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RESEARCH ARTICLE

δ_I -SEMI-CONNECTED AND COMPACT SPACES IN IDEAL TOPOLOGICAL SPACES

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Abstract

In this paper, we introduce $\delta_{\mathbf{I}}$ -s-sep sets, $\delta_{\mathbf{I}}$ -s-con and $\delta_{\mathbf{I}}$ -s-com spaces also study some of its properties in topological spaces via ideals.

Introduction:-

The notion of ideal in topological spaces was studied by Kuratowski [8] & Vaidyanathaswamy [12]. Applications to various fields in ideal topological spaces were investigated by Jankovic and Hamlett [7], Dontchev et al. [3], Mukherjee et al. [9], Arenas et al. [2], Navaneethakrishnan et al. [11], Nasef and Mahmoud [10], etc. In 2008, Ekici and Noiri [4] introduced the notion of connectedness in ideal topological spaces.

Preliminaries

Throughout this paper, (X, τ, I) and (Y, σ, I) (or simply X and Y), always mean ideal topological spaces on which no separation axioms are assumed.

Definition 2.1. [1] A subset A of an ideal topological space (X, τ, I) is said to be $\delta_{\mathbf{I}}$ -s-o if $A \subseteq cl^*(int_\delta(A))$. The complement of $\delta_{\mathbf{I}}$ -s-o set is called $\delta_{\mathbf{I}}$ -s-cl set.

Definition 2.2. [1] Let A be a subset of an ideal topological space (X, τ, I) and x be a point of X. Then

1. x is called a $\delta_{\mathbf{I}}$ -s-clu point of A if $A \cap U \neq \emptyset$ for every $U \in \delta_{\mathbf{I}}SO(X)$,
2. the family of all $\delta_{\mathbf{I}}$ -s-clu points of A is called $\delta_{\mathbf{I}}$ -s-clo of A and is denoted by $scl_{\delta_{\mathbf{I}}}(A)$.

Definition 2.3. [5] A function $f : (X, \tau, I) \to (Y, \sigma, I)$ is said to be $\delta_{\mathbf{I}}$ -s- irresolute if inverse image of every $\delta_{\mathbf{I}}$ -s-o set in Y is $\delta_{\mathbf{I}}$ -s-o set in X.

Definition 2.4. [6] A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be contra $\delta_{\mathbf{I}}$ -s-continuous if $f^{-1}(V)$ is $\delta_{\mathbf{I}}$ -s-cl in X for each open set V of Y.

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δ_I-semi-separated

Definition 3.1. Let (X, τ, I) be an ideal topological space. Two non-empty subsets M and N are said to be δ_I-semi-separated {simply written as δ_I-s-sep} if and only if $M \cap scl_{\delta_I}(N) = \emptyset$ and $scl_{\delta_I}(M) \cap N = \emptyset$. i.e., $[M \cap scl_{\delta_I}(N)] \cup [scl_{\delta_I}(M) \cap N] = \emptyset$.

Definition 3.2. If $X = M \cup N$ such that M and N are non-empty δ_I-s-sep sets in (X, τ, I) then M, N form a δ_I-s-separation of X.

Example 3.3. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{b, d\}, \{a, b, d\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Consider $P = \{a\}$, $Q = \{b\}$ and $R = \{d\}$. Then the sets P and Q are δ_I-s-sep but the sets Q and R are not δ_I-s-sep.

Definition 3.4. A point $x \in X$ is said to be an δ_I-s-adherent point of a subset A of an ideal topological space (X, τ, I) if every δ_I-s-o set containing x, contains atleast one point of A.

Remark 3.5. Two δ_I-s-sep sets are always disjoint. But two disjoint sets need not be δ_I-s-sep. In Example 3.3, the sets Q and R are disjoint but not δ_I-s-sep.

Theorem 3.6. Two sets are δ_I-s-sep if and only if they are disjoint and neither of them contains δ_I-s-clu point of the other.

