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RESEARCH ARTICLE

APPLYING BAYESIAN WEIGHTED LINEAR DISCRIMINANT ANALYSIS FOR THE CLASSIFICATION OF COMMERCIAL AND PERSONAL LOANS IN THE LIBERIA BANKING SECTOR

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Abstract

This paper presents the development and application of a Bayesian weighted linear discriminant analysis (BwLDA) model aimed at classifying commercial and personal loans in Liberia's banking sector. Initially, a weighted linear discriminant analysis (wLDA) model was formulated to enhance traditional linear discriminant analysis (LDA) by introducing class weighting to mitigate imbalance and improve classification accuracy. However, wLDA revealed notable misclassification and inconsistencies with actual bank records. To address these limitations, Bayesian principles were integrated, resulting in the BwLDA model. By incorporating prior information and employing Markov Chain Monte Carlo sampling, BwLDA produced more robust posterior estimates and improved classification performance. The model demonstrated greater consistency between predicted default probabilities and actual bank outcomes, especially in high-risk institutions such as Access Bank Liberia Limited and Eco Bank Liberia Limited. Despite minor overand underestimations, BwLDA exhibited strong adaptability and reliability across various performance metrics. The findings suggest that BwLDA offers a more precise, flexible, and data-informed approach to credit risk classification and is recommended for adoption to support risk management and regulatory decision-making within Liberia's financial sector.

Introduction: -

The fundamental task in banking is credit risk classification, particularly in emerging economies where banking institutions face limited data, class imbalance, and volatile market conditions. Linear discriminant analysis (LDA), referred to as a traditional classification model, has been widely used in credit risk modeling because of its simplicity and interpretability (Alvin, 2002, and Raubenheiner 2004). However, LDA often assumes homoscedasticity and equal prior probabilities, which may not be held in real-world banking datasets. These limitations can be addressed by integrating the weights of each classification into the LDA, to be considered as weighted linear discriminant analysis (wLDA) in order to accommodate class imbalance and improve classification performance (Zhou and Liu, 2010).

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However, wLDA still lacks the ability to incorporate prior knowledge and quantify uncertainty, which are critical in environments with limited historical data or evolving credit risk patterns. In recent times, studies have advocated for Bayesian approaches in financial modeling, emphasizing the capacity of prior knowledge to be integrated to provide robust posterior estimates through probability frameworks (Geweke 2005 and Rossi et al., 2005). This paper builds on this perspective by extending wLDA into a Bayesian framework, resulting in the Bayesian weighted linear discriminant analysis (BwLDA). The proposed model is applied to classify commercial and personal loans in Liberia's banking sector, aiming to enhance predictive accuracy, reduce misclassification and support better regulatory and credit risk decisions.

Over the past decades, credit risk modeling has evolved significantly with early methods grounded in statistical models such as LDA and logistic regression. LDA, introduced by Fisher (1936), has been extensively used for binary classification tasks, including credit scoring. Additionally, the integration of structural models like Merton's (1974) framework into classification models introduces an asset-based perspective that enhances default prediction. Hybrid models that combine statistical and structural elements are gaining traction for their ability to reflect firm-specific and systemic risk more comprehensively (Duffie and Singleton, 2003).

This paper is structured as follows. Section 2 presents the theoretical framework which served as the basis for the analysis. Section 3 provides a brief methodology used in the paper. Section 4 provides the results and discussion while Section 4gives the summary and conclusions. Finally, Section 5 provides recommendations for the next steps to undertake.

Theoretical Framework

Bayesian Weighted Least Discriminant Analysis

Let us define the set of loan observations for each bank "h" and loan type "k" as $Y_{ihk} = \{x_{ihk} | i = 1, \cdots, n_{hk}\}$, where $n_{hk} \in \mathbb{N}^+$ denotes the number of observations. The two loan types considered are commercial $(k = 1)$ and personal $(k = 2)$, each forming a distinct class with its own distribution of financial indicators.

