



RESEARCH ARTICLE

THEORETICAL STUDY OF THERMODYNAMICS PROPERTIES OF LANDAU LEVELS IN INAS TWO-DIMENSIONAL ELECTRON GAS

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Abstract

This study theoretically analyzes the two-dimensional electron gas (2DEG) of InAs in the matter of thermodynamics proprieties and the effects of the magnetic field, Landau levels, and temperature on its corresponding behavior. InAs is a combination of two groups of semiconductors (III and V groups) which require chemical reaction to process the binding between these two and emphasize the importance of investigating and calculating the chemical potential and the required magnetization amount. The electron of a Two-Dimensional Gas Electron (2DEG) can move freely in two dimensions which creates the third direction motion by different quantized energy levels. This study will emphasize the thermodynamic properties of InAs and provide a suitable calculation to identify its behavior regarding heat capacity, chemical potential, and other factors. This behavior leads to the use InAs in the production of diode laser.

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Introduction:-

Generally, the semiconductors' electrical properties are in between the properties of the insulators and conductors. One of the essential semiconductors' group is the III-V group which combine a member from the III group and a member of the V group as well in which the InAs is a member. The combination demands a chemical reaction and supportive energy to complete the binding process [1].

Like all types of semiconductors, InAs have unique characteristics regarding thermal and chemical capabilities [2]. The excellence of the band structure on InAs due to the interaction of Coulomb ions leads the electron to be characterized by unique high mobility ($\mu_n \geq 1 \times 10^4 \text{ cm}^2/\text{Vs}$) and great presence of saturation speed [3].

The InAs structure is cubic crystals in the zinc-blende structure with a bandgap of energy less than 0.35 eV which is considered very small. Moreover, the InAs effective mass could be observed by using the actual E-K curve with its well-known parabolic design which provides the average effective mass of $0.023 m_0$.

The general different properties of InAs are listed in Table 1 to illustrate the exact value of each property.

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Table 1:- InAs Thermodynamic Properties.

Properties	Values
Molar mass	189.740 g/mol
Density	5.67 g/cm ³
Melting point	215 K
Band gap	0.354 eV (300 K)
Electron mobility	40000 cm ² /(V*s)
Thermal conductivity	0.27 W/(cm*K) (300 K)
Refractive index(n_D)	3.51
Crystal structure	Zinc Blende
Lattice constant	a = 6.0583 Å
Heat capacity	47.8 J·mol ⁻¹ ·K ⁻¹
molar entropy	75.7 J·mol ⁻¹ ·K ⁻¹
enthalpy offormation	-58.6 kJ·mol ⁻¹

The wavelength of InAs is dramatically short which makes it a suitable choice for mid-infrared wavelength applications with its minimum losses and tenability facilities to be applicable for application that requires high power or the production of diode laser with fixed temperature [4].

The second gas electron's discretized energy spectrum which reflects the presence of Landau levels undermined the high amount of magnetic field in such a semiconductor reaction [5]. The appearance of these levels in the density of states has followed the principle of function of Dirac-delta and its free electron state in the gaps. The DOS could be subjected to the influence of the magnetization factors or heat capacity within a particular temperature [6, 7].

In this study, the fixed particle density is considered the ideal situation to illustrate the different properties of InAs. Correspondingly, the effect of de Haas-van Alphen will be addressed along with the effect of the chemical potential and heat capacity's behavior.

Formulization

General Formulization

To perform and calculate the partial differential analysis and investigate the behavior of the semiconductors, the nonlinear Schrödinger equation should be used in the presence of the magnetic field as well as in its absence [8].

The required value of energy in the absence of the magnetic field is derived from the general form of the Schrodinger equation as follows:

$$H\Psi = E\psi \quad [1]$$

Leading to after several operations to obtain the energy:

$$E = \frac{\hbar^2 k^2}{2m} \quad [2]$$

Where, \hbar is Planck's constant divided by 2π , and m^* is the the electron's effective mass.

