCH (1,1)	STD	3.314431	3.117352	3.292944	-682.0014
3.	EGARCH (1,2)	GED	3.291430	3.109379	3.290431	-679.4223
4.	EGARCH (1,2)	SSTD	3.278414	3.091437	3.283167	-676.3972
5.	EGARCH (1,2)	SGED	3.258311	3.061117	3.186667	-672.1172
1.	**TGARCH (1,2)**	**ND**	**3.152467**	**3.016342**	**3.126607**	**-653.4120**
2.	TGARCH (1,1)	STD	3.344431	3.147342	3.273644	-680.2019
3.	TGARCH (4,1)	GED	3.302239	3.111319	3.240461	-677.4443
4.	TGARCH (1,2)	SSTD	3.285319	3.097714	3.221673	-673.3112
5.	TGARCH (1,4)	SGED	3.251191	3.045317	3.183197	-668.8272
1.	**PGARCH (1,3)**	**ND**	**3.131162**	**3.026641**	**3.114271**	**-649.2181**
2.	PGARCH (1,1)	STD	3.316931	3.152441	3.277946	-672.5011
3.	PGARCH (3,2)	GED	3.271461	3.102231	3.251131	-668.4003
4.	PGARCH (1,2)	SSTD	3.241482	3.041411	3.203134	-656.3112
5.	PGARCH (1,2)	SGED	3.201144	3.011764	3.134672	-647.1266

Note: 1. The bolded face denotes the model selected by the search criteria.

To evaluate the best fit under each error distribution the model selection was done based on the various information criteria and statistical significance of the ARCH and GARCH terms. The results presented in Table 6 show the estimations of different order of symmetric and asymmetric volatility models across the various error distributions. From the estimations the information criteria optimally select GARCH (1, 1) and PGARCH (1, 3) both under normal error distribution for symmetric and asymmetric volatility models respectively. Nevertheless, to estimate the risk parameter (λ), GARCH-M (1, 1) under normal error distribution was selected as the best. Hence, the paper proceeds with the estimation of those models.

Table 7a:- GARCH Result.

	Mean Equation	Intercept	0.026019*
		AR	0.232911*
		Intercept	-0.000814
GARCH (1,1)		A	0.461737*
	Variance Equation	B	0.481344*
		$\alpha + \beta$	0.943081
		θ	0.000139
ARCH-LM test statistic (TR^2)	0.098517		
Prob. Chi-Square (I)	0.7143		

Table 7b:- PGARCH Result.

	Mean Equation	Intercept	0.016996*
		AR	0.261149*
		Intercept	0.000214*
PGARCH (1,3)		A	0.264121*
	Variance Equation	B	0.681619*
		α + β	0.945740
		θ	-3.86E-039**
		δ	2.031766*
ARCH-LM test statistic (TR^2)	0.251744		
Prob. Chi-Square (I)	0.6231		

Table 7c:- GARCH-M Result.

	Mean Equation	Intercept	0.028117*
		AR	0.241149*
		λ	-0.243719**
		Intercept	-0.001214
GARCH-M (1,1)		A	0.364177*
	Variance Equation	B	0.514335*
		α + β	0.878512
		θ	0.000211
ARCH-LM test statistic (TR^2)	0.001799		
Prob. Chi-Square (I)	0.9617		

Source: Author's Computation
* and ** denote 1% and 5% level of significance respectively.

Table 7a and 7b show the estimation of GARCH (1, 1) and PGARCH (1, 3) both under normal error distribution. The diagnostic test shows that both the models accept the null hypothesis of no evidence of remaining heteroscedasticity. However, from the PGARCH (1, 3) estimate there is evidence of significant leverage effect in the market returns, hence, between these two models the best model for analyzing the volatility of NSE market returns is the PGARCH (1, 3) model under normal error distribution. The model shows that the leverage effect is positive and significant at 5% level, thus, positive shocks induce a larger increase in volatility when compared to the negative shocks of equal magnitude. The ARCH term (α) has the correct sign and is significant at 1% level which means that news about volatility from the previous periods has an explanatory power on current volatility. The GARCH term (β) has the correct sign and is significant at 1% level and this shows that large changes in the market returns followed by large changes and small changes followed by small changes, hence , volatility clustering. The sum of the terms (α + β) is greater than 0.5, meaning that changes of the market returns are explosive. The logged lag trading volume augmented in the model is significant at 5% level which implies that it is important in explaining the stock returns volatility. Moreover, the estimated power parameter (δ) is significant at 1% level and since $\delta = 2$, it means the model is a standard GARCH model that allows for leverage effect. However, Table 7c displays that the risk parameter (λs) of the best GARCH-M model is negative and significant at 5% level. This implies that the market risk and returns are inversely related and thus, the higher the risk of the stock the lower the probability of high returns.

