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## RESEARCH ARTICLE

### A SURVEY ON BULK TRANSPORTATION PROBLEM.

Sungeeta Singh<sup>1</sup>, Sudhir Chauhan<sup>2</sup> and Kuldeep Tanwar<sup>1</sup>.

1. Department of Mathematics, Amity University, Gurgaon, Haryana, India
2. Department of Mathematics, Amity School of Engineering and Technology, Bijwasan, New Delhi, India.

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#### Abstract

Bulk Transportation problem(BTP) is a special type of transportation problem having wide industrial applications. In a BTP the requirement of each destination has to be met from only one source; however, subject to the availability of a commodity, a source can supply to any number of destinations. The paper presents an outline survey of the methods used in solving a BTP. The paper is motivated by the importance of BTP and the need for researchers to be acquainted with all existing methods in order to solve such problems or develop new improved heuristic methods.

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#### Introduction:-

The classical transportation problem consists of 'm' sources each producing a finite available units of a certain commodity and 'n' destinations each having finite demand of that commodity. However, in a BTP the supply of finite available units of a certain commodity from 'm' sources to 'n' destinations is restricted by the condition that the demand of any destination must be fulfilled from only one source. Further, subject to the availability of the commodity a source may supply to any number of destinations. In a BTP, a bulk transportation cost is assigned from each source to each destination. The additional condition named as bulk arises often in practical situations. BTP has a lot of applications in the areas where demand is required to be fulfilled from a single source such as the vehicle routing problems, the scheduling and facility location problems, assigning software development programs to computer programmers, fixed charge source location models in which customer's demand must be fulfilled by a single source.

The specific BTPs studied by researchers are the single criterion BTP, Multi-Criteria BTP, Multi-Index BTP and Non Convex BTP. In single criterion BTP, there is only one objective to minimize the total cost or the total time of transportation. In a Multi-Criteria BTP, there is more than one objective such as minimization of cost, time and deterioration of goods during transportation etc. In a Multi-Index BTP, there are more than two indices such as source, destination, different types of commodities, modes of transportation and facilities etc.

Section 2 deals with the Single criterion BTP; the single criterion being the cost or time. Section 3 presents the Bi Criteria BTP and the work done therein. In Section 4 the Multi-Index BTP is considered and the contributions are highlighted. Section 5 briefly deals with the Non Convex BTP. Section 6 concludes the paper.

**Corresponding Author:- Sungeeta Singh.**

Address:- Department of Mathematics, Amity University, Gurgaon, Haryana, India.

**Single Criterion Bulk Transportation Problem:-**

A single criterion BTP involves only one criterion viz. Cost or Time, The objective of the BTP is thus to minimize cost or to minimize time; the former is known as the Cost minimizing BT while the latter is called the Time minimizing BTP.

**Cost Minimizing Bulk Transportation Problem:-**

Cost Minimizing BTP was formulated by Maio and Roveda [9] with the objective of minimizing the total cost of bulk transportation. In this BTP the requirement of each destination is to be satisfied from only one source. However, a source can supply to any number of destinations according to the availability of the commodity.

Here the number of sources and destinations are 'm' and 'n' respectively. The entire requirement of a destination is met from one source alone, however, a source can supply to any number of destinations.

**The mathematical formulation of the cost minimizing bulk transportation problem is as follows:**

Minimize

$$C = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (1)$$

subject to the constraints

$$\sum_{j=1}^n b_j x_{ij} \leq a_i \quad (i = 1, 2, \dots, m) \quad (2)$$

$$\sum_{i=1}^m x_{ij} = 1 \quad (j = 1, 2, \dots, n) \quad (3)$$

$$x_{ij} = 0 \text{ or } 1 \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, n). \quad (4)$$

where  $a_i$  ( $i=1, 2, 3, \dots, m$ ) is the number of units of a commodity available at source  $i$ ,  $b_j$  ( $j=1, 2, 3, \dots, n$ ) is the units of commodity required at destination  $j$ ,  $c_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) is the per unit cost of bulk transportation of the commodity from source  $i$  to destination  $j$  and  $x_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) the variable assuming the value 1 or 0 according to whether the requirement of destination 'j' is met from source 'i' or not. Also,  $a_i$ 's,  $b_j$ 's and  $c_{ij}$ 's are all non-negative real values and C denote the total cost of bulk transportation.

