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RESEARCH ARTICLE

DIAGNOSTIC PROBLEM SOLVING IN GEOMETRY AS A TOOL FOR DIAGNOSING STUDENT DIFFICULTIES.

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Abstract

We propose a successful diagnostic tool that will assist teachers in uncovering their students' difficulties in solving geometry problems by incorporating diagnostic problems into mathematics instruction. Through the diagnostic problems, the teacher can quickly and accurately diagnose the specific knowledge gaps of a student so that the student can fill these gaps and progress successfully in learning the material in the mathematics curriculum. Using this tool, the teacher will be able to identify the material that needs to be reviewed, clarified, or reinforced, as well as the material that has been well-learned and mastered by the students.

Introduction:-

To succeed in any field, one needs personal experience in choosing the right tools. For example, an experienced engineer, who possesses the necessary understanding and tools, can disassemble a faulty device, understand the malfunction during the disassembly process, identify the causes of the malfunction, propose repair options, and even fix the device. Similarly, an experienced doctor can diagnose a patient by gathering and cross-referencing data from various tests, such as laboratory tests, X-rays, and so on, related to the areas where a health problem has arisen. In contrast, a teacher, whose profession is teaching – a profession commonly referred to as the "profession of the future"– lacks a “pedagogical ultrasound” tangible tool, or tomography reflecting the understanding of their students. In this article, we propose a successful diagnostic tool that will assist teachers in uncovering their students' difficulties in solving geometry problems by incorporating diagnostic problems into mathematics instruction. Through the diagnostic problems, the teacher can quickly and accurately diagnose the specific knowledge gaps of a student so that the student can fill these gaps and progress successfully in learning the material in the mathematics curriculum. Using this tool, the teacher will be able to identify the material that needs to be reviewed, clarified, or reinforced, as well as the material that has been well-learned and mastered by the students. Additionally, based on this information, the teacher can build an effective lesson on almost any topic in the curriculum, both through self-directed learning and in a way that engages students in active learning.

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The basic pedagogical idea is as follows

1. The teacher does not provide complete solutions to the problems. He helps the students to solve them.
2. The problems can be solved using the material that the students have learned, but the solutions may not be visible without the help of the teacher.
3. For each such task, a precise path to a successful breakthrough idea is constructed, mainly presented in the form of friendly recommendations.

Typically, when a student is unable to solve a particular problem, the teacher provides a complete solution. In this educational process, the student remains passive. The process proposed in this article makes the student active. When the teacher does not provide a complete solution but rather stages of the solution, the student will be able to reach the solution independently. During this process, both the diagnostic of the student's knowledge level and assistance to help the student reach the solution are simultaneously enabled.

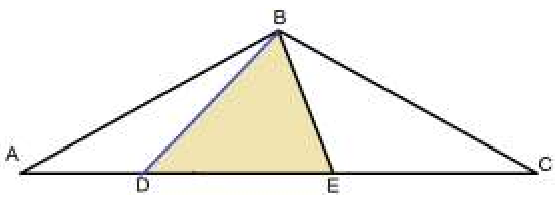
This article proposes three diagnostic problems in plane geometry. For each problem we suggest hints in form of stages of the solution. For the first problem, we also describe our personal experience in solving the problem, and a guide for the teachers of how to diagnose the students' difficulties in moving between the stages.

Problem 1

Given a triangle ABC, with points D and E located on side AC.

$\sphericalangle DBE = 60°, CE = 2, AD = 1, \sphericalangle A = \sphericalangle C = 30°$

What is the length of the segment DE?

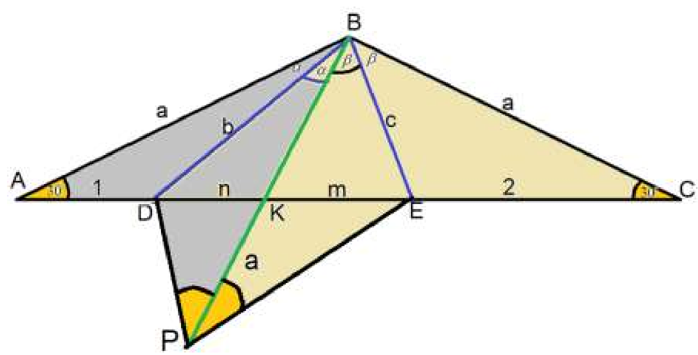


Suggested stages

1. Use an auxiliary construction: $BP = BA = a, \sphericalangle ABD = \sphericalangle DBK = \alpha$
2. Prove that: $\sphericalangle BAD = \sphericalangle DPB = 30°$