To address class imbalance and emphasize discriminative features, the study computes the weighted means for each loan type within a bank:

$$\hat{\mu}_{w,hk} = \frac{\sum_{i=1}^{n_{h1}} w_{ihk}\, x_{ihk}}{\sum_{i=1}^{n_{h1}} w_{ihk}}, \qquad k \in \{1, 2\}.$$

The corresponding weighted variances are:

$$\hat{S}^2_{whk} = \frac{\sum_{i=1}^{n_{h1}} w_{ihk} \left(x_{ih1} - \hat{\mu}_{w,hk}\right)^2}{\sum_{i=1} w_{ihk}}$$

The difference between the two classes is quantified using the weighted Fisher ratio (wFR)

$$\beta_w(\theta_j) = \frac{\left(\hat{\mu}_{w,h1} - \hat{\mu}_{w,h2}\right)^2}{\hat{S}^2_{wh1} + \hat{S}^2_{wh2}}.$$

Projecting the data onto the direction "θ_j" that maximizes β_w, the projective means represented by

$$\hat{\mu}^{proj}_{w,hk} = \theta_j^T \hat{\mu}_{w,hk}.$$

Maximizing β_w leads to the optimal discriminant vector:

$$\hat{\theta}_j = \left(\hat{\mu}_{w,h1} - \hat{\mu}_{w,h2}\right)\hat{S}^{-1}_{w,hk,W},$$

where $\hat{S}^{-1}_{w,hk,W}$ is the pooled within-class weighted covariance matrix. This formulation ensures optimal linear separation between low loan risk and high loan risk classes across loan types.

The proposed Bayesian weighted linear discriminant function for borrower "i", bank "h", and loan type "k" is defined as:

$$g_{ihk}(x_j) = \sum_{j=1}^{n_{hk}} \hat{\theta}_j x_j + \log(\pi_{ihk}),$$

where x_j represent the j-th financial feature, $\hat{\theta}_j$ is the corresponding weight or coefficient derived from the wLDA model, and π_{ihk} denotes the inclusion marginal probability for the borrower. A higher value of $\hat{\theta}_j$ implies a stronger influence of the feature on risk classification, while values near zero indicate minimal impact. The commercial loan feature vector is defined as

$$x = \left[\widehat{LTVR}_{ihk}, \widehat{LR}_{ihk}, \widehat{DSCR}_{ihk}, DD_{ihk}, G(\widehat{A^t_{ihk}}), r_{ihk}, t_{ihk}\right],$$

while the personal loan feature vector is defined as

$$x = \left[\widehat{DTIR}_{ihk}\widehat{LTIR}_{ihk}, DD_{ihk}, G(\widehat{A^t_{ihk}}), r_{ihk}, t_{ihk}, ph_{ihk}\right].$$

Integrating the Bayesian statistics and consistent with the approach of Mohamed and Saad (2019), the posterior distribution of the parameter vector $\widehat{\theta}_j$ given the observed data Y_{ihk} is defined as:

$$P(\theta_j/Y_{ihk}) = \frac{P_r(Y_{ihk}/\theta_j)\, P_r(\theta_j)}{P_r(Y_{ihk})},$$

where $P(\theta_j/Y_{ihk})$ is the posterior distribution, $P(Y_{ihk}|\theta_j)$ is the likelihood, $P(\theta_j)$ is prior, and $P(Y_{ihk})$ is the marginal likelihood or normalizing constant. The likelihood for each observation under the logistic assumption is

$$P\left(Y_{ihk} = 1\middle|\theta_j\right) = \frac{1}{1 + e^{-g_{ihk}(x_j)}},$$

where $g_{ihk}(x_j)$ is the discriminant score derived from the weighted linear discriminant function and $Y_{ihk} =$ 1indicate high risk. Assuming independence across borrowers, the joint likelihood becomes

$$L\left(\theta_j\middle|Y_{ihk}\right) = \Pi_{i=1}^{n_{h1}} P_r\left(Y_{ihk}\middle|\theta_j\right)^{Y_{ihk}} \cdot (1 - P_r\left(Y_{ihk}\middle|\theta_j\right)^{1-Y_{ihk}}$$

The Bayesian estimator for each parameter θ_j under squared error loss is the posterior mean given as:

$$\widehat{\theta}_j = E\left[\theta_j\middle|Y_{ihk}\right] = \int \theta_j P_r\left(\theta_j\middle|Y_{ihk}\right) d\theta_j,$$

and the posterior variance is given by

$$\sigma_j^2 = E\left[\sigma_j^2\middle|Y_{ihk}\right] = \int \sigma_j^2 P_r\left(\sigma_j^2\middle|Y_{ihk}\right) d\sigma_j^2.$$

Owing to the intractability of these integrals, posterior distributions are approximated using the Markov Chain Monte Carlo (MCMC) method, as implemented in frameworks such as PyMC3 or Stan. These techniques iteratively sample from the posterior, yielding estimates for $\widehat{\theta}_j$ and σ_j^2. However, as noted by Hoeting *et al*., (1999), MCMC methods may pose challenges in terms of convergence diagnostics and interpretation, especially in high-dimensional parameter spaces. To address this, this study adopted the convergence assessment approach proposed by Gelman and Rubin (1992). Several standard diagnostic tools were employed: trace plots were examined to evaluate mixing and stationarity of the chains; the Gelman–Rubin statistic ($\hat{R}$) was computed to ensure that all chains converged toward a common distribution, with acceptable values close to 1.0; and effective sample size (ESS) was examined to confirm adequate sampling of the posterior. These diagnostics collectively ensured the reliability and stability of the estimated parameters used in the Bayesian weighted linear discriminant analysis (BwLDA) model.