Maio and Roveda [9] presented a solution procedure based on zero-one implicit enumeration. The main drawback of their algorithm is that a lot of calculations are involved for checking the feasibility of the solution and several solutions have to be recorded till the optimal solution is identified.

An algorithm comprising of two phases for solving this problem is proposed by Srinivasan and Thompson [18]. The problem is converted into a different transportation problem whose basic feasible solution is the optimum solution of the actual problem. In the first phase, the problem is solved by ignoring the constraints while in the second phase, a branch and bound technique is used to obtain a unique solution. The proposed algorithm is more efficient when the number of destinations exceeds the number of sources by a large number. Murthy [10] proposed a method based on lexicographic search for solving cost minimizing bulk transportation problem. Solutions are generated as a string of numbers systematically, in some hierarchy of their values.

Verma and Puri [19] considered the cost minimizing bulk transportation problem and proposed a method based on the branch and bound technique.

**Time Minimizing Bulk Transportation Problem:-**

In this section, a single criterion time minimizing BTP is discussed. The single criterion here is time instead of cost as discussed in the preceding subsection and consequently the objective is to minimize the total time 'T' of the BTP.

Here 'm' and 'n' denotes number of sources and destinations respectively. The entire requirement of a destination is met from one source only; but a source can supply to any number of destinations.

The mathematical formulation of the time minimizing bulk transportation problem is as follows:

Minimize

$$T = \max_{i,j} \{t_{ij} : x_{ij} = 1\} \quad (1)$$

subject to the constraints

$$\sum_{j=1}^n b_j x_{ij} \leq a_i \quad (i = 1, 2, \dots, m) \quad (2)$$

$$\sum_{i=1}^m x_{ij} = 1 (j = 1, 2, \dots, n) \quad (3)$$

$$x_{ij} = 0 \text{ or } 1 (i = 1, 2, \dots, m; j = 1, 2, \dots, n) \quad (4)$$

where  $a_i$  ( $i=1, 2, 3, \dots, m$ ) is the available units of a product available at source  $i$ ,  $b_j$  ( $j=1, 2, 3, \dots, n$ ) is the units of a product required at destination  $j$ ,  $t_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) is the bulk transportation time of the product from source  $i$  to destination  $j$  and  $x_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) the variable assuming the value 1 or 0 according as the requirement of destination  $j$  is met from source  $i$  or not.  $a_i$ 's,  $b_j$ 's and  $t_{ij}$ 's are non-negative real values and  $T$  is the total time of bulk transportation.

A new method for obtaining an initial basic feasible solution of zero-one time minimizing transportation problem is proposed by Bhatia [3]. The initial feasible solution is improved step by step until an optimal solution is obtained. The proposed method is also helpful in checking the feasibility of the problem.

Foulds and Gibbons [5] proposed two methods for The Bulk, Zero-One Time Mini-Max Transportation Model. One method was based on branch and bound method and other was based on backtracking technique. A comparison between the two proposed methods showed that the branch and bound method provides better results as compared to backtracking technique

### Bi-Criteria Bulk Transportation Problem:-

Bi-Criteria BTP is a type of bulk transportation problem with two objectives of minimizing the total cost and minimizing the total time of transportation. Here, as there are two criteria cost and time which are to be minimized, the optimum solution can only be a trade off between the two. Thus, here Pareto Optimal efficient pairs are obtained as the solution pairs.