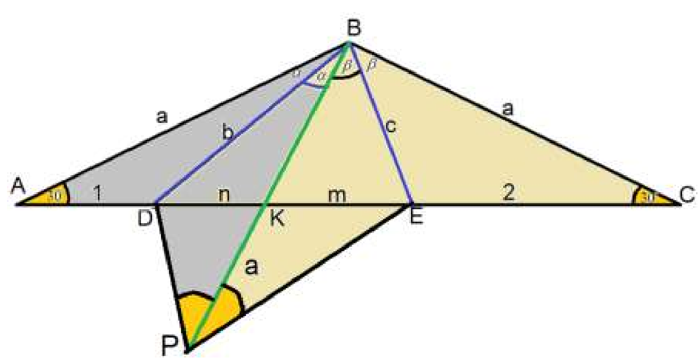
$$AD = DP = 1$$

3. Prove that: $\sphericalangle BPE = \sphericalangle BCE = 30°$
4. Prove that: $\sphericalangle BPE = \sphericalangle BCE = 30°$
5. Prove that: $EP = EC = 2$
6. Prove that $\sphericalangle DPE = 60°$ and $\sphericalangle PDE = 90°$
7. Write the solution.



Personal experience in solving the problem

1. From the given data, it follows that triangle ABC is isosceles, as it has two equal angles. Initially, I thought of finding the length of AC to perform segment subtraction, but it was challenging to find.



2. I looked at the first hint, which involves constructing the auxiliary line BA=BP, BD bisects angle ABP, and I started thinking about how to prove the second hint.

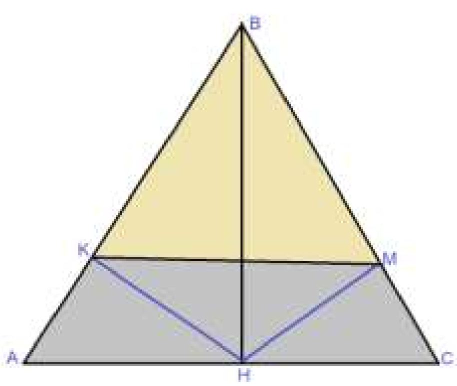
3. To prove the second hint, it is necessary to prove that $\Delta PBD \cong \Delta ABD$. Therefore, it follows that $\sphericalangle A = \sphericalangle P = 30°$ according to the given information and the corresponding angles in congruent triangles. Also, according to the given data and the corresponding sides in the congruent triangles, it follows that DP=1, which is required by the third hint.
4. At this stage, I tried to explain why BE bisects angle PBC. This will allow to the use of the fourth hint.
5. $\alpha+\beta=60°$, $\sphericalangle ABC=120°$. It follows that angle PBE is equal to angle CBE.
6. To prove Hint 4, one needs to prove that $\Delta PBE \cong \Delta CBE$. From the congruence, the results for Hint 4 and also Hint 5 follow.
7. According to the proof for Hint 2 and Hint 4, $\sphericalangle DPE=60°$ by summing the angles.
8. I obtained triangle DPE, where the length of one side is 1, and the second side is 2, and the angle between them is 60°. The claim is that the angle opposite the side of length 2 is a right angle. In such a triangle, the angle cannot be greater than 90° or less than 90°; this leads to a contradiction, so the angle must be 90°. At this stage, using the Pythagorean theorem, DE can be determined.

Teachers Guide:

1. The student should record the given data on the diagram.
2. If a student cannot show that$\Delta PBD \cong \Delta ABD$, work with him on congruence of triangles.
3. If the student cannot use the congruence, he will not be able to prove stages 2 and 3. In this case review conclusions from congruence.
4. If a student cannot deduce why angle PBE is equal to angle CBE, he does not understand how to relate the angle ABC, which is 120°, and $\alpha + \beta = 60°$. As a result, he will not know how to prove the congruence of triangles PBE and CBE. In this case, the student is advised to formulate appropriate equations to show $\sphericalangle PBE = \sphericalangle CBE$
5. If a student can prove that angle PBE equals angle CBE but does not deduce hints 4 and 5, he lacks knowledge of triangle congruence. In this case work with the student on congruence of triangles.
6. If a student has proven the congruence of triangles but does not deduce hints 4 and 5, he lack understanding of the characterization of equal sides and angles resulting from the congruence. In this case review conclusion from congruence.
7. If a student cannot relate what was proven in hints 2 and 4, he cannot deduce that angle DPE is 60°. In this case, the student is asked to combine corresponding angles.
8. If a student does not understand why angle PDE is a right angle, he cannot apply the properties of a right triangle with one angle of 30°. They also will not be able to deduce why it is impossible for angle PDE to be greater than or less than 90°. What is the contradiction encountered when the angle is greater than 90°, and what is the contradiction encountered when the angle is less than 90°?
9. If a student finds that the triangle is a right triangle but does not know how to find the length of DE, he do not remember the Pythagorean theorem. In this case, remind the student of the Pythagorean theorem.