In this study, the logistic assumption underlying the likelihood function is rooted in its well-established application to binary classification problems where the outcome is dichotomous, such as high-risk and low-risk. In the credit rating model, the assumption is well established, particularly in logistic regression and discriminant analysis frameworks, where the probability of default (PD) is modeled as a logistic transformation of a linear combination of predictors (Hosmer and Lemeshow 2000, Hand and Henley 1997). In this context, the discriminant score produced by the BwLDA model serves as the input to the logistic function, yielding a probability estimate bounded between 0 and 1. The following logistic function is applied as:

$$\widehat{PD}_{ihk} = \frac{1}{1 + e^{-g_{ihk}(x_j)}}$$

This approach is also consistent with previous work by Maria and Erick (2007) and Valentyn (2018), who applied logistic regression in estimating credit default probabilities.

The classification threshold "$\hat{\Delta}_{ihk}$" is established such that i) $g_{ihk}(x_j) \geq \hat{\Delta}_{ihk}$, the loan is classified as low risk and ii) if $g_{ihk}(x_j) < \hat{\Delta}_{ihk}$, the loan is classified as high risk. This study adopted a data-driven approach by computing the mean discriminant scores as the cutoff point for classifying borrowers into low-risk and high-risk groups. Particularly, the threshold is defined as:

$$\hat{\Delta}_{ihk} = \frac{1}{n_{h1}} \sum_{i=1}^{n_{hk}} g_{ihk}(x_j),$$

and to classify the default probabilities, the threshold is defined as $\hat{\Delta}_{ihk}^{PD} = \frac{1}{1+e^{-\hat{\Delta}_{ihk}}}$.

Integration of the Merton Model into Discriminant Analysis

The distance-to-default (DD) from the Merton model is integrated into a Bayesian weighted discriminant function as a forward-looking, market-based indicator to enhance the capacity of credit risk models (Crosbie and Bohn 2003). This approach treats firm or borrower assets as stochastic processes and evaluates the risk of default based on asset dynamics related to debt obligations.

Assume that the asset value A_{ihk}^{t} follows a geometric Brownian motion governed by the stochastic differential equation:

$$dA_{ihk}^{t} = H_{ihk}^{t}dt + D_{ihk}^{t}d\mathcal{B}_t,$$

with solution

$$A_{ihk}^{t} = A_{ihk}^{0}exp\left(\left(r_{ihk} - q_{ihk} - \frac{1}{2}\sigma_{ihk}^{2}\right)t + \sigma_{ihk}\mathcal{B}_t\right).$$

.

This expression models the evolution of borrower assets over time, incorporating the drift $(r_{ihk} - q_{ihk})$ and volatility "σ_{ihk}" . From this, the distance-to-default is the number of standard deviations by which current asset exceed liabilities, is computed as

$$DD_{ihk} = \frac{ln\left(\frac{A_{ihk}^{0}}{F_{ihk}}\right) + \left(r_{ihk} - q_{ihk} + \frac{\sigma_{ihk}^{2}}{2}\right)T}{\sigma_{ihk}\sqrt{T}},$$

where F_{ihk} is the face value of liabilities. The expected firm or individual value at maturity, conditional on default, is

$$G(A_{ihk}^{t}) = exp\left(\mu_{ihk} + \frac{\sigma_{ihk}^{2}}{2}\right)\Phi\left(\frac{ln\left(\frac{A_{ihk}^{0}}{F_{ihk}}\right) + \left(\mu_{ihk} + \frac{\sigma_{ihk}^{2}}{2}\right)T}{\sigma_{ihk}\sqrt{T}}\right).$$

By incorporating $G(A_{ihk}^{t})$ and DD_{ihk} into the Bayesian weighted discriminant function, the BwLDA model integrates market-based asset volatility and debt structure, improving the classification of default risk. This hybrid approach strengthens credit risk modeling by combining structural financial theory and statistical classification, offering a more robust decision-support tool for banking institutions.

