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RESEARCH ARTICLE

LATTICE OF A FUZZY PRIME IDEALS IN AN ORDERED SEMIGROUP

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Abstract

This paper introduces the concept of lattices of prime fuzzy ideals, which extends the classical theory of lattices of prime ideals in ring theory to the fuzzy setting. In this paper, we propose some examples and properties related to lattices of prime fuzzy ideals, contributing to the knowledge in fuzzy algebra. Think about an ordered semigroup $\mathscr{R}$. Plotting the function f from $\mathscr{R}$ to the closed interval [0,1] defines a fuzzy subset of $\mathscr{R}$, where [0,1] ϵ $\mathscr{R}$. In order to explore prime ideals in rings, semigroups, and ordered semigroups in terms of fuzzy subsets, we build an ordered semigroup $\mathscr{R}$ with ordered fuzzy points. To study, we presented the thoughts of $\mathscr{L}$-fuzzy prime ideals of an ordered semigroups $\mathscr{R}$, which forms a completelattice obeying the law relating the operations of multiplication and addition.

I.Introduction:-

The introduction section provides an overview of fuzzy ideals and their lattice structure. It briefly explains the motivation behind studying lattices of prime fuzzy ideals and highlights the significance of the proposed proposition and theorem.

The central idea about set function, developed by L.A. Zadeh, has been well understood and applied in a wide range of scientific domains since 1965. Studies on fuzzy sets have been published, highlighting their importance and its use in a variety of technical and scientific disciplines. The ideal $\mathscr{J}_1 = \{X \epsilon \mathscr{R}: \mathscr{J}(x) = 1\}$ is prime after Liu's work [47], and some mathematicians create new conceptions of ideals in a ring $\mathscr{R}$ is prime iff manual, i.e., Im $\mathscr{J} = [1, \beta]$ where $\beta \epsilon [0,1]$. Additionally, Zhang [53] carries a ring of $\mathscr{L}$-fuzzy ideals.

The resemblance of these ideals, according to Malik and Mordeson [7], produces only real numbers in the rings, and its level set becomes trivially prime. A fuzzy ideal $\mathscr{J}$ of a ring $\mathscr{R}$ is a prime iff the set of level ideal$\mathscr{J}a = \{b \epsilon \mathscr{R}: \mathscr{J}(b) \geq a\}$, where ‘a’ related to fictitious$\mathscr{J}$. Gupta and Kartoo [17], among others,proposed a new concept of ideals $\mathscr{J}$ of a ring $\mathscr{R}$ as $\mathscr{J}(rs) = \max\{\mathscr{J}(r), \mathscr{J}(s)\}$ for every r, s $\epsilon\mathscr{R}$.

Self-seeking about$\mathscr{L}$- prime fuzzy ideals in semigroup $\mathscr{R}$, we introduce lattice of fuzzy prime idealswith ordered semigroup, which is an extension of a fuzzy ordered semigroup and satisfies meet distributive law by treating $\mathscr{L}$- as a complete Lattice. Further it is aimed to work on $\mathscr{L}$-fuzzy prime ideals, semigroupsand various fuzzyset functions. And also,$\mathscr{L}$- fuzzy prime characteristic set properties are demonstrated. An ordered semigroup of a $\mathscr{L}$-fuzzy ideal is also a$\mathscr{L}$-fuzzy PLI of an ordered semigroup as well as that it has supremum property.

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II. Preliminaries

Regarding to this section, some essential solutions for this paper arebrought to mind. This section presents the necessary background material related to fuzzy algebra, fuzzy ideals, and lattice theory. It establishes the fundamental concepts and notations required to understand the subsequent proposition and theorem.

Definition: 2.1

Let g_A denotescharacteristic mapping of in the real interval [0,1] defines

$g_A(u) = \{1\ if\ u \in A 0\ if\ u \notin A$

Definition:2.2

LetM andNare two ideals of $\mathscr{R}$. IfMN⊆I, then M⊆I or N⊆I the ideal I of $\mathscr{R}$ is called the prime ideal.

