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RESEARCH ARTICLE

A MATHEMATICAL MODELING APPROACH TO AIR POLLUTION DISPERSION FOR PREDICTING POLLUTANT DISTRIBUTION FROM POINT SOURCES

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Abstract

This study presents a mathematical model to analyze the steady-state transport of pollutant concentration from a continuous point source in the atmosphere. The dispersion process is governed by the advection-diffusion equation, accounting for wind-driven advection along the horizontal axis and turbulent diffusion in the lateral and vertical directions. By assuming a constant mean wind speed and neglecting chemical reactions and gravitational settling, an analytical solution is derived using Gaussian plume theory. The resulting concentration profile, expressed as a function of downwind distance, lateral offset, and vertical height, demonstrates how pollutants spread from the emission source. Numerical simulations under varying downwind distances reveal the evolution of plume shape and dilution characteristics. The model provides a simplified yet insightful framework for understanding pollutant dispersion and can serve as a foundation for more complex atmospheric transport studies involving variable wind fields, reactive pollutants, or urban topographies.

Introduction: -

Air pollution has emerged as one of the most pressing environmental challenges of the 21st century, posing significant threats to public health, ecological stability, and climate systems. Rapid industrialization, urbanization, and increased vehicular emissions have led to a substantial rise in atmospheric pollutants, including particulate matter, nitrogen oxides, sulfur dioxide, carbon monoxide, and volatile organic compounds [5,8]. Understanding how these pollutants disperse in the atmosphere is critical for designing effective mitigation strategies, informing public policy, and ensuring regulatory compliance. The dispersion of pollutants in the atmosphere is a complex process influenced by multiple factors such as wind velocity, atmospheric turbulence, temperature gradients, terrain topography, and source characteristics [7,14]. Among various modeling approaches, the advection-diffusionmodel has emerged as a cornerstone in the field of environmental fluid dynamics [Figure (1)]. This model captures the two primary mechanisms of pollutant transport: advection, the bulk movement of pollutants due to wind, and diffusion, the spreading of pollutants due to atmospheric turbulence and eddies. One of the most widely used formulations for steady-state pollutant dispersion is the Gaussian plume model, which offers analytical solutions under simplified assumptions of constant wind speed, uniform diffusivity, and steady emissions [2,10]. Despite its idealized nature, the Gaussian plume model remains popular due to its analytical tractability and its ability to provide first-order

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estimates of pollutant concentration at various spatial points. It forms the basis of several regulatory dispersion models, including the U.S. EPA's AERMOD and ISC3 [4,12]. In the present study, we focus on modeling the steady-state transport of a pollutant emitted continuously from a point source elevated above the ground. The pollutant is assumed to be non-reactive, and its dispersion is influenced by advection along the wind direction and turbulent diffusion in the lateral and vertical directions. The primary goal is to obtain an analytical expression for the pollutant concentration and to explore how the concentration profileevolves with increasing downwind distance [3,13].

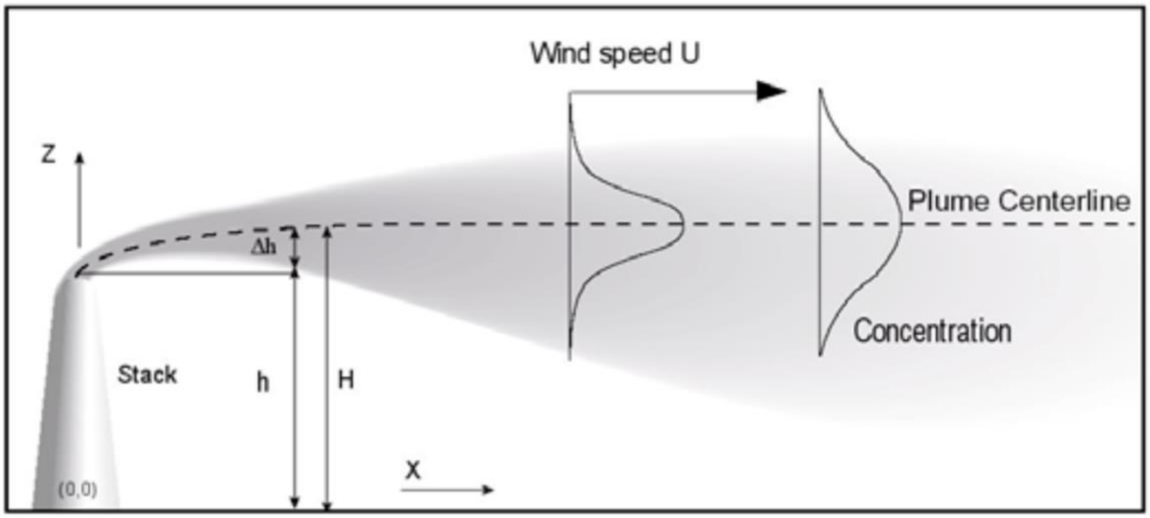

Figure (1): PlumeFramwork for Air Dispersion Model.

