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RESEARCH ARTICLE

SUM CORDIAL LABELING OF CORONA PRODUCT GRAPH OF PATH TO PATH

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Abstract

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In this paper the researcher studies on the labeling of Corona Product Graph of Path to Path by admitting the certain condition of Sum cordial labeling. A sum cordial labeling of a graph G is a binary labeling with vertex set V, with edge labeling f:E(G)→{0,1},defined by f(uv)=(f(u)+f(v))(mod2) if $|v_f(0)-v_f(1)|\leq 1$and $|e_f(0)-e_f(1)|\leq 1$. A graph G is Sum cordial if it admits Sum cordial labeling.

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Introduction:-

We know that Graph labeling is used in radio-astronomy, development of radar and missile guidance codes, spectral characterization of material using x-ray crystallography, communication networks and transportation network. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Labeling helps to distinguish between any two adjacent vertices or edges. Graph labeling was first introduced in the year 1967 by Rosa [1]. Rosa defined a function as $f: V(G) \rightarrow \{0,1,2,3..........q\}$, f is an injection such that, when each edge xy is assigned the label $\left|f(x)-f(y)\right|$, the resulting edge labels are distinct.

The Sum cordial labeling was introduced in 2014 by V.R.Visavaliya, M.I.Bosmia and B.M. Patel [2]. They investigated sum cordial labeling of flower graph, web graph, tadpole, triangular snake and shell graph. And its further result was defined in 2016 by M.I. Bosmia, V.R. Visavaliya and B.M. Patel [3]. They proved that wheel, closed helm, quadrilateral snake, double quadrilateral snake and gear graphs are sum cordial graphs. In December 2016 A. Lourdusamy and F. Patrick [4] worked the Sum divisor cordial labeling for star and ladder related graphs. In December 2016 P. Lawrence Rozario Raj and S. Hema surya [5] proved that the switching of a pendent vertex in path P_n, switching of any vertex in cycle C_n, $DS(B_{n,n})$, $G^*K_{2,n}$ and $G^*K_{3,n}$ are Sum divisor cordial graph. In 2018 A. Sugumaran and K.Rajesh [6] proved that the some of the special graphs such as plus graph Pln, umbrella graph U(n,n) (n is odd), Path union of r copies of C_n (n is odd), (n,n)-kite graph and complete binary tree BT_n are sum cordial graphs.

Theorem-

The corona product graph of path to path $P_m \odot P_n$ admits Sum cordial labeling, where m, n≥3.

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Proof-Case1- When m and n both are even-
Steps to construct a graph G= Pm□ Pn:-

First we join all the base vertices of P_n and then the rest vertices of P_m join with the base vertices of P_n and with each other.

Label the Graph G= Pm□ Pn in the following steps-

Label all the base vertices of P_n with (1, 0) alternatively.
Label other vertices of P_m as follows-
Label 1st three vertices of P_m with (1, 1, 0) respectively.
Label $(2m+4)^{th}$ and $(2m+5)^{th}$ vertices of P_m with 0, where m=2i,i=0,1,2,3.......n
Label $(2m+6)^{th}$ and $(2m+7)^{th}$ vertices of P_m with 1, where m=2i,i=0,1,2,3.......n

In figure1.1 graph P_6 □ P_6 is labeled as follows-

Label all the base vertices of P_6 with (1, 0) alternatively.
Label other vertices of P_6 as follows-
Label 1st three vertices of P_6 with (1, 1, 0) respectively.
Label $(2m+4)^{th}$ and $(2m+5)^{th}$ vertices of P_6 with 0, where m=2i,i=0,1,2,3.......n
Label $(2m+6)^{th}$ and $(2m+7)^{th}$ vertices of P_6 with 1, where m=2i,i=0,1,2,3.......n