Methodology: -

Sampling Design

This study employed a stratified random sampling design targeting banks in Liberia that maintain both commercial and personal loan portfolios. Only banks with 600 or more loan records were considered, forming the sampling domains. The strata were defined by the cross-classification of qualifying banks and two loan types, resulting in ten strata.

Sampling within each stratum followed a probability proportional to size (PPS) approach, using loan amounts as the size measure. Larger loans had a higher probability of inclusion. Rather than sampling individual borrowers directly, loan records were sampled within each bank's domain. Participating banks were asked to anonymize borrower data, with guidance from the researcher where necessary. This approach aligns with Luis and Terrance (2021), who advocate stratified designs for efficient representation in complex populations.

Sampling Weights and Marginal Probability

The calculation of the inclusion probability for each loan is

$$\pi_{ihk} = \frac{n_{hk}}{N_{hk}},$$

where N_{hk} is the total sample size and $n_{hk}$is the sample size within each stratum. Corresponding sampling weights were calculated as

$$w_{ihk} \propto \frac{1}{\pi_{ihk}},$$

ensuring appropriate representation in the wLDA. In the Bayesian extension (BwLDA), the inclusion probabilities were incorporated in logarithmic form into the discriminant function, enhancing both computational stability and model interpretability. This adjustment filters out low-relevance variables and strengthens the separations of risk classes under high-dimensional imbalanced data scenarios.

Sample Allocation

Using Yamane (1967) formula with a 3% margin of error, the sample sizes for each bank were determined by

$$n_h = \frac{N_{hk}}{1 + N_{hk}e^2}.$$

The data came from five banking institutions in Liberia such as Ecobank Liberia (EBLL), Access Bank Liberia (ABLL), International Bank Liberia (IBLL), Guaranty Trust Bank Liberia (GTBLL), and United Bank for Africa Liberia (UBALL), and covers the period from January 2022 to December 2023.

Variable Selection and Feature Construction

Variables were selected based on their theoretical and practical relevance to credit risk assessment, capturing borrower solvency, leverage, liquidity and market-based risks. Variables selected for commercial loans include loan amount (LA_{ihk}), loan tenure (t_{ihh}), risky interest rate (r_{ihk}), loan amount outstanding (LAO_{ihk}), expected value of the firm at maturity ($G(A_{ihk}^t)$, distance-to-default (DD_{ihk}), leverage ratio ($\widehat{LR}_{ihk}) = \frac{F_{ihk}^t}{H_{ihk}^t}$, where $H_{ihk}^t = G(\widehat{A_{ihk}^t}) - F_{ihk}^t$, Loan-to-value ratio($\widehat{LTVR}_{ihk}) = \frac{LAO_{ihk}}{F_{ihk}^t}$, debt service coverage ($\widehat{DSCR}_{ihk}) = \frac{NOI_{ihk}}{F_{ihk}^t}$. where NOI is estimated as a percentage of principal based on industry benchmarks (10% for retail, 25% real estate, 15% manufacturing (Jones and Mingo1998).

The variables selected for personal loans includeloan amount (LA_{ihk}), loan tenure (t_{ihk}), payment history, interest rate (r_{ihk}), loan amount outstanding (LAO_{ihk}), and gross monthly income (GMI_{ihk}), estimate via

$$\widehat{GMI}_{ihk} = \beta_0 + \beta_1(LA_{ihk}) + \beta_2(ES_{ihk}) + \beta_3(LOE_{ihk}) + \beta_4(ES_{ihk} \times LOE_{ihk}) + \epsilon,$$

where ES_{ihk} = education status ($1 = informal\ and\ 0 = formal$),$LOE_{ihk}$ = length of employment ($0 > 5$ years, $1 < 5$) , debt-to-income ratio ($\widehat{DTIR}_{ihk}) = \frac{M_{ihk}}{\widehat{GMI}_{ihk}}$, where M_{ihk} is the monthly debt payment and is computed as

$$M_{ihk} = \frac{LA_{ihk} \times r_{ihk} \times (1 + r_{ihk})^{t_{ihk}}}{(1 + r_{ihk})^{t_{ihk}} - 1},$$

loan-to-income ratio ($\widehat{LTIR}_{ihk}$) $= \frac{LA_{ihk}}{\widehat{GMI}_{ihk} \times t_{ihk}}$, and payment history (ph_{ihk})(0 = strong payment, $1 = poor$). These variables were inputted to the Bayesian weighted discriminant function for both loan types, enabling the classification of high and low risk borrowers in Liberia's banking sector. The selection is consistent with international guidelines from BASEL II (BCBS) (2006).