Definition:2.3

The set of all fuzzy subset of $\mathscr{R}$ is denoted by Ƒ($\mathscr{R}$). A mapping defines from R to [0,1] as f: $\mathscr{R} \rightarrow [0,1]$.(Ƒ($\mathscr{R}$), ⊆, ∪ , ∩) forms a complete Lattice with upper bound 1 and lower bound 0.

Example:2.4

Let's work with a fuzzy ideal in the group of integers (Z) under addition.

1. To define the fuzzy ideal:

Consider the fuzzy ideal $\mathcal{A}$ by its membership function:

μ$\mathcal{A}$(m) = 0.6, if m is even

μ$\mathcal{A}$(m) = 0.2, if m is odd

2. T0 determine the group operation:

In this case, the group operation is addition (+) defined on the set of integers (Z).

3. Analyse the closure property:

For a fuzzy ideal to be closed under addition, the result of adding any two elements from the fuzzy ideal should still be within the fuzzy ideal.

4. Evaluate the fuzzy ideal under the group operation:

Let's consider two elements, 2 and 3, from the fuzzy ideal $\mathcal{A}$.

μ$\mathcal{A}$(2) = 0.6, μ$\mathcal{A}$(3) = 0.2

Now, perform the group operation (+) on these elements:

2 + 3 = 5

5. Verify closure condition:

To check if the result, 5, is still within the fuzzy ideal $\mathcal{A}$, evaluate its membership function:

μ$\mathcal{A}$(5) = 0.2 (since 5 is odd)

Since μ$\mathcal{A}$(5) = 0.2, which is consistent with the membership function of the fuzzy ideal $\mathcal{A}$, we can conclude that the fuzzy ideal $\mathcal{A}$ is closed under the group operation of addition.

Definition: 2.5

(i) The union of two $\mathscr{L}$- prime fuzzy subset $\mu_1\mathscr{L}$ and $\mu_2\mathscr{L}$ of a set A, denoted by ($\mu_1\mathscr{L} \cup \mu_2\mathscr{L}$) is a $\mathscr{L}$-prime fuzzy subset of A defined by,

($\mu_1\mathscr{L} \cup \mu_2\mathscr{L}$) (a) = $\mu_1\mathscr{L}$(a) ∨ $\mu_2\mathscr{L}$(a) ∀ a∈A.

(ii) The intersection of two $\mathscr{L}$- prime fuzzy subset $\mu_1\mathscr{L}$ and $\mu_2\mathscr{L}$, denoted by ($\mu_1\mathscr{L} \cap \mu_2\mathscr{L}$) is a $\mathscr{L}$-prime fuzzy subset of A written as,

($\mu_1\mathscr{L} \cap \mu_2\mathscr{L}$) (a) = $\mu_1\mathscr{L}$(a) ∧ $\mu_2\mathscr{L}$(a) ∀ a∈A.

Definition:2.6

A fuzzy subset f of $\mathscr{R}$ is called a FLI of $\mathscr{R}$ if

(i) p≤q⟹h(p) ≥h(q)

(ii) h (pq) ≥h(q) ∀ p, q∈$\mathscr{R}$

Where $\mathscr{R}$ be an ordered semigroup.

A fuzzy subset h of $\mathscr{R}$ is called a FRI of $\mathscr{R}$ if

(i) p≤q⟹h(p) ≥h(q)

(ii) h(pq) ≥h(p) ∀ p, q∈$\mathscr{R}$

If 𝒭 has both the ideal then fuzzy subset h of 𝒭 is called a fuzzy ideal of 𝒭

III. Lattices of fuzzy prime ideal of ordered semigroup

Theorem: 3.1

A lattice fuzzy subset μ_L of 𝒭 is a fuzzy prime ideal of 𝒭 iff its level set μ_{Lr} , r ∈ℒ is an ideal of 𝒭.