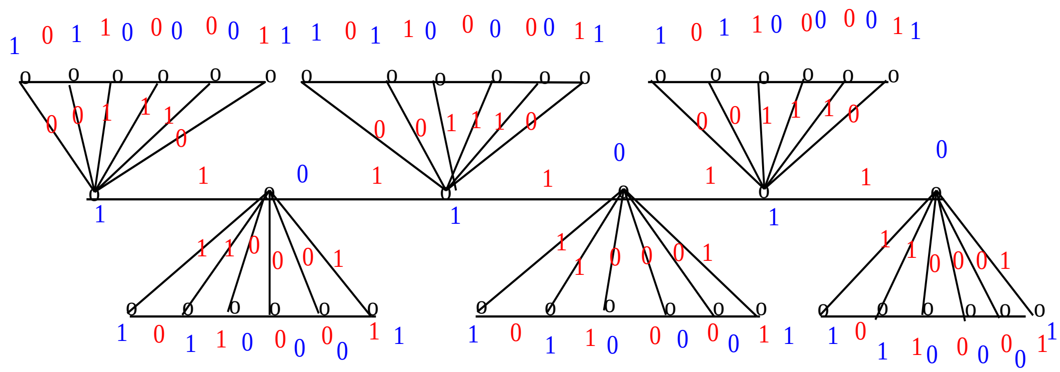

FIG.1.1 SUM CORDIAL LABELING OF P_6□ P_6

$\sum v(1) = (m+1)\, i$
$\sum v(0)= (m+1)\, i$
Where m=3, 5, 7........... And i= 2, 3, 4, 5.......
$|\sum v(0)- \sum v(1)|=| \{(m+1)\, i \}- \{(m+1)i\}|= 0\leq 1$
$\sum e(1) = 2mi-1$
$\sum e(0)= 2mi$
Where m=4, 6, 8....... And i=2, 3, 4..........
$|\sum e(0)- \sum e(1)|=| \{2mi\}-\{ 2mi-1\}|= 1\leq 1$

Case2- When m is odd and n is even-
Steps to construct a graph G= Pm□ Pn:-

First we join all the base vertices of P_n and then the rest vertices of P_m join with the base vertices of P_n and with each other.

Label the Graph G= Pm□ Pn in the following steps-

Label first three vertices of P_n with (0, 0, 1) respectively. And rest will be labeled with (1,1, 0,0) alternatively.
Label the vertices of P_m as follows-
Label 1^{st} and 2^{nd} vertices of p_m with (1, 0) and (1, 1) alternatively.
Label $(2m+3)^{th}$ and $(2m+4)^{th}$ vertices of P_m with 0, where m=2i, i=0,1,2......n
Label $(2m+5)^{th}$ and $(2m+6)^{th}$ vertices of P_m with 1, where m=2i, i=0,1,2......n

In figure2.1 graph $P_7 \square P_6$ is labeled as follows-

Label first three vertices of P_6 with (0, 0, 1) respectively. And rest will be labeled with (1, 1, 0, 0) alternatively.
Label the vertices of P_7 as follows-
Label 1^{st} and 2^{nd} vertices of P_7 with (1, 0) and (1, 1) alternatively.
Label $(2m+3)^{th}$ and $(2m+4)^{th}$ vertices of P_7 with 0, where m=2i, i=0,1,2......n
Label $(2m+5)^{th}$ and $(2m+6)^{th}$ vertices of P_7 with 1, where m=2i, i=0,1,2......n

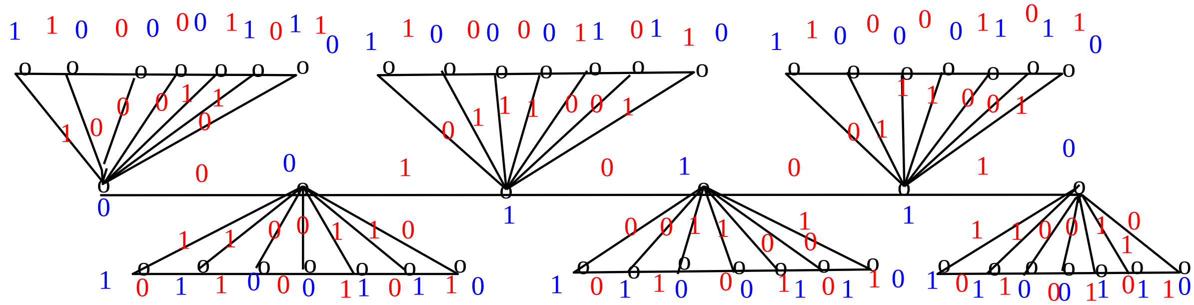

FIG.2.1 SUM CORDIAL LABELING OF $P_7 \square P_6$

$\sum v(1) = (m+1)\, i$
$\sum v(0) = (m+1)\, i$
Where m=3, 5,7........... And i= 2, 3, 4,5.......
$|\sum v(0) - \sum v(1)| = |\{(m+1)\, i - (m+1)\, i\}| = 0 \leq 1$
$\sum e(0) = 2m\,(i+1)$
$\sum e(1) = 2m\,(i+1) - 1$
Where m=3, 5, 7....... And i=2, 3, 4..........
$|\sum e(0) - \sum e(1)| = |\{2m\,(i+1)\} - \{2m\,(i+1) - 1\}| = 1 \leq 1$