Proof. Let A and B be δ_I-s-sep. Now, $A \cap scl_{\delta_I}(B) = \emptyset \Leftrightarrow A \cap (B \cup B_1) = \emptyset$, where the set B_1 denotes the set of all δ_I-s-clu points of $B \Leftrightarrow A$ and B are disjoint and A contains no δ_I-s-clu point of B. Similaly, $scl_{\delta_I}(A) \cap B = \emptyset$ if and only if A and B are disjoint and B contains no δ_I-s-clu point of A.

Theorem 3.7. Subsets of δ_I-s-sep sets are δ_I-s-sep.

Proof. Let C and D be subsets of two δ_I-s-sep sets A and B respectively. Then $A \cap scl_{\delta_I}(B) = \emptyset$ and $scl_{\delta_I}(A) \cap B = \emptyset$. Then we have $C \cap scl_{\delta_I}(D) \subseteq A \cap scl_{\delta_I}(B) = \emptyset$ and $scl_{\delta_I}(C) \cap D \subseteq scl_{\delta_I}(A) \cap B = \emptyset$. Thus C and D are δ_I-s-sep.

Theorem 3.8. Two δ_I-s-cl subsets of X are δ_I-s-sep if and only if they are disjoint.

Proof. By Remark 3.5 δ_I-s-cl separated sets are disjoint.

Conversely, let A and B be two δ_I-s-cl disjoint sets. Then we have $scl_{\delta_I}(A) = A$, $scl_{\delta_I}(B) = B$ and $A \cap B = \emptyset$. Consequently, $A \cap scl_{\delta_I}(B) = \emptyset$ and $scl_{\delta_I}(A) \cap B = \emptyset$. Hence A and B are δ_I-s-sep.

Theorem 3.9. Two δ_I-s-o subsets of X are δ_I-s-sep if and only if they are disjoint.

Proof. By Remark 3.5 δ_I-s-o separated sets are disjoint.

Conversely, let P and Q be two δ_I-s-o disjoint sets. Suppose that $P \cap scl_{\delta_I}(Q) \neq \emptyset$ and let $x \in P \cap scl_{\delta_I}(Q)$. Then $x \in P$ and x is a δ_I-s-adherent point of Q. Since P is a δ_I-s-o set containing x and x is a δ_I-s-adherent point of Q, therefore P must contain atleast one point of Q. Thus we have $P \cap Q \neq \emptyset$ which is a contradicton. Therefore $P \cap scl_{\delta_I}(Q) = \emptyset$. Similarly, $scl_{\delta_I}(P) \cap Q = \emptyset$. Hence P and Q are δ_I-s-sep.

Theorem 3.10. If the union of two δ_I-s-sep sets is a δ_I-s-cl set then the individual sets are δ_I-s-clo of themselves.

Proof. Let M and N be two δ_I-s-sep sets such that $M \cup N$ is δ_I-s-cl. Now, $M \cup N = scl_{\delta_I}(M \cup N) \supseteq scl_{\delta_I}(M) \cup scl_{\delta_I}(B)$. Therefore $scl_{\delta_I}(M) = scl_{\delta_I}(M) \cap [scl_{\delta_I}(M) \cup scl_{\delta_I}(N)] \subseteq scl_{\delta_I}(M) \cap [M \cup N] = M$. Thus we have $scl_{\delta_I}(M) = M$. Similarly, $scl_{\delta_I}(M) = M$.

Theorem 3.11. If the union of two δ_I-s-sep sets is δ-open, then the individual sets are δ_I-s-o.

Proof. Let M and N be two δ_I-s-sep sets such that $M \cup N$ is δ-open. Therefore we have $M \cup N \cap [scl_{\delta_I}(N)]^c$ is δ_I-s-o and so $M \cup N \cap [scl_{\delta_I}(N)]^c = M$. This implies M is δ_I-s-o. Similarly, we can prove N is δ_I-s-o.

δ_I-semi-connected

Definition 4.1. A space (X, τ, I) is δ_I-s-con if and only if X has no δ_I-s-separation.

If $\mathbf{X}$ is not $\delta_{\mathbf{I}}$-s-con then it is $\delta_{\mathbf{I}}$-s-discon.