On the other hand, the required value of energy in the presence of the magnetic field is derived from the general form of the Schrodinger equation along with the Hamiltonian as follows [9, 10 , 11]:

$$\frac{1}{2m} \left[(-i\hbar \vec{\nabla} - q\vec{A})^2 \right] \psi(x, y, z) = E\psi(x, y, z) \quad [3]$$

Leading to after several operations a harmonic oscillator equation in which the solution could form the energy as follow:

$$E = \left(n + \frac{1}{2} \right) \hbar \omega \quad [4]$$

Where, $\omega_c = \frac{qB}{m}$ is the electron cyclotron frequency.

To state the different state number of a particular energy level, the density of state represents the mathematical physics approach to identify [12]. Its calculations determine the exact spaces and bands between the energy levels on the exact moment of reaching the small value of the energy band gap.

The Landau level energy that obtained by the Schrodinger equation in case of magnetic field is present leads to identify the DOS as follow [13]:

$$D(B, E) = \sum_{n, k_y} \delta(E - E_n) \quad [5]$$

Thus,

$$D(B, E) = \frac{eB}{\pi \hbar} \sum_n \frac{1}{\sqrt{2\pi\Gamma}} \exp\left[-\frac{(E - E_n)^2}{2\Gamma^2}\right] \quad [6]$$

Where \hbar is Planck's constant/ 2π , e is the electron charge, E_n is the energy of the LLs, and Γ is the broadening parameter.

Chemical Potential Formulization

The behavior of the system and the amount of energy required to maintain the balance at the moment of the number of particles is changed is determined by the chemical potential. This balanced behavior influences the number of particles, energy, and volume of the system depending on the types of different ensembles [14].

There are three types of ensembles with specific features and properties for each one of them. The first one is a microcanonical ensemble which represents the isolated system with the same particles and energy as well. The second one is a canonical ensemble which capable to adjust its energy with a fixed temperature in presence of a heat reservoir. The third type is a grand-canonical ensemble in which the temperature and chemical potential are fixed allowing the energy to be exchanged when the number of particles changes[15]. Table.2 illustrates the mathematical representation of these different ensembles.

Table 2:- Different ensamples expressions The required energy to adjust the balance of systems and their corresponding behavior could be represented.

	Phase space integration	Thermodynamic potential
Microcanonical ensemble	$\Gamma(E, V, N) = \int_{E < H < E + \Delta} d^{3N}q d^{3N}p$ $\Omega(E, V, N) = \frac{\partial \Gamma}{\partial E}$ $= \int d^{3N}q d^{3N}p \delta(E - H)$	$S(E, V, N) = k_B \ln \left(\frac{\Gamma}{h^{3N} N!} \right)$
Canonical ensemble	$Z_N(T, V) = \int \frac{d^{3N}q d^{3N}p}{h^{3N} N!} e^{-\beta H}$ $= \frac{1}{h^{3N} N!} \int_0^\infty dE \Omega(E) e^{-\beta H}$	$F(T, V, N) = -k_B T \ln Z_N(T, V)$ $Z_N = e^{-\beta F}$
Grand canonical ensemble	$\mathcal{Z}(T, \mu) = \sum_{N=0}^\infty e^{\mu N / k_B T} Z_N(T)$	$\Omega(T, V, \mu) = -k_B T \ln \mathcal{Z}(T, V, \mu)$ $\mathcal{Z} = e^{-\beta \Omega} = e^{\frac{\mu N}{k_B T}}$

by the DOS as follows:

$$N = \int_0^\infty f(E, \mu, T) D(B, E) dE \quad [7]$$

Where, B is the magnetic field, T is the temperature, $\mu = \mu(B, T)$ is the chemical potential, and $D(B, E)$ is the Gaussian DOS is reflecting the density of state, and $f(E, \mu, T)$ is:

$$f(E, \mu, T) = \frac{1}{1 + \exp\left(\frac{E - \mu}{k_B T}\right)} \quad [8]$$

Leading to have:

$$\frac{\partial \mu}{\partial T} = \frac{- \int D \frac{\frac{E-\mu}{k_B T} \exp\left(\frac{E-\mu}{k_B T}\right)}{\left(1 + \exp\left(\frac{E-\mu}{k_B T}\right)\right)^2} dE}{\int D \frac{1}{\frac{k_B T}{\exp\left(\frac{E-\mu}{k_B T}\right)} + 1} dE} \quad [9]$$