Results and Discussion: -

Estimated BwLDA Model Parameters using Commercial Loans

Table 1 presents the estimated posterior coefficients θ_j, representing the adjustment parameters of the discriminant function and corresponding posterior variance σ_j^2 with key financial indicators across five banks under the BwLDA model. The results show notable variance in parameter estimates by banks, reflecting how each commercial loan portfolio in each bank reacts to different risk factors. For example, GTBLL consistently exhibits the highest θ_j values, particularly for indicators like loan tenure ($\theta_j = 1.1138$) and expected asset value ($\theta_j = 0.9763$), coupled with relatively low variances, suggesting strong and stable contributions to loanclassifications. In contrast, IBLL shows moderate θ_j values across all indicators with slightly higher variance in some parameters (for example,t_{ihk}=0.3926), indicating a more balanced but less decisive feature influence. EBLL also demonstrates high θ_j values, especially for DD_{ihk} ($\theta_j = 0.9718$), showcasing its reliance on default risk in classification. Overall, the estimated values for θ_j and σ_j^2 across banks suggest that the BwLDA model adapts flexibly to credit risk patterns, offering tailored discriminant power for each loan portfolio.

Table 1: - Estimated BwLDA model parameters across banks using commercial loans.

Commercial Loans (CL) Indicators	IBLL		GTBLL		ABLL		UBA		EBLL	
	$\theta_j\sigma_j^2$		$\theta_j\sigma_j^2$		$\theta_j\sigma_j^2$		$\theta_j\sigma_j^2$		$\theta_j\sigma_j^2$	
$LTVR_{ihk}$	0.7804	0.3351	0.8259	0.3424	0.7622	0.3464	0.7677	0.3588	0.8328	0.3616
LR_{ihk}	0.8094	0.3343	0.8717	0.3616	0.7996	0.3826	0.7915	0.3353	0.8447	0.3614
$DSCR_{ihk}$	0.7641	0.3319	0.8813	0.3631	0.7803	0.3741	0.7782	0.3454	0.8691	0.3815
DD_{ihk}	0.7929	0.334	0.9539	0.3795	0.7512	0.3794	0.7415	0.3661	0.9718	0.3327
$G(\widehat{A_{ihk}^t})$	0.7644	0.3673	0.9763	0.3362	0.7871	0.3775	0.8271	0.3398	0.9965	0.3496
r_{ihk}	0.7408	0.3654	0.9964	0.3458	0.8135	0.3636	0.7805	0.496	1.0632	0.3408

t_{ihk}	0.8172	0.3926	1.1138	0.3403	0.7861	0.3751	0.8082	0.354	1.0612	0.343

Estimated Classification Counts for Commercial Loans using BwLDA Model

The performance of the BwLDA model in classifying commercial loan risk is assessed by comparing its predicted risk categories with the actual classifications recorded by each bank (IBLL, GTBLL, ABLL, EBLL, and UBA). The confusion matrices of the model demonstrated perfect internal consistency, with each high-risk borrower corresponding to a high predicted probability of default (PD) and each low-risk borrower aligning with the predicted PD. The outputs of the BwLDA model achieved 100% accuracy, precision, and recall across all banks with no misclassifications recorded.

However, when comparing the BwLDA results with the actual bank risk classifications, a slight discrepancy exists. For instance, in IBLL, the bank classified 68 loans as high-risk compared to 90 classified as high-risk by BwLDA, an overestimation of approximately 32.4%. Likewise, the ABLL bank reported 271 high-risk loans, whereas the BwLDA classified 263, an underestimation of 2.95%. The EBLL also showed a minor underestimation of 3.57%, whereas the UBA reflected an overestimation of 4.29%. These discrepancies suggest that while the model exhibits perfect internal predictive performance, it may diverge slightly from how banks define or record risk owing to different thresholds, internal scoring systems, or expert-driven adjustments.

In practice, banks in Liberia tailor their risk assessment frameworks to reflect their portfolio characteristics and institutional risk appetites. For instance, some banks may use internal credit-scoring models and apply predefined thresholds to categorize borrowers into low-risk (performing) or high-risk (non-performing) groups. These thresholds are often dynamic and subject to periodic revisions in response to changing credit policies, market conditions, and regulatory guidance. While some adopt fixed cutoffs (e.g., 0.5 on the PD scale), others rely on adaptive thresholds or expert judgment, particularly when dealing with heterogeneous loan portfolios or during periods of economic uncertainty. In contrast, this study applied a data-driven threshold, calculated as