Definition 4.2. A subset of $(\mathbf{X}, \tau, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-con if it is $\delta_{\mathbf{I}}$-s-con as a subspace.

Theorem 4.3. An ideal topological space $(\mathbf{X}, \tau, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-discon if and only if there exist a non-empty proper subset of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl.

Proof. Necessity: Let $(\mathbf{X}, \tau, \mathbf{I})$ be $\delta_{\mathbf{I}}$-s-discon. Then there exist non-empty $\delta_{\mathbf{I}}$-s-sep subsets M and N of $\mathbf{X}$ such that $M \cup N = \mathbf{X}$. Therefore $scl_{\delta_{\mathbf{I}}}(M) \cup N = \mathbf{X}$, $M \cup scl_{\delta_{\mathbf{I}}}(N) = \mathbf{X}$ and $M \cap N = \emptyset$. Thus we have $M = \mathbf{X} - N$, $M = \mathbf{X} - scl_{\delta_{\mathbf{I}}}(N)$ and $N = \mathbf{X} - scl_{\delta_{\mathbf{I}}}(M)$. This shows that, M is non-empty proper subset of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl.

Sufficiency: Let M be a non-empty proper subset of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s- cl. Then, M^c is a non-empty proper subset of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-cl and $\delta_{\mathbf{I}}$-s-o. Thus $M \cap M^c = \emptyset$, $scl_{\delta_{\mathbf{I}}}(M)=M$ and $scl_{\delta_{\mathbf{I}}}(M^c)=M^c$ and therefore $scl_{\delta_{\mathbf{I}}}(M) \cap M^c = M \cap M^c = \emptyset$ and $M \cap scl_{\delta_{\mathbf{I}}}(M^c) = M \cap M^c = \emptyset$. Also $\mathbf{X} = M \cup M^c$. Hence $\mathbf{X}$ is $\delta_{\mathbf{I}}$-s-discon.

Theorem 4.4. An ideal topological space $(\mathbf{X}, \tau, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-discon if and only if $\mathbf{X}$ is the union of non-empty disjoint $\delta_{\mathbf{I}}$-s-o sets.

Proof. Necessity: Let $\mathbf{X}$ be $\delta_{\mathbf{I}}$-s-discon. Then there exist a non-empty proper subset M of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl. Therefore M^c is a non-empty proper subset of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl. This shows that $\mathbf{X} = M \cup M^c$ and $M \cap M^c = \emptyset$. This implies that $\mathbf{X}$ is the union of two non-empty disjoint $\delta_{\mathbf{I}}$-s-o sets.

Sufficiency: Let $\mathbf{X}$ be the union of two non-empty disjoint $\delta_{\mathbf{I}}$-s-o sets M and N. Then $N^c = M$. Now N is $\delta_{\mathbf{I}}$-s-o, it follows that M is $\delta_{\mathbf{I}}$-s-cl. Since $N \neq \emptyset$, it implies that M is a non-empty proper subset of $\mathbf{X}$ which is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl. This shows that $\mathbf{X}$ is $\delta_{\mathbf{I}}$-s-discon.

Theorem 4.5. An ideal topological space $(\mathbf{X}, \tau, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-con if and only if $\mathbf{X}$ cannot be written as the union of non-empty disjoint $\delta_{\mathbf{I}}$-s-o sets.

Proof. Obvious.

Corollary 4.6. A space $(\mathbf{X}, \tau, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-con (resp. $\delta_{\mathbf{I}}$-s-discon) if and only if $\mathbf{X}$ cannot be written as (resp. can be written as) the union of non-empty disjoint $\delta_{\mathbf{I}}$-s-cl sets.

Theorem 4.7. An ideal topological space $(\mathbf{X}, \tau, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-con if and only if the only subsets of $\mathbf{X}$ which is $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl are $\emptyset$ and $\mathbf{X}$.

Proof. Let F be a $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl subset of $\mathbf{X}$. Then $\mathbf{X} - F$ is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl. Since $\mathbf{X}$ is $\delta_{\mathbf{I}}$-s-con, $\mathbf{X}$ can not be expressed as union of two disjoint non empty $\delta_{\mathbf{I}}$-s-o sets F and $\mathbf{X} - F$, which implies $\mathbf{X} - F$ is empty.