Affects Formulation

At the moment, the intensity of the magnetic field is being altered, the effect along the magnetic susceptibility is called the Hass-van Alphen effect. It plays a significant role in the quantization of the electron orbits along with the magnetic field with a certain amount to keep the balance for both temperature and the equilibrium's constant depending on the following equation[16]:

$$\frac{d \ln K}{d\left(\frac{1}{T}\right)} = - \frac{\Delta_r H}{R} \quad [10]$$

This type of effect leads to an obvious magnetic field's increment in which the levels of Fermi is crossed by Landau levels.

Additionally, one of the most essential factor that affect the behavior of InAs is the heat capacity which indicates to the sufficient amount of heat capable to increase the temperature by one degree. This effect could be obtained by maintaining either the volume or the pressure to be constant [17]. Therefore, and depending on the small band gap of InAs and its low atomic weight, the energy levels will be able to have their important role in the temperature's variation. The energy levels in the low atomic weight play a very significant role in the heat capacity as well as the variation of the temperature as follow [18, 19]:

$$C_v(B, T) = \frac{\partial}{\partial T} \int_0^\infty F(E, \mu, T) (E - \mu) D(B, E) dE \quad [11]$$

Where, $F(E, \mu, T)$ and $D(B, E)$ are represented by Eq.8 and Eq.6 respectively.

Thus,

$$[12] \quad C_v(B, T) = \int_0^\infty (E - \mu) \left(\frac{\partial \mu}{\partial T} B_2 + B_1 \right) dE - \int_0^\infty D(B, E) F(E, \mu, T) \frac{\partial \mu}{\partial T} dE$$

Where B_1 and B_2 have been determined after a certain number of calculations.

Results and Outcomes:-

The heat capacity has been represented with different filling factors of Landau levels which given by:

$$v = \frac{hN}{eB} \quad [13]$$

Fig.1 illustrate the relation between the temperature and the heat capacity of the InAs. It is obvious that a single peak of heat capacity happened at very low temperature (6 K). The heat capacity stated to increase at the time of temperature increased until it reached its peak value then it started a sharp decreased until a particular temperature degree. After that, the heat capacity still decreasing but with small values even when the temperature keeps its increment. This relation is direct depending on the energy levels and the filling Factors. When it is very small the value of the peak heat capacity will be larger and the sharp decreasing will be less obviously which explain the InAs as second gas electron behavior.

The energy curve's fluctuation without a magnetic field can be compared to the energy curve's variation in Fig. 2 (a, b) with a magnetic field. The distance between Landau Levels appears to increase as the magnetic field strength rises.

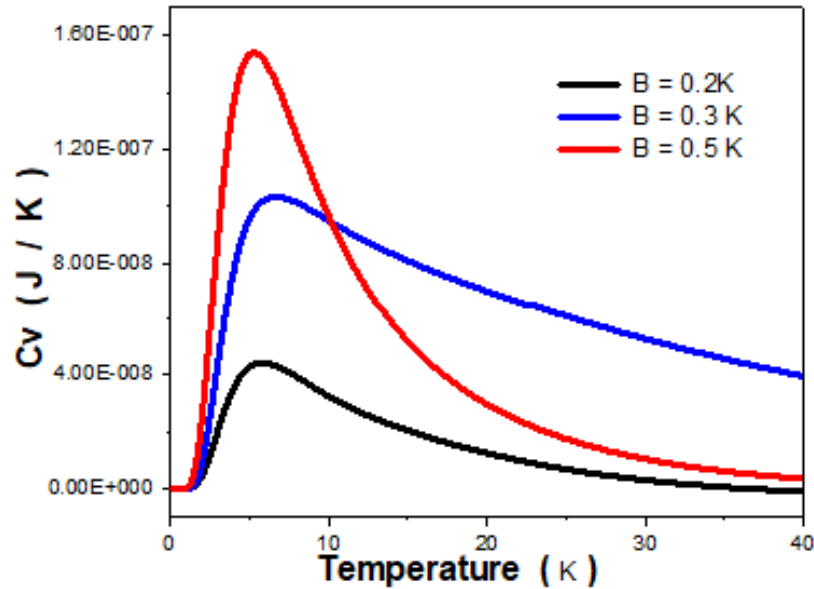


Figure 1:- InAs specific heat at different magnetic field as a function of temperature.

