Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI:10.21474/IJAR01/20245
DOI URL: http://dx.doi.org/10.21474/IJAR01/20245

RESEARCH ARTICLE

PERFECT S-GEODETIC FUZZY GRAPHS

Sameeha Rehmani
Department of Mathematics, Sullamussalam Science College, Areacode-673639, Kerala, India.

Manuscript Info

Manuscript History
Received: 14 November 2024
Final Accepted: 16 December 2024
Published: January 2025

Key words:-
s-Geodetic Basis, s-Geodetic Number, Pseudo s-Geodetic Set, Pseudo s-Geodetic Number, Perfect s-Geodetic Fuzzy Graph

Abstract

This study introduces the concept of a ”Pseudo s-geodetic set” within the context of fuzzy graphs. This set comprises all nodes that are not members of any s-geodetic basis for a given fuzzy graph G. The size of this set is termed the ”Pseudo s-geodetic number” of G. We specifically define fuzzy graphs possessing a Pseudo s-geodetic number of zero as ”Perfect s-geodetic fuzzy graphs.” Several examples of such graphs are provided. Furthermore, we demonstrate that complete fuzzy graphs with two nodes, as well as fuzzy cycles where each arc has identical strength, fall under the category of Perfect s-geodetic fuzzy graphs.

Introduction:-

Fuzzy set theory, a mathematical framework for representing uncertainty in everyday situations, was pioneered by Zadeh in 1965 [18]. Building upon this foundation, Rosenfeld [11], along with Yeh and Bang [17], developed the theory of fuzzy graphs in 1975. Rosenfeld also established fuzzy analogs of various graph-theoretic concepts, including paths, cycles, trees, and connectedness, exploring their properties [11]. Subsequent research has explored related concepts such as fuzzy trees [10], fuzzy interval graphs [7], and cycles and co-cycles in fuzzy graphs [8]. Bhattacharya [1] introduced fuzzy groups and the notion of a metric in fuzzy graphs. The concept of strong arcs was introduced by Bhutani and Rosenfeld in 2003 [4], who also defined fuzzy end nodes and examined their properties [2]. Geodesic distance in fuzzy graphs was defined by Bhutani and Rosenfeld [3], which Suvarna and Sunitha [16] used to introduce and study the geodesic iteration number and geodesic number. These concepts were later extended using μ-distance by Linda and Sunitha [5]. The concept of sum distance and its metric properties was introduced by Mini Tom and Sunitha [6]. More recently, Sameeha and Sunitha [13] introduced the s-geodetic iteration number and s-geodetic number based on sum distance. The set of nodes in a fuzzy graph G = (V, σ, μ) that are not included in any s-geodetic basis of G is defined as the pseudo s-geodetic set of G. The cardinality of this set is the pseudo s-geodetic number of G [14]. This paper defines fuzzy graphs with a pseudo s-geodetic number of zero as perfect sgeodetic fuzzy graphs, provides examples of such graphs, and explores some of their properties. Specifically, we prove that complete fuzzy graphs and fuzzy cycles are perfect s-geodetic fuzzy graphs.

Preliminaries

In this section, a brief summary of some basic definitions in fuzzy graphs taken from [3, 4, 9, 10, 16] are given.

A fuzzy graph [9] is a triplet $G:(V,\sigma,\mu)$ where σ is a fuzzy sub set of a set V of nodes and μ is a fuzzy relation on σ. ie, $\mu(u,v) \leq \sigma(u)\wedge\sigma(v), \forall u,v \in V$. We assume that V is finite and non-empty, μ is reflexive $(i.e., \mu(x,x)=\sigma(x), \forall x)$ and symmetric $(i.e., \mu(x,y)=\mu(y,x), \forall(x,y))$. Also we denote the underlying crisp graph by $G^*:(\sigma^*,\mu^*)$ where $\sigma^*=\{u\in V/\sigma(u)>0\}$and $\mu^*=\{(u,v)\in V\times V/\mu(u,v)>0\}$. Here we assume $\sigma^*=V$. A fuzzy graph is called a complete fuzzy graph [9] if $\mu(u,v)=\sigma(u)\wedge$

Corresponding Author:-Sameeha Rehmani
Address:-Department of Mathematics, Sullamussalam Science College, Areacode-673639, Kerala, India.