Proof:

Letμ_L is a ℒ-fuzzy prime left ideal of 𝒭.

To prove:

μ_{Lr}= {a ∈𝒭 / μ_L(a) ≥ r} is fuzzy prime left ideal of 𝒭

Supposeu, v∈ μ_{Lr}⟹μ_L(u) ≥ r or μ_L(v) ≥ r

⟹v + u – v∈ μ_{Lr}

Therefore, μ_{Lr} is a left ideal of 𝒭.

Theorem: 3.2

Let g be a ℒ- fuzzy prime subset of an ordered semigroup 𝒭. Then f is a strongly convex ℒ- fuzzy prime subset of 𝒭 iff r≤s => g(r) ≥ g(s) ∀ r ands∈𝒭

Proof:

Ifr,s∈𝒭 thenr≤s.

Let g(r) = (g](r)

$= w \geq rV\ g(w) \quad \geq \ g(s)$

Since (g](r) = $V_{r \geq R}$ g(w) and 𝒭≤ r,

h(r) ≤(h](r) ∀ r∈𝒭

Therefore,

g $\subseteq (g]$→ (1)

Conversely,

we can prove,

(g] ⊆ g →(2)

From equation (1) and (2),

we have

g = (g]

Definition: 3.3

Let g be a ℒ-prime fuzzy subset of 𝒭. We define (g] by the rule

(g](r) = $s \geq rVf(s)$

A fuzzy subset of 𝒭 is called strongly convex if g = (g]

Proposition 3.4

If ℒ- Lattice of fuzzy prime ideal of an ordered semigroup 𝒭is convex,

then g= $\cup_{y_\lambda}$∈ fy_λ

Definition 3.5

Let 𝒭 be an ordered semigroup a ∈𝒭 and **λ**∈ [0,1]. An ordered fuzzy point a_λ of 𝒭 is defined as a_λ (x) = $\{\lambda, if\ x\ \epsilon\ (a]0,\ if\ x\ \notin (a]$

Example 3.6

Lattice of Prime Fuzzy Ideals in a Ring.

Consider a commutative ring R with elements {a, b, c, d} and operations + and ×. Define the fuzzy ideals as follows:

- Fuzzy Ideal P: Assign membership values 0.2, 0.4, 0.6, and 0.8 to elements a, b, c, and d, respectively.
- Fuzzy Ideal Q: Assign membership values 0.3, 0.6, 0.9, and 1.0 to elements a, b, c, and d, respectively.
- Fuzzy Ideal R: Assign membership values 0.4, 0.5, 0.7, and 0.9 to elements a, b, c, and d, respectively.

The lattice formed by these prime fuzzy ideals is characterized by the containment relationships among them: P ⊆ R ⊆ Q.

Theorem: 3.7

Let 𝒭 be an ordered semigroup. 𝒥 is a fuzzy prime idealof 𝒭,if and only if g_s is a ℒ-fuzzy prime ideal of 𝒭.

Proof:

Let 𝒥- an ideal of 𝒭. If g is a ℒ -fuzzy prime ideal of 𝒭 such that a ° b ⊆ g. If a doesn't contained in g, there exist ordered fuzzy point X_λ∈ g (**λ**>0) such that X_λ∉ g. For any Yμ ∈ h (μ ≠ 0)

since X_λ° Yμ⊆a ° b ⊆g and z ∈𝒭

(X_λ)° (Yμ) (z) = $\lambda \cap \mu > 0$,

= 0, otherwise

Since $\mathscr{I}$ is prime (x) (y) $\subseteq \mathscr{I}$

On the contrary,

Let A and B be two ideals of s such that AB $\subseteq \mathscr{I}$. Since g is a $\mathscr{L}$- fuzzy prime ideal of s we have $k_A \subseteq g$ and $k_B \subseteq g$

$\therefore$ A $\subseteq \mathscr{I}$ (or) B $\subseteq \mathscr{I}$

$\Rightarrow$AB $\subseteq \mathscr{I}$

Example: 3.8

Lattices of prime fuzzy ideals in a group

consider a group with elements {e, a, b}, where 'e' denotes the identity element, and 'a' and 'b' are other elements of the group.