Conversely, suppose $\mathbf{X} = U \cup V$ where U and V are disjoint non-empty $\delta_{\mathbf{I}}$-s-o sets of $\mathbf{X}$. Then U is both $\delta_{\mathbf{I}}$-s-o and $\delta_{\mathbf{I}}$-s-cl. Therefore by assumption, either $U = \emptyset$ or $\mathbf{X}$, which contradicts the assumption that U and V are disjoint non-empty $\delta_{\mathbf{I}}$-s-o subsets of $\mathbf{X}$. Therefore $\mathbf{X}$ is $\delta_{\mathbf{I}}$-s-con.

Corollary 4.8. If $f: (\mathbf{X}, \tau, \mathbf{I}) \rightarrow (Y, \sigma, \mathbf{J})$ is a $\delta_{\mathbf{I}}$-s-irresolute surjective function and $\mathbf{X}$ is $\delta_{\mathbf{I}}$-s-con, then Y is δ-s-con.

Theorem 4.9. If the sets P and Q form a $\delta_{\mathbf{I}}$-s-separation of $(\mathbf{X}, \tau, \mathbf{I})$ and if $(Y, \sigma, \mathbf{I})$ is $\delta_{\mathbf{I}}$-s-con subspace of $\mathbf{X}$, then Y lies entirely within either P or Q.

Proof. Since P and Q form a $\delta_{\mathbf{I}}$-s-separation of $\mathbf{X}$. If $P \cap Y$ and $Q \cap Y$ were both non-empty, they would form a $\delta_{\mathbf{I}}$-s-separation of Y, which is a contradiction. Therefore one of them is empty. Hence Y must lie entirely in P or in Q.

Theorem 4.10. A contra $\delta_{\mathbf{I}}$-s-continuous image of a $\delta_{\mathbf{I}}$-s-con space is connected.

Proof. Let $f:(X,\tau,I)\to(Y,\sigma)$ be a contra δ_I-s-continuous function of a δ_I-s-con space (X,τ,I) onto a topological space (Y,σ). Suppose Y is disconnected. Let A and B form a separation of Y. Then A and B are clopen and $Y=A\cup B$ where $A\cap B=\emptyset$. Since f is contra δ_I-s-continuous, $X=f^{-1}(Y)=f^{-1}(A\cup B)=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty δ_I-s-o sets in X. Also $f^{-1}(A)\cap f^{-1}(B)=\emptyset$. Hence X is not δ_I-s-con. This is a contradiction. Therefore Y is connected.

Theorem 4.11. If A is δ_I-s-con and $A\subseteq B\subseteq scl_{\delta_I}(A)$, then B is δ_I-s-con.

Proof. Let A be δ_I-s-con and let $A\subseteq B\subseteq scl_{\delta_I}(A)$. Suppose that B is not δ_I-s-con, then C and D form a δ_I-s-seperation of B. By Theorem 4.9, A must lie entirely in C or in D. Suppose that $A\subseteq C$ implies $scl_{\delta_I}(A)\cap D\subseteq scl_{\delta_I}(C)\cap D=\emptyset$. Also, $D\subseteq B\subseteq scl_{\delta_I}(A)$ implies $scl_{\delta_I}(A)\cap D=D$. This shows that $D=\emptyset$, which is a contradiction. Similarly, we will have a contradiction for $A\subseteq D$. Therefore B is δ_I-s-con.

Corollary 4.12. The δ_I-s-clo of a δ_I-s-con set is δ_I-s-con.

Theorem 4.13. If every two points of a set E are contained in some δ_I-s-con subset of E, then E is δ_I-s-con.

Proof. Suppose that E is not δ_I-s-con. Then, E is the union of non-empty disjoint δ_I-s- sep sets A and B. Since A and B are non-empty disjoint sets, let $a\in A$ and $b\in B$ and a,b are two distinct points of E. By hypothesis, there exists a δ_I-s-con subset C of E such that $a, b\in C$. By Theorem 4.9, we have $C\subseteq A$ or $C\subseteq B$. This is not possible, since A and B are disjoint and C contains atleast one point of A and one that of B. Thus a contradiction. Hence E is δ_I-s-con.