The mathematical formulation of a Bi-criteria BTP is

Minimize

$$C = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (1)$$

$$T = \max\{t_{ij} x_{ij} : i = 1, 2, \dots, m; j = 1, 2, \dots, n\} \quad (2)$$

subject to the constraints

$$\sum_{j=1}^n b_j x_{ij} \leq a_i (i = 1, 2, \dots, m) \quad (3)$$

$$\sum_{i=1}^m x_{ij} = 1 (j = 1, 2, \dots, n) \quad (4)$$

$$x_{ij} = 0 \text{ or } 1 (i = 1, 2, \dots, m; j = 1, 2, \dots, n) \quad (5)$$

where  $m$  and  $n$  denotes number of sources and destinations respectively and  $a_i$  ( $i=1, 2, 3, \dots, m$ ) be the units of commodity available at source  $i$ ,  $b_j$  ( $j=1, 2, 3, \dots, n$ ) the units of commodity required at destination  $j$ ,  $c_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) the units of cost of bulk transportation of the commodity from source  $i$  to destination  $j$ ,  $t_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) the unit of time of bulk transportation of the commodity from source  $i$  to destination  $j$ , and  $x_{ij}$  ( $i=1, 2, \dots, m; j=1, 2, \dots, n$ ) the variable assuming the value 0 or 1 according as the entire requirement of destination  $j$  is to met or met from source  $i$ . All the parameters  $a_i$ 's,  $b_j$ 's,  $c_{ij}$ 's and  $t_{ij}$ 's are free to take any non-negative real values. Let  $C$  and  $T$  denote the total cost and duration of bulk transportation respectively.

Prakash and Ram [14] proposed a solution for the Bi-Criteria BTP with the objectives of minimizing the cost and duration of transportation as primary and secondary objectives. In the proposed method, priority factors are used to convert the Multi Objective BTP into a single objective BTP.

Gupta et al. [6] studied cost-time trade-off relations in BTP which further generated a sequence of efficient cost-time pairs using which the decision maker has a flexibility in decision making by choosing the appropriate efficient pair.

A variety of BTP were studied by Bhavani [4]. Prakash et al. [13] proposed two methods, one based on branch and bound technique and the other based on preemptive priority factors for solving the cost-time BTP with the objective of minimizing the total cost and duration of BTP without priorities. They proposed two algorithms to get the Pareto Optimal solutions or Cost time trade off pairs. They studied the problem of Prakash and Ram [14] wherein the priorities were not assigned to the objectives. A comparison between the two proposed methods is also shown and it is shown that the algorithm using preemptive priority factors takes less time as compared to the algorithm based on the branch and bound technique.

Prakash et al. [16] proposed the Extremum Difference Method to obtain the Pareto Optimal solutions of the cost-time trade-off BTP. .

Prakash et al. [15] proposed a method based on the lexicographic minimum to obtain Pareto Optimal solutions of the cost-time BTP, by considering a sequence of prioritized bi-criterion BTPs whose solution gives the set of Pareto Optimal solutions of the actual problem. Lexicographic minimum is used to solve the prioritized Bi-criteria bulk transportation problems.

#### Multi-Index Bulk Transportation problem:-

Multi-Index BTP is an extension of bulk transportation problem which involves transporting different types of commodities through various modes of transportations like truck, rail, ship etc. having multiple sources and multiple destinations. The objective of the problem is to find the minimum bulk transportation cost.

Here, there are 'm' sources each producing 'p' commodities, 'n' destinations and 'k' facilities. The bulk transportation cost from a source 'i' to the destination 'j' at a given facility 'k' is  $C(i, j, k)$ . Let  $S(i, p)$  denote the number of units of *p*th commodity available at source 'i' and  $D(j, p)$  denote the number of units of *p*th commodity required at destination 'j'.

Further,  $D^1(i, p)$  denotes the number of units of *p*th commodity supplied from *i*th plant to some destination.  $D^1(j, p)$  denotes the number of units of the *p*th commodity supplied to *j*th destination from some source.

The mathematical formulation of the Multi-Index BTP is as follows:

Minimize

$$Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l C(i, j, k) X(i, j, k) \quad (1)$$

subject to the constraints

$$\sum_{j=1}^n \sum_{k=1}^l D^1(j, p) X(i, j, k) \leq S(i, p), \quad \text{for all } i, p \quad (2)$$

$$\sum_{i=1}^m \sum_{k=1}^l D^1(i, p) X(i, j, k) = D(j, p), \quad \text{for all } j, p \quad (3)$$

$$X(i, j, k) = 0 \text{ or } 1 \quad (4)$$

Constraint(1) represents the total cost of the bulk supply of the commodities from source 'i' to destination 'j' availing facility 'k'. Constraint (2) shows that a source can supply its commodities to more than one destinations while constraint (3) shows that a destination may get its requirement from more than one plant respectively. If constraint (4) has value 1, then commodities are supplied from source 'i' to destination 'j' availing facility 'k' else not.