Problem 2

In an acute triangle ABC, BH is the altitude to the side AC, BH = 4, HK is perpendicular to side AB, HM is perpendicular to side BC. The radius of the circumcircle of the triangle ABC is equal to 5. Find the ratio between the area of triangle ABC and the area of the quadrilateral APBD.



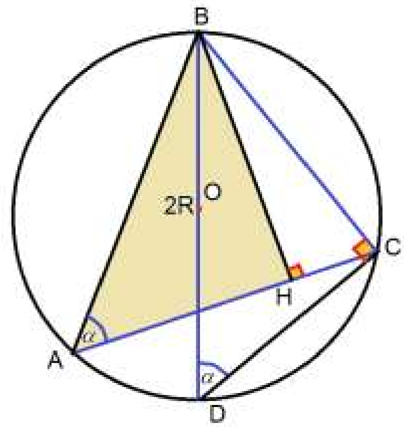
Suggested stages

1. Verify that the quadrilateral AKHM is a cyclic quadrilateral.
2. Check that $\sphericalangle AKM = \sphericalangle ACB$ and $\sphericalangle BMK = \sphericalangle BAC$
3. Check that $\Delta BMK \sim \Delta BAC$
4. Check that $\frac{BM}{AB} = \frac{BK}{BC}$
5. Check by right triangles HBA and HBC that: $4^2 = HB^2 = BC \cdot BM = BK \cdot BA$
6. Check that: $(BC \cdot BA) \cdot (BK \cdot BM) = 16^2$
7. Check that: $\frac{AB}{2R} = \frac{BH}{BC}$, namely $AB \cdot BC = 40$
8. Check that: $BK \cdot BM = \frac{16^2}{40}$
9. Check that:

$$\frac{S_{BKM}}{S_{ABC}} = \frac{BK \cdot MM}{AB \cdot AC} = \frac{4}{25}$$

10. Check that:

$$\frac{S_{BKM}}{S_{AKMC}} = \frac{\frac{S_{BKM}}{S_{ABC}}}{1 - \frac{S_{BKM}}{S_{ABC}}} = \frac{4}{21}$$

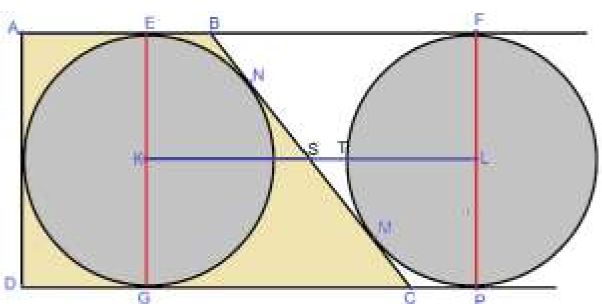


Problem 3

Given right-angled trapezoid ABCD, $AB \parallel CD, AD \perp CD$. Point K is the center of the incircle of the trapezoid, and point L is the center of the second circle that is tangent to the extensions of the bases of the trapezoid, as well as tangent to the longer leg BC of the trapezoid.

a. Prove that KL = BC.

b. Given that BN = 2, CN = 50. Find the length of the segment DL.



Suggested stages

Part A

1. Draw a diagram according to the given data.
2. Explain why the two circles have equal radiiand KS The midsegment of a trapezoidEBCG
3. Explain why $EB = BN$ and $KS = \frac{BC}{2}$

4Similarly, explain why

$$LS = \frac{BF + CP}{2} = \frac{BM + MC}{2} = \frac{BC}{2}$$

5. Check that $KL = BC$

Part B

1. Check whether the angle BKC is necessarily a right angle.
2. Check whether it is necessarily $KN \perp BC$

3. Explain why $R^2 = KN^2 = BN \cdot NC = 100$
4. Check that DP $= 10 + 52 = 62$
5. Use the fact that triangle DLP is a right triangle to find the length of DL.
6. Write the solution.