Theorem 4.14. The union of any family of δ_I-s-con sets having a non-empty intersection is δ_I-s-con.

Proof. Let $\{E_\alpha\}$ be any family of δ_I-s-con sets such that $\cap_\alpha E_\alpha\neq\emptyset$. Let $E=\cup_\alpha E_\alpha$. Suppose that E is not δ_I-s-con, then A and B constitute a δ_I-s-seperation of E. Since $\cap_\alpha E_\alpha\neq\emptyset$, let $x\in\cap_\alpha E_\alpha$. Then x belongs to each E_α and so $x\in E$. Consequently, $x\in A$ or $x\in B$. Suppose that $x\in A$, $E_\alpha\cap A\neq\emptyset$ for every α. From Theorem 4.9, $E_\alpha\subseteq A$ or $E_\alpha\subseteq B$. Since A and B are disjoint and $E_\alpha\cap A\neq\emptyset$ for every α . We must have $E_\alpha\subseteq A$ for each α. Consequently, $\cup_\alpha E_\alpha\subseteq A$ or $E\subseteq A$. This shows that $B=\emptyset$, which is a contradiction. Hence E is δ_I-s-con.

Corollary 4.15. Let $\{E_\alpha|\alpha\in\Lambda\}$ be a family of δ_I-s-con subsets of (X,τ,I) such that one of the members of this family intersects every other member. Then $\bigcup\{E_\alpha|\alpha\in\Lambda\}$ is δ_I-s-con.

Proof. Let E_{α_o} be a member of the given family such that $E_{\alpha_o}\cap E_\alpha\neq\emptyset$ for every $\alpha\in\Lambda$. Then By Theorem 4.14, $C_\alpha=E_{\alpha_o}\cup E_\alpha$ is δ_I-s-con for each α. Now, $\bigcup\{C_\alpha|\alpha\in\Lambda\}=\bigcup\{E_{\alpha_o}\cup E_\alpha|\alpha\in\Lambda\}=E_{\alpha_o}\cup(\bigcup\{E_\alpha|\alpha\in\Lambda\})=\bigcup\{E_\alpha|\alpha\in\Lambda\}$ and $\bigcap\{C_\alpha|\alpha\in\Lambda\}=\bigcap\{E_{\alpha_o}\cup E_\alpha|\alpha\in\Lambda\}=E_{\alpha_o}\cup(\bigcap\{E_\alpha|\alpha\in\Lambda\})\neq\emptyset$. Thus $\bigcup\{C_\alpha|\alpha\in\Lambda\}$ is the union of δ_I-s-con sets having a non-empty intersection is δ_I-s-con. Therefore $\bigcup\{E_\alpha|\alpha\in\Lambda\}$ is δ_I-s-con.

δ_I-semi-compact

Definition 5.1. A collection $\{A_\alpha|\alpha\in\Lambda\}$ of δ_I-s-o sets in an ideal topological space (X,τ,I) is called δ_I-s-o cover of a subset B of X if $B\subseteq\bigcup\{A_\alpha|\alpha\in\Lambda\}$ holds.

Definition 5.2. An ideal topological space (X,τ,I) is said to be δ_I-semi-compact{simply written as δ_I-s-com} if every δ_I-s-o cover of X has a finite subcover.

Definition 5.3. A subset B of an ideal topological space (X,τ,I) is called δ_I-s-com relative to X if for every collection $\{A_\alpha|\alpha\in\Lambda\}$ of δ_I-s-o subsets of X such that $B\subseteq\bigcup\{A_\alpha|\alpha\in\Lambda\}$, there exists a finite subset Λ_o of Λ such that $B\subseteq\bigcup\{A_\alpha|\alpha\in\Lambda_o\}$

Proposition 5.4. A δ_I-s-cl subset of a δ_I-s-com space (X,τ,I) is δ_I-s-com relative to (X,τ,I).