Case3- When m is even and n is odd-
Steps to construct a graph G= $P_m \square P_n$:-

First we join all the base vertices of P_n and then the rest vertices of P_m join with the base vertices of P_n and with each other.

Label the Graph G= $P_m \square P_n$ in the following steps-

Label all the base vertices of P_n with (1, 0) alternatively.
Label the vertices of P_m as follows-
Label 1^{st} three vertices of P_m with (1, 1,0) respectively.
Label $(2m+4)^{th}$ and $(2m+5)^{th}$ vertices of P_m with 0, where m=2i, i=0,1,2......n
Label $(2m+6)^{th}$ and $(2m+7)^{th}$ vertices of P_m with 1, where m=2i, i=0,1,2......n

In figure3.1 graph $P_6 \square P_7$ is labeled as follows

Label all the base vertices of P_7 with (1, 0) alternatively.
Label the vertices of P_6 as follows-
Label 1^{st} three vertices of P_6 with (1, 1, 0) respectively.
Label $(2m+4)^{th}$ and $(2m+5)^{th}$ vertices of P_6 with 0, where m=2i, i=0,1,2......n
Label $(2m+6)^{th}$ and $(2m+7)^{th}$ vertices of P_6 with 1, where m=2i, i=0,1,2......n

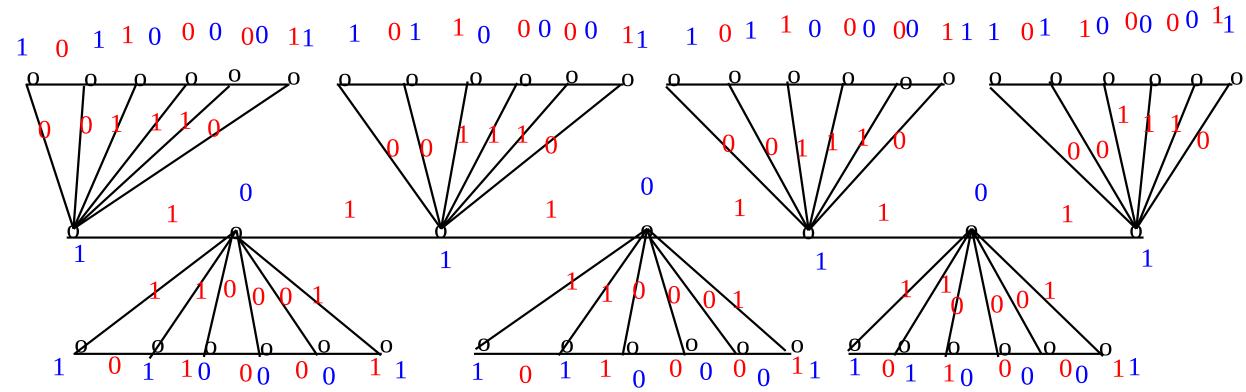

FIG.3.1 SUM CORDIAL LABELING OF P_6 □ P_7

$\sum v(1) = m(i+1)$
$\sum v(0)= m(i+1)-1$
Where m=4, 6, 8........... And i= 1, 2, 3, 4, 5.......
$|\sum v(0)- \sum v(1)|=| \{m(i+1)-1\}- \{m(i+1)\}|=|-1| \leq 1$
$\sum e(1) = m(2i+1)-1$
$\sum e(0)= m(2i+1)$
Where m=4, 6, 8....... And i=1, 2,4,6,8..........
$|\sum e(0)- \sum e(1)|=| \{m(2i+1)\}-\{m(2i+1)-1\}|= 1\leq 1$
Case 4- When m and n both are odd-

Steps to construct a graph G= Pm□ Pn:-

First we join all the base vertices of P_n and then the rest vertices of P_m join with the base vertices of P_n and with each other.