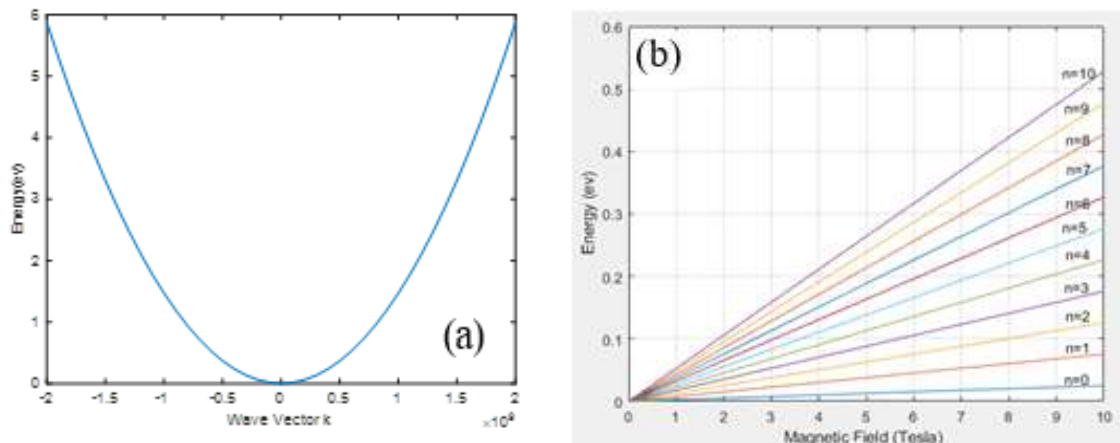


Figure 2:- Energy diagram of the Landau levels for a two-dimensional electron gas in the presence of a magnetic field B (b), and without magnetic field (a).

The orbits of electron are quantized due to the presence of the magnetic field along with the density of state of the gas will be divided by the cyclotron frequency into separated Landau levels as follow:

$$w_c = \frac{eB}{hN} \quad [14]$$

This separation will provide a unique explanation of the behavior of the InAs in its equilibrium behavior. Fig.3 provides an obvious illustration on the relation between the magnetic intensity and the heat capacity of InAs at constant temperature. It describes the heat capacity's oscillatory behavior with a presented of shape U width[20,21].

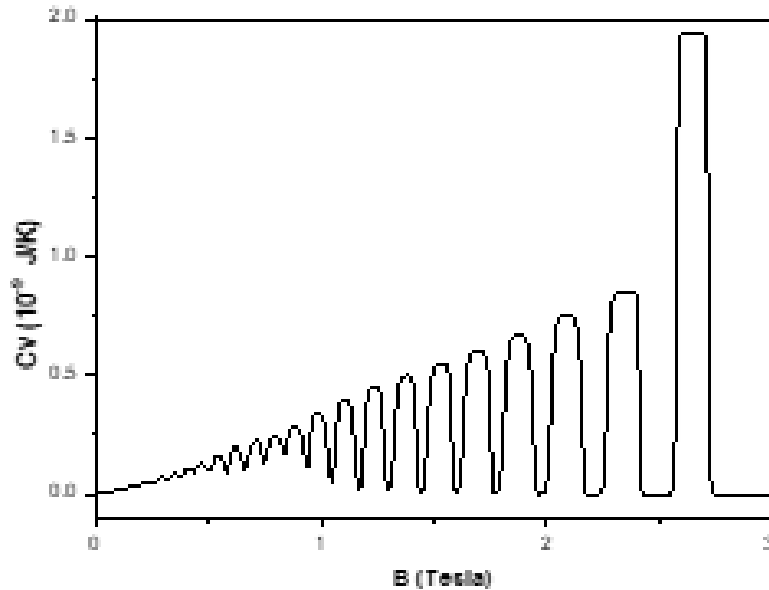


Figure 3:- InAs specific heat at constant temperature $T = 3K$.