Proof: Let A be any δ_I-s-cl subset of an ideal topological space (X,τ,I). Then A^c is δ_I-s-o in (X,τ,I). Let $S=\{A_i|i\in\Lambda\}$ be a δ_I-s-o cover of A. Then $S^*=S\cup A^c$ is a δ_I-s-o cover of X. That is $X=(\cup_{i\in\Lambda}A_i)\cup A^c$. By assumption X is δ -s-com and hence S^* is reducible to a finite subcover of X say $X=A_{i_1}\cup A_{i_2}\cup\ldots\cup A_{i_n}\cup A^c$ where $A_{i_k}\in S^*$. But A and A^c are disjoint. Hence $A\subseteq A_{i_1}\cup A_{i_2}\cup\ldots\cup A_{i_n}\in S$. Thus δ_I-s-o cover S of A contains a finite subcover. Hence A is δ_I -s-com relative to X.

Proposition 5.5. If a map $f : (X, \tau, I) \to (Y, \sigma, J)$ is δ_I -s-irresolute and a subset B of X is δ_I -s-com relative to X, then $f(B)$ is δ -s-com relative to Y.

Proof: Let $\{A_\alpha | \alpha \in \Lambda\}$ be a collection of δ -s-o sets in Y such that $f(B) \subseteq \bigcup_{\alpha\in\Lambda} A_\alpha$. Then $B \subseteq \bigcup_\alpha f^{-1}(A_\alpha)$, where $\{f^{-1}(A_\alpha) | \alpha \in \Lambda\}$ is a δ -s-o set in X. Since B is δ_I -s-com relative to X, there exists finite subcollection $\{f^{-1}(A_1), f^{-1}(A_2), ..., f^{-1}(A_n)\}$ such that $B \subseteq \bigcup_{\alpha=1}^{n} f^{-1}(A_\alpha)$. That is $f(B) \subseteq \bigcup_{\alpha=1}^{n} A_\alpha$. Hence $f(B)$ is δ -s-com relative to Y.

Proposition 5.6. Every finite union of δ_I -s-com sets is δ_I -s-com.

Proof. Let U and V be any δ_I -s-com subsets of (X, τ, I). Let F be a δ_I -s-o cover of $U \cup V$. Then F will also be a δ_I -s-o cover of both U and V. By assumption, there exists a finite subcollection of F of δ_I -s-o sets, say $\{U_1, U_2, ..., U_n\}$ and $\{V_1, V_2, ..., V_n\}$ covering U and V respectively. Then the collection $\{U_1, U_2, ..., U_n, V_1, V_2, ..., V_n\}$ is a finite collection of δ_I -s-o sets covering $U \cup V$. By induction, every finite union of δ_I -s-com sets is δ_I -s-com.

Proposition 5.7. Let A be a δ_I -s-com subset of a space (X, τ, I) and B be a δ_I -s-cl subset of X. Then $A \cap B$ is δ_I -s-com in X.

Proof. Let $\{G_\alpha\}$ be a δ_I -s-o cover of $A \cap B$. Since B is δ_I -s-cl, $\{G_\alpha, B^c\}$ is δ_I -s-o. Then $\{G_\alpha, B^c\}$ is a δ_I -s-o cover of A. By assumption A is δ_I -s-com, there exists a finite subcollection, say, $\{G_k, B^c\}$. Then $\{G_k\}$ is a finite δ_I -s-o subcover of $A \cap B$. Thus $A \cap B$ is δ_I -s-com in X.

Theorem 5.8. An ideal topological space (X, τ, I) is δ_I -s-com if and only if every family of δ_I -s-cl subsets of X having finite intersection property has a non-empty intersection.