Prime fuzzy ideals	Fuzzy Closure (%)
{e}	100
{e, a}	80
{e, b}	80
{e, a, b}	70

The first row represents the prime fuzzy ideal {e}, which includes only the identity element with a fuzzy closure of 100% (i.e) the closure of this fuzzy ideal under the group operation is perfect.

The second row represents the prime fuzzy ideal {e, a}, which includes the identity element and the element 'a' with a fuzzy closure of 80%. That is, fuzzy closure of this fuzzy ideal under the group operation is 80%.

Similarly, the subsequent rows represent prime fuzzy ideals with different combinations of elements and their corresponding fuzzy closures. The fuzzy closure percentage decreases as more elements are included in the fuzzy ideals.

IV. Properties of Fuzzy Prime ideals in Lattice:

a.Fuzzy Prime Ideal Property: A fuzzy ideal F in a lattice L is a prime fuzzy ideal if for any elements a, b in L, if a ∨ b belongs to F, then either a or b belongs to F.

b. Prime Implicative Property: A fuzzy ideal F in a lattice L is prime if and only if for any elements a, b in L, if a ∧ b belongs to F, then a or b belongs to F.

c. Non-empty and Proper: A prime fuzzy ideal is non-empty, containing at least one element of the lattice. It is also a proper fuzzy ideal, meaning it does not contain the identity element.

d. Irreducible: A prime fuzzy ideal cannot be expressed as the intersection of two or more distinct fuzzy ideals. It is not reducible to smaller fuzzy ideals.

e. Maximal: A prime fuzzy ideal is maximal among fuzzy ideals that do not contain it properly. There are no fuzzy ideals strictly containing a prime fuzzy ideal.

f. Prime Generator Property: If a fuzzy ideal F is prime, then for any elements a, b in L, if a∧b belongs to F, then there exist prime fuzzy ideals F1 and F2 such that a belongs to F1 and b belongs to F2, and F is the intersection of F1 and F2.

g. Localization Property: Prime fuzzy ideals have a close relationship with the localization of lattices. They can be used to define a localized lattice by localizing the lattice with respect to the prime fuzzy ideal.

h. Quotient Lattice: Prime fuzzy ideals can be used to construct quotient lattices. The quotient lattice is formed by factoring out the lattice by the prime fuzzy ideal, resulting in a new lattice structure.

Example 4.1:

Lattice of Prime Fuzzy Ideals in a Partially Ordered Set

Consider a partially ordered set with elements {x, y, z} and the relation ≤ defined as x ≤ y ≤ z. Define the fuzzy ideals as follows:

- Fuzzy Ideal X: Assign membership values 0.2, 0.4, and 0.6 to elements x, y, and z, respectively.
- Fuzzy Ideal Y: Assign membership values 0.3, 0.5, and 0.7 to elements x, y, and z, respectively.
- Fuzzy Ideal Z: Assign membership values 0.4, 0.6, and 0.8 to elements x, y, and z, respectively.

The lattice formed by these prime fuzzy ideals is characterized by the containment relationships among them: X ⊆ Y ⊆ Z.

Conclusion:-

We are aware that the work of an ordered semigroup structures relies heavily on ideals of ordered semigroups. The conclusion section summarizes the main contributions of the paper, emphasizing the importance of the proposition and theorem in advancing the theory of lattices of prime fuzzy ideals. It may also suggest potential directions for future research in this area.

The idea of the $\mathscr{L}$-fuzzy prime ideal of various semigroup is discussed in this paper. Additionally, we demonstrated that the homomorphic image of an ordered semigroup's $\mathscr{L}$-fuzzy prime ideal also possesses the supremum property. The creation of the $\mathscr{L}$-fuzzy prime maximal ideal of ansemigroup can benefit from these.

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