$\sigma(v)\ \forall u, v\ \in \sigma *$.A sequence of distinct nodes $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0,\ i = 1,2,\ldots,n$ is called a path P_n[9] of length n. An arc of G with least nonZero membership value is the weakest arc of G. The degree of membership of a weakest arc in the path is defined as the strength of the path. The path becomes a cycle if $u_0 = u_n,\ n \geq 3$ and a cycle is called a fuzzy cycle [10] if it contains more than one weakest arc. The strength of connectedness [9] between two nodes u and v is the maximum of the strengths of all paths between u and v and is denoted by $CONN_G(u, v)$. The fuzzy graph G is said to be connected if $CONN_G(u, v) > 0$ for every u, v in σ*. An arc (u,v) of a fuzzy graph is called strong [4] if its weight is at least as great as the strength of connectedness of its end nodes u,v when the arc (u, v) is deleted and a $u - v$ path P is called a strong path if P contains only strong arcs.

For any path $P: u_0 - u_1 - u_2 - \ldots - u_n$,**length** of P, $L(P)$, is defined as the sum of the weights of the arcs in P. That is , $L(P) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i)$.
If n=0, define L(P)=0 and for n≥1, L(P)>0.
For any two nodes u,v in G:(V,σ,μ), if P={P_i: P_i is a u-v path, i=1,2,3,...}, then the **sum distance** between u and v is defined as $d_s(u,v)$ = Min{L(P_i) : P_i∈P,i = 1,2,3,...}[6].

Let S be a set of nodes of a connected fuzzy graph G. The s-geodetic closure[13] (S) of S is the set of all nodes in S together with the nodes that lie on s-geodesics between nodes of S. S is said to be convex if S contains all nodes of every $u - v$ s-geodesic for all u,v in S. i.e, if $(S) = S$.

S is said to be s-geodetic cover (s-geodetic set) of G if $(S) = V(G)$ and anyCover of G with minimum number of nodes is called an s-geodetic basis for G. The s-geodetic number [13] of a fuzzy graph $G: (V, \sigma, \mu)$ is the number of nodes in a s-geodetic basis of G and is denoted by $s - gn(G)$.

The following results have been taken from [16].

Corollary2.1.[16] For a complete fuzzy graph G on 2 nodes, $s - gn(G) = 2$.

Remark2.2.[13] Let $C_n, n \geq 3$, be fuzzy cycles each of whose arcs are having same strength. When n is even, the set of any two s-peripheral nodes is an s-geodetic set of C_n. But when n is odd, no 2 nodes form an s-geodetic set and in fact there exists an s-geodetic set on 3 nodes. Therefore, for cycles having each arc of same strength,

$$s - gn(Cn) = \begin{cases} 2; \text{ when n is even} \\ 1; \text{ when n is odd} \end{cases}$$

Perfect s-geodetic fuzzy graph

In graph theory, the concept of Perfect edge geodetic graph was introduced by Stalin in [15] and in fuzzy graph theory, the concept of Perfect geodesic fuzzy graphs was developed using geodesic distance in [12].

In this paper, the vertex version of this concept in fuzzy graph theory is developed using sum distance and is termed as Perfect s-geodetic fuzzy graph.

Definition3.1.[14] Let $G: (V, \sigma, \mu)$ be a connected fuzzy graph and S be an s-geodetic basis of G. Then the set of nodes which do not belong to anys-geodetic basis of G is the Pseudo s-geodetic set S' of G.
The cardinality of Pseudo s-geodetic set S' is called Pseudo s-geodetic numberand is denoted by $s - gn'(G)$.

Definition3.2.A connected fuzzy graph $G : (V, \sigma, \mu)$ is said to be a Perfect s-geodetic fuzzy graph if every node of G lies in any one of the s-geodetic basis of G.
In other words, G is a Perfect s-geodetic fuzzy graph if its pseudo s-geodetic number $s - gn'(G) = 0$.

Example3.3. Consider the fuzzy graph G given in Fig.1.

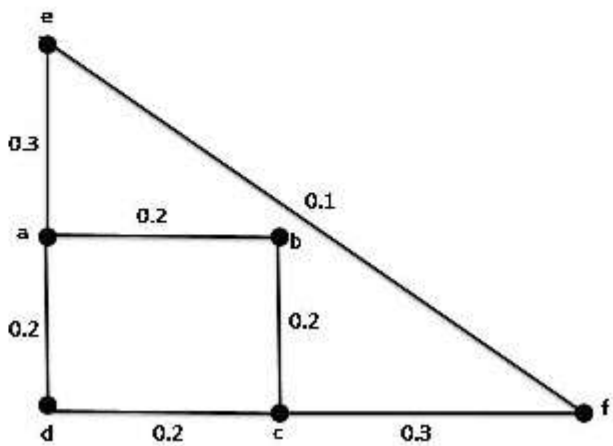

Fig.1:-

Here, the arc (e, f) is the weakest arc of G. $S_1 = \{e, a, c, f\}$and $S_2 = \{e, b, d, f\}$are both s-geodetic basis for G since $(S_1) = (S_2) \; = V(G)$.