Label the Graph G= Pm□ Pn in the following steps-

Label all the base vertices of P_n with 1.
Label the vertices of P_m as follows-
Label 1st two vertices of P_m with (1, 0) respectively.
Label $(2m+3)^{th}$ and $(2m+4)^{th}$ vertices of P_m with 0, where m=2i, i=0,1,2......n
Label $(2m+5)^{th}$ and $(2m+6)^{th}$ vertices of P_m with 1, where m=2i, i=0,1,2......n

In figure 4.1 graph P_7 □ P_7 is labeled as follows-

Label all the base vertices of P_7 with 1.
Label the vertices of P_7 as follows-
Label 1st two vertices of P_7 with (1, 0) respectively.
Label $(2m+3)^{th}$ and $(2m+4)^{th}$ vertices of P_7 with 0, where m=2i, i=0,1,2......n
Label $(2m+5)^{th}$ and $(2m+6)^{th}$ vertices of P_7 with 1, where m=2i, i=0,1,2......n

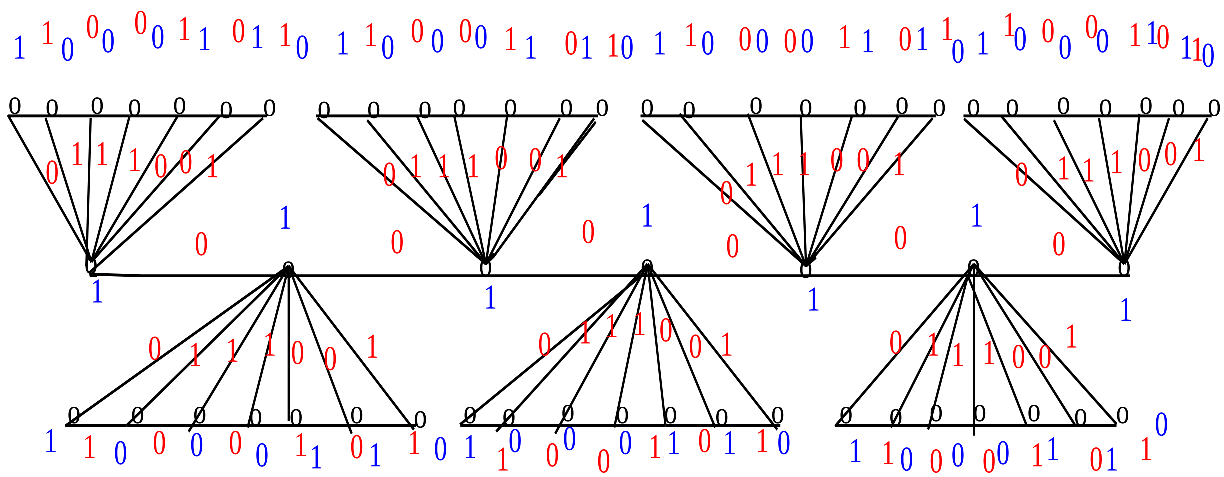

FIG.4.1 SUM CORDIAL LABELING OF $P_7 \odot P_7$

$\sum v(1) = \{(m+1)(2i+1)\}/2$
$\sum v(0) = \{(m+1)(2i+1)\}/2$
Where m=3, 5, 7........... And i= 1, 2, 3, 4, 5.......
$|\sum v(0) - \sum v(1)| = |\{(m+1)(2i+1)\}/2 - \{(m+1)(2i+1)\}/2| = 0 \leq 1$
$\sum e(1) = m(2i+1)$
$\sum e(0) = m(2i+1)-1$
Where m=3, 5, 7....... And i=1,2, 3, 4..........
$|\sum e(0) - \sum e(1)| = |\{m(2i+1)\}-1-\{m(2i+1)\}| = 1 \leq 1$
Therefore, $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.
Hence the rooted product graph of path to path **Pm⊙ Pn** admits Sum cordial labeling, where m, n≥3
Hence proved.

Conclusion:-

In this paper we find that the Corona product Graph of path to path satisfies the condition of Sum Cordial Labeling that is $f:E(G)\to\{0,1\}$,defined by $f(uv)=(f(u)+f(v))(\mathrm{mod}\,2)$ if $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.

A lot of work has been accomplished in this area by many researchers and still work is being carried out for this Graph.

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