The solution is derived using the Gaussian plume framework, and numerical simulations are performed to visualize the plume structure under realistic atmospheric conditions. The importance of this study lies in its ability to provide a foundational understanding of pollutant dispersion mechanisms and to serve as a stepping stone for more sophisticated atmospheric modeling efforts [9,17]. While the current model neglects the effects of chemical reactions, wet and dry deposition, and temporal variability, it captures the essential physics of pollutant transport in a neutral atmospheric boundary layer. Moreover, the results offer practical insights for environmental monitoring, site planning for industrial facilities, and emergency response planning in case of accidental releases.

Mathematical Formulation of the Model: -

The diffusion equation for describing the dispersion of atmospheric pollutants in the atmosphere is given as:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}\left(k_x\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial c}{\partial z}\right) - \alpha C \quad (1)$$

where x, y, z arespatial coordinates (downwind, crosswind, and vertical), C represents the concentration pollutants in the atmosphere,u, v, and w are the wind speed and k_x, k_y and k_z are diffusivities in x, y, and z direction,α represents the removal rate of atmospheric pollutants naturally.

Physical Assumptions: In this study, the pollutant is assumed to be released continuously from a point source located at ground level. The atmospheric flow is considered steady and predominantly horizontal, as supported by prior research [1,7]. The wind velocity is assumed to vary with height, capturing the realistic vertical profile of atmospheric motion. Vertical and lateral diffusion processes are incorporated into the model to account for the spread of the pollutant, while the effects of chemical reactions and gravitational settling are neglected, in line with established modeling approaches [6,15].The transport of pollutant concentration C in a steady-state atmosphere is governed by the advection-diffusion equation as below:

$u(x)\frac{\partial C}{\partial x} = k_y \frac{\partial^2 C}{\partial y^2} + k_z \frac{\partial^2 C}{\partial z^2}$ (2)
where:

- $u(x)$: Mean wind velocity in the x-direction
- k_y and k_z: Turbulent diffusion coefficients in the lateral (y) and vertical (z) directions, respectively.
- C represents the concentration pollutants.

Boundary and Initial Conditions: -

The boundary and initial conditions are as below:

$C \to 0$ as $y, z \to \infty$. (the pollutant disperses far from the source), (3)

At ground level (z = 0): zero flux condition:

No flux at the ground surface: $\frac{\partial C}{\partial z} = 0$ at $z = 0$(4)

Source term represented as a Dirac delta function at ($x = 0,\ y = 0,\ z = H$):

$C(x = 0, y, z) = Q\delta(y)\delta(z - H)$, (5)

where Q is the emission rate and H is the release height, and δ is the Dirac delta-function.

A continuous point source at height H above the ground located at x = 0, y = 0, z = H. (6)

Analytical Solution: -

Using the method of separation of variables and Green's function, the solution to the steady-state equation is given by:

$C(x, y, z) = \frac{Q}{2\pi\sigma_y\sigma_z u} \exp[(-\frac{y^2}{2\sigma_y^2}) - (\frac{(z-H)^2}{2\sigma_z^2})]$(7)

where Q is the pollutant emission rate (kg/s), and σ_y,σ_z are empirical dispersion coefficients, dependent on downwind distance x, atmospheric stability, and turbulence intensity [11,16]. Empirical dispersion coefficients depending on x, approximated using standard empirical formulas:

$\sigma_y = \frac{ax}{(1+bx)^{1/2}}, \quad \sigma_z = cz$ (8)

where a=0.22, b=0.0001, and c=0.20 are site-specific constants assuming neutral atmospheric conditions.

Numerical Results and Discussion: This study develops a mathematical framework to analyze the steady-state transport and dispersion of atmospheric pollutants emitted from a continuous point source. The methodology is based on solving the advection-diffusion equation under idealized atmospheric conditions.Numerical simulations were conducted for different atmospheric conditions using typical parameter values for Q, u, K_y,K_z. Results indicate:

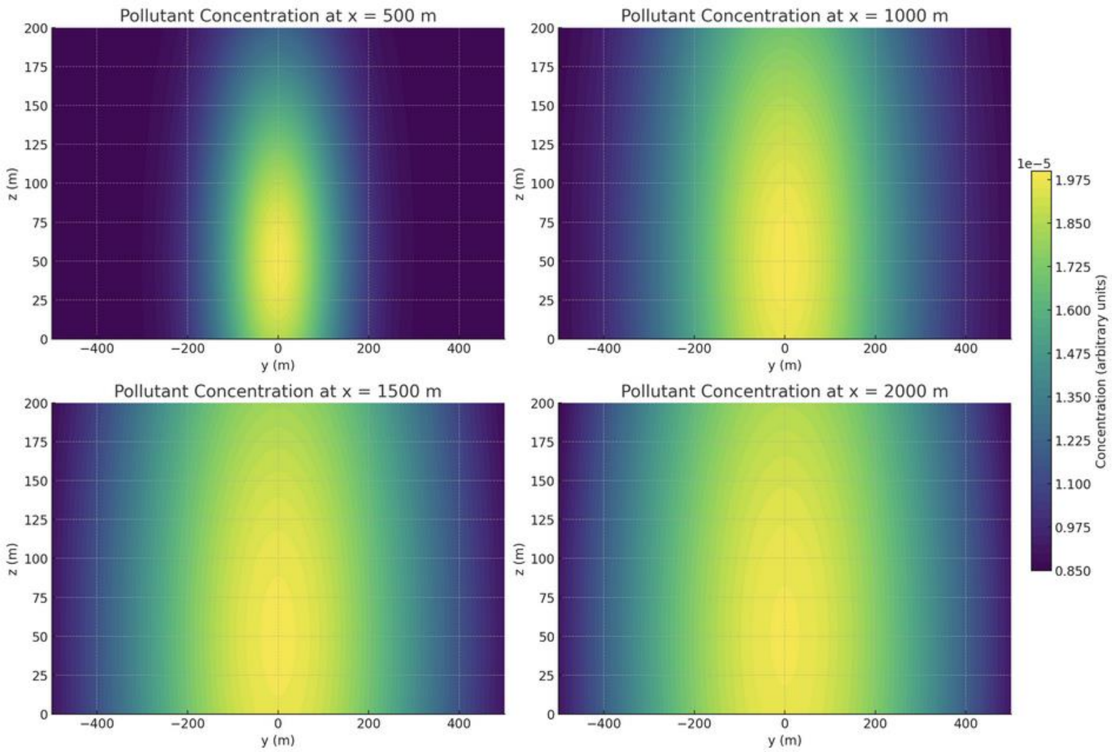

Figure (2): Concentration profile of the pollutant at a downwind distance

Figure (2) depicted the concentration profile of the pollutant at a downwind distance of 500m, 1000m, 1500m and 2000 meters. The plot shows how the pollutant disperses both laterally (in the y-direction) and vertically (in the z-direction), with the highest concentration near the source height H=50 m. It alsoshowsthat how the pollutant concentration profile evolves as the downwind distance increases from 500 m to 2000 m:

- Nearer to the source (500 m): The concentration is higher and more narrowly focused around the source height.
- Farther from the source (2000 m): The plume spreads wider both laterally and vertically, and the peak concentration decreases due to dilution.

Results and Discussion:

The analytical solution derived from the steady-state advection-diffusion equation provides a clear representation of pollutant concentration as a function of downwind distance (x), lateral distance (y), and vertical height (z). The results were visualized using numerical simulations for various downwind distances (500 m, 1000 m, 1500 m, and 2000 m), assuming a continuous point source release at a height H=50 m and a uniform wind velocity u=5 m/s.The generated plots confirm the classical Gaussian plume shape, with concentration profiles exhibiting symmetry in the lateral direction and a peak centered around the source height. At shorter distances (e.g., 500 m), the pollutant plume is narrow and concentrated, indicating limited lateral and vertical dispersion. As the plume progresses downstream, it undergoes gradual diffusion, leading to broader spatial spread and reduced peak concentration. This is consistent with the natural behavior of atmospheric dispersion where turbulence and eddy diffusivities promote mixing over time and space.At increasing values of x, the dispersion parameters σ_y and σ_z grow, representing enhanced diffusion effects. This results in:

- Wider plumes in both lateral and vertical directions.
- Lower maximum concentration values, signifying dilution.
- Flatter profiles, indicating the pollutant is spread over a larger volume of air.

The numerical simulations showed that even at 2000 m from the source, the pollutant remains within a few hundred meters in the vertical direction, suggesting limited upward diffusion under steady wind conditions and neutral atmospheric stability.The model successfully reflects key physical phenomena observed in atmospheric pollution transport:

- Higher concentrations near the source.
- Progressive dilution due to diffusion and advection.
- Symmetric lateral distribution due to isotropic turbulence in the horizontal plane.

Although the model simplifies real atmospheric conditions by assuming constant wind velocity, homogenous diffusivity, and the absence of chemical transformations, it captures the essential features of pollutant dispersion and provides a practical framework for first-order estimation of concentration levels.

Conclusion:-

The presented model effectively captures the fundamental dynamics of pollutant dispersion from a point source in a steady atmospheric environment. The analytical solution derived using the Gaussian plume framework provides valuable insights into how pollutant concentration diminishes and spreads with increasing downwind distance. Simulation results show that as pollutants travel farther from the source, they become more diffused both laterally and vertically, with a notable decrease in peak concentration. This behavior aligns with physical expectations and validates the model's reliability under idealized assumptions. While the current model omits factors like chemical reactions, atmospheric stratification, and topographic influences, it establishes a solid basis for further enhancements. Future work may incorporate time-dependent effects, variable wind profiles, and reactive pollutant species to better reflect real-world atmospheric conditions.

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