Then the pseudo s-geodetic set S' is $V(G) - \{a, b, c, d, e, f\} = \varphi$ and hence $s - gn'(G) = 0$. Therefore G is a Perfect s-geodetic fuzzy graph.

Proposition3.4. If$S_1, S_2, \ldots, S_n$arethes-geodeticbasesofafuzzygraph$G$: (V, σ, μ),thenthepseudos-geodeticnumber$s - gn'(G) = |\bigcap_{i=1}^{n} {S_i}^c|$.

Proof.LetS'bethepseudos-geodeticsetofG.Toshowthat$s - gn'(G) = |\bigcap_{i=1}^{n} {S_i}^c|$,

itisenoughtoshowthat$S' = \bigcap_{i=1}^{n} {S_i}^c$

Let$v$beanodeofGsuchthat$v \in S'$.Thenbydefinition3.1,$v$doesnotbelongtoanys-geodeticbasisofG. $i.e, v \notin S_i \forall i \; = \; 1,2, \ldots, n$.

$\Rightarrow v \in {S_i}^c \; \forall i = 1,2, \ldots, n$.

$\Rightarrow v \in \bigcap_{i=1}^{n} {S_i}^c$.

$\Rightarrow S' \subseteq \bigcap_{i=1}^{n} {S_i}^c$ …………………….. (1)

Conversely,let$u$beanodeofGsuchthat$u \in \bigcap_{i=1}^{n} {S_i}^c$.

Then$u \in {S_i}^c \forall i = 1,2, \ldots, n$.

$\Rightarrow u \notin S_i \forall i = 1,2, \ldots, n$.

Hencebydefinition3.1,$u \in S'$ andso$\bigcap_{i=1}^{n} {S_i}^c \subseteq S'$.

From(1)and(2),$S' \; = \bigcap_{i=1}^{n} {S_i}^c$.

Proposition 3.5.A complete fuzzy graph on 2 nodes is a perfect s-geodetic fuzzy graph.

Proof.ByCorollary2.1,thes-geodeticnumberofacompletefuzzygraphG on2 nodes is$s - gn(G) \; = 2$.Therefore$S = V(G)$ is the unique s-geodetic basisofGandsobyProposition3.4,thepseudos-geodeticset$S' = S^c = \varphi$. Thus we get $s - gn'(G) \; = \; 0$.Hence by Definition 3.2, G is a perfect s-geodetic fuzzy graph.

Proposition 3.6.A fuzzy cycle G on n nodes, each of whose arcs are having same strength, is a perfect s-geodetic fuzzy graph.

Proof.ConsiderthefollowingCases:

Case(1):$n$iseven.
ByRemark2.2,thes-geodeticnumber$s-gn(G)=2$if$n$iseven.
Clearly$S_i=\left\{v_i, v_{\left(i+\frac{n}{2}\right)mod\ n}\right\},\ (1\leq i\leq n)$, aretheonlys-geodeticbasesofG.Then byProposition3.4,thepseudos-geodeticset$S'=\varphi$andso$s-gn'(G)=0$. HenceGisa perfect s-geodeticfuzzygraph.

Case(2):$n$isodd.
ByRemark2.2,thes-geodeticnumber$s-gn(G)=3$if$n$isodd.
Clearly$S_i=\left\{v_i, v_{\left(i+\frac{(n-1)}{2}\right)\ mod\ n}, v_{\left(i+\frac{(n+1)}{2}\right)mod\ n}\right\}, (1\leq i\leq n)$aretheonlys-geodeticbasesofG.ThenbyProposition3.4,thepseudos-geodeticset$S'=\ \varphi$andso$s-gn'(G)=0$.HenceGisaperfect s-geodeticfuzzygraph.

Conclusion:-

Thepseudos-geodeticsetofafuzzygraphGisdefinedasthesetofnodes of G which do not belong to any s-geodetic basis of G and its cardinality iscalled the pseudo s-geodetic number of G.In this paper, perfect s-geodetic fuzzy graphs are defined to be those fuzzy graphs whose pseudo s-geodetic numberis0.ThatisifallnodesofthefuzzygraphGliesinatleastone of the s-geodetic bases of G.A complete fuzzy graph on 2 nodes and fuzzy cycles,eachofwhosearcsarehavingsamestrength,arefoundtobeperfect s-geodetic fuzzy graphs.

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