Proof. Suppose (X, τ, I) is δ_I -s-com. Let $\{A_\alpha | \alpha \in \Lambda\}$ be a family of δ_I -s-cl sets with finite intersection property, Suppose $\bigcap_{\alpha\in\Lambda}\{A_\alpha\} = \emptyset$. Then $[\bigcap_{\alpha\in\Lambda}\{A_\alpha\}]^c = X$. This implies $\bigcup_{\alpha\in\Lambda}\{A_\alpha^c\} = X$. Thus the cover $\{A_\alpha^c | \alpha \in \Lambda\}$ is a δ_I-s-o cover of (X, τ, I). Then by assumption,the δ_I-s-o cover $\{A_\alpha^c | \alpha \in \Lambda\}$ has a finite subcover, say $\{A_\alpha^c | \alpha = 1,2, ... n\}$. This implies $X = \bigcup_{\alpha=1}^{n} \{A_\alpha^c\} = [\bigcap_{\alpha=1}^{n}\{A_\alpha\}]^c$ and so $\emptyset = \bigcap_{\alpha=1}^{n}\{A_\alpha\}$. This contradicts the assumption. Hence $\bigcap_{\alpha\in\Lambda}\{A_\alpha\} \neq \emptyset$.

Conversely, suppose (X, τ, I) is not δ_I -s-com. Then there exists a δ_I -s-o cover of (X, τ, I) say $\{G_\alpha | \alpha \in \Lambda\}$ having no finite subcover. This implies for any finite subfamily $\{G_\alpha | \alpha = 1, 2, ..., n\}$ of $\{G_\alpha | \alpha \in \Lambda\}$ we have $\bigcup_{\alpha=1}^{n} G_\alpha \neq X$. Now $\emptyset \neq [\bigcup_{\alpha=1}^{n}\{G_\alpha\}]^c = \bigcap_{\alpha=1}^{n} \{G_\alpha^c\}$. Then the family $\{G_\alpha^c | \alpha \in \Lambda\}$ of δ_I -s-cl sets has a finite intersection property. Also by assumption $\{\bigcap_{\alpha\in\Lambda} G_\alpha^c\} \neq \emptyset$ and so $\bigcup_\alpha G_\alpha \neq X$. This implies $\{G_\alpha | \alpha \in \Lambda\}$ is not a δ_I -s-cover of (X, τ, I). This contradicts the fact that $\{G_\alpha | \alpha \in \Lambda\}$ is a δ_I -s-cover for (X, τ, I). Therefore δ_I -s-o cover $\{G_\alpha | \alpha \in \Lambda\}$ of X has a finite subcover $\{G_\alpha | \alpha = 1, 2, ..., n\}$. Hence (X, τ, I) is δ_I -s-com.

Corollary 5.9. An ideal topological space (X, τ, I) is δ_I -s-com if and only if every family of δ_I -s-cl sets of X with empty intersection has a finite sub-family with empty intersection.

Proposition 5.10. The image of a δ_I -s-com space under δ_I -s-irresolute surjective function is δ -s-com.

Proof. Let $f : (X, \tau, I) \to (Y, \sigma, I)$ is a δ_I -s-irresolute function from δ_I -s-com space (X, τ, I) onto an ideal topological space (Y, σ, I). Let $\{A_\alpha | \alpha \in \Lambda\}$ be a δ -s-o cover of Y. Then $\{f^{-1}(A_\alpha) | \alpha \in \Lambda\}$ is a δ_I -s-o cover of X, since f is δ_I -s-irresolute. As X is δ_I -s-com, δ_I -s-o cover $\{f^{-1}(A_\alpha) | \alpha \in \Lambda\}$ of X has a finite subcover, say, $\{f^{-1}(A_\alpha) | \alpha = 1, 2, ..., n\}$. Therefore $X = \bigcup_{\alpha=1}^{n}\{f^{-1}(A_\alpha) | \alpha = 1, 2, ..., n\}$. Thus $\{A_1, A_2, ... A_n\}$ is a finite subcover for Y. Hence Y is δ -s-com.

Definition 5.11. An ideal toplogical space (X, τ, I) is called locally δ_I -s-com if every point in X has atleast one δ_I -s-neighborhood whose closure is δ_I -s-com.

Proposition 5.12. Every δ_I -s-com space is locally δ_I -s-com.

Proof. Let (X, τ, I) be a δ_I -s-com space. Let $x \in X$. Then X is a δ_I -s-neighborhood of x such that $cl(X) = X$ is δ_I-s-com. Hence X is locally δ_I-s-com.

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