Babu and Murthy [2] developed a Lexi-Search algorithm for solving a variant BTP. He showed that Lexi-Search algorithm is more efficient than the Branch and Bound method.

Purosotham and Murthy [17] proposed a Lexi-Search algorithm based on pattern recognition technique with the objective of minimizing the cost of multi-product BTP.

Latha [7,8] proposed a method to solve a Multi commodity BTP and a three dimensional time minimization BTP by formulating the model into a zero-one programming problem and solving it by the pattern recognition techniques coupled with the Lexi-search approach.

Naganna[11], Naganna and Murthy [12] proposed a method to solve time dependent multi-index BTP through a Lexi-Search algorithm based on Pattern recognition technique.

#### Non Convex Bulk Transportation Problem:-

Arora and Ahuja [1] proposed a method for solving non convex BTP. A fractional BTP was formed by the authors whose feasible solution was used to find the optimal solution of the non convex BTP. Parametric programming was used to obtain the feasible solution of the fractional BTP.

**Conclusion:-**

The present survey is done to acquaint the reader with the BTP its genesis and application as well as the methods outlined in solving a BTP and its extensions. The authors hope that the present survey will be beneficial to researchers in providing new insights to solve a BTP and its extensions, which in turn, may lead to the development of new and possibly simpler methods to solve BTPs and its extensions. The existing methods in literature to solve a BTP are rather lengthy and time consuming.

**Table 1:-** Contributions in Bulk Transportation Problem

	Author	Type of BTP	Contribution
1.	Maio and Roveda [9]	Cost BTP	A solution procedure based on zero- one implicit enumeration is developed.
2.	Srinivasan and Thompson [18]	Cost BTP	An algorithm comprising of two phases for solving this problem is proposed. In the second phase Branch and Bound procedure is used
3.	Murthy [10]	Cost BTP	A lexicographic search based method is proposed.
4.	Verma and Puri [19]	Cost BTP	A method based on branch and bound technique is proposed.
5.	Bhatia [3]	Time BTP	A new method for obtaining an initial basic feasible solution of time minimizing BTP is proposed which is improved step by step until an optimal solution is obtained.
6.	Foulds and Gibbons [5]	Time BTP	Two methods for the Time Minimizing BTP based on branch and bound method and backtracking technique are proposed.
7.	Prakash and Ram [14]	Cost-Time BTP	Priority factors are proposed to convert the Multi Objective BTP into a single objective BTP
8.	Gupta et al. [6]	Cost-Time BTP	Studied cost-time trade-off relations in BTP which further generated a sequence of efficient cost-time pairs
9.	Bhavani [4]	Cost-Time BTP	A variety of BTPs were studied
10.	Prakash et al. [13]	Cost-Time BTP	Two methods, one based on branch and bound technique and the other on preemptive priority factors, were proposed.
11.	Prakash [14]	Cost-Time BTP	Priority factors are used to convert the Multi Objective BTP into a single objective BTP.
12.	Prakash and Ram [15]	Cost-Time BTP	A method based on the lexicographic minimum is proposed.
13.	Prakash et al. [16]	Cost-Time BTP	Extremum Difference Method is proposed to obtain the Pareto Optimal solutions of the cost-time trade-off BTP.
14.	Babu and Murthy [2]	Multi -Index BTP	A Lexi-Search algorithm for solving a variant BTP is proposed and is shown as more efficient than Branch and Bound Method.
15.	Purosotham and Murthy [17]	Multi -Index BTP	A Lexi-Search algorithm based on pattern recognition technique is proposed.
16.	Latha [7,8]	Multi -Index BTP	A method based on the pattern recognition technique coupled with the Lexi-search approach is proposed.
17.	Naganna[11],Naganna and Murthy [12]	Multi -Index BTP	A method to solve time dependent multi-index BTP by a 'Lexi-Search' algorithm based on 'Pattern Recognition Technique' is

			proposed.
18.	Arora and Ahuja [1]	Non Convex BTP	A fractional BTP was formed by the authors whose feasible solution was used to find the optimal solution of the non convex BTP.

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