Forecasting

Basically, using the various information criteria employed by this paper, the best model to forecast the market returns under symmetric models is GARCH (1, 1) in normal error distribution and PGARCH (1, 3) model under asymmetric models in normal error distribution where the latter was found to be the best between the two models. However, as an alternative, the paper goes further to adopt the three different statistical error functions for testing the forecasting performance models, namely the root mean square error (RMSE), mean absolute error (MAE), and mean absolute percent error (MAPE) to still select the best between the two models. The rule is that, the model with the lowest value of the forecast evaluation statistics is said to be the best. The forecasting performance of these two models using the three statistical error functions is reported in Table 8.

Table 8:- Forecast Performance of the Selected Models.

Error Drstribution	Models	RMSE	MAE	MAPE
Normal Distribution	GARCH (1, 1)	0.067334	0.040141	NA
	PGARCH (1, 3)	0.067113	0.040117	NA

Source: Author's Computation

Table 8 presents the result of the forecast performance between the two best models selected by the symmetric and asymmetric models respectively. From the relative values reported, the statistical error functions under PGARCH (1, 3) model yields the lowest value and therefore, it was found to be the best forecasting model for the volatility of the market returns.

Conclusion and Policy Recommendations:-

This paper modeled and forecasted the volatility of the Nigerian Stock Exchange Market while incorporating trade volume from January 1988 to December 2021. The Nigerian Stock Exchange is a market where local and foreign financial assets are being traded and the market provides returns to investors and this is the difference between the closing market price at the current day and the closing market price at the previous day. Therefore, the market returns could be positive or negative. Hence, there is need for information that could be used in maximizing and minimizing the market gains and losses respectively. This call for the need of volatility modelling which is used to predict the rise or fall of the market risk and uncertainty and such information is not only useful to investors but also the monitors of the economy.

For this analysis the estimation techniques employed include both symmetric and asymmetric volatility models such as ARCH, GARCH, GARCH-M, EGARCH, TGARCH, and PGARCH in various error distributions, namely normal distribution (ND), student's-t distribution (STD), generalized error distributions (GED), skewed (SSTD), and skewed GED (SGED) evaluated based on information criteria, namely Log likelihood (LogL), Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan Quinn Criterion (HQC).

The results revealed the following facts. First, there is evidence of positive leverage effect which reflects that positive shocks induce a larger increase in volatility when compared to the negative shocks of equal magnitude. Second, the volatility of the Nigerian Stock Exchange market could best be modelled and forecasted using PGARCH (1, 3) model in normal error distribution. Third, news about volatility from the previous periods has an explanatory power on current volatility. Fourth, there is volatility clustering in the market returns. Fifth, changes in the market returns are explosive. Sixth, the market risk and returns are inversely related. Last, the out-sample forecast of the market volatility for the next 2years shows that there will be a low volatility from March to September of the year 2023 and then upward there will be an increase in volatility up to December 2024.

However, for policy and investment decision the paper recommends taking the news on volatility into consideration while forming expectations in the market, investors could invest more within March to September 2023 as the market risk and returns are inversely related, following the forecast estimate that the volatility will be on increase from October 2023 up to December 2024 within which investors are likely to invest less as the market risk and returns are inversely related; monetary authority could contract the money supply so as to control the problem of idle cash if the percentage of the investors in the risk-averse category is greater than that of risk-lovers and risk diversifiers categories.

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