This behavior is not only affects the heat capacity but also the chemical potential behavior. Fig.4 illustrate the behavior of InAs in the matter of the chemical potential. They follow the increment sharp behavior of very small intervals which extremely providing the particular properties of the InAs.

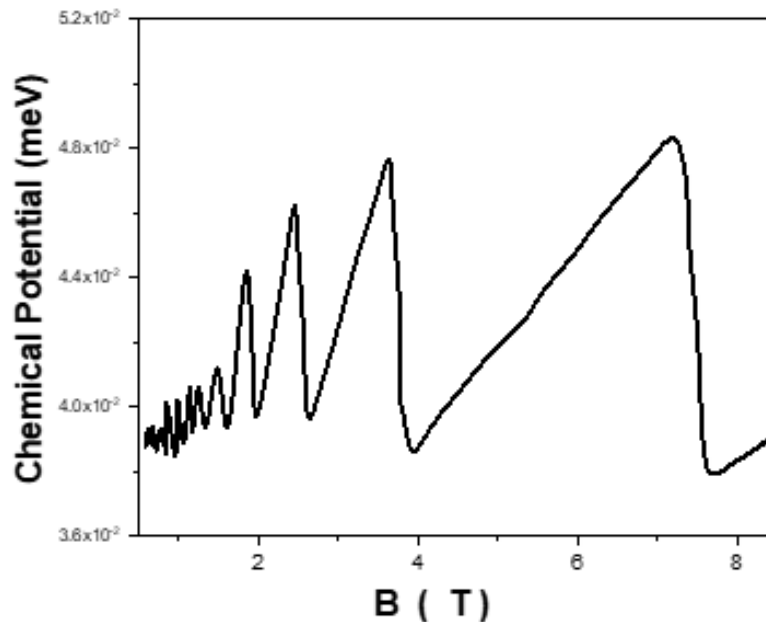


Figure 4:- InAs chemical potential at constant temperature.

Using Equation (6), Figure 5 shows the DOS as a function of energy for widening parameter value $\Gamma = 0.2$ meV. We see that the DOS is clearly influenced by the flaws in the samples used [22].

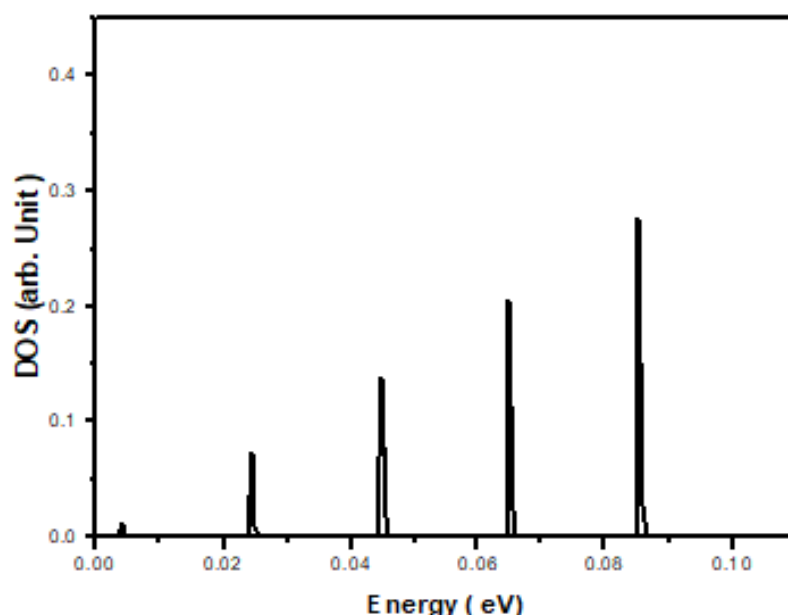


Figure 5:- Density of state for a broadening $\Gamma = 0.2$ meV.

Conclusion:-

In this study, the thermodynamic properties of the second gas electron of InAs has been addressed along with the behaved investigation along different circumstances. The required energy for the system to maintain its equilibrium state with different influences of the chemical potential have been discussed. The several types characteristics of InAs semiconductor leads to predict even more innovation by investigating more studies and perform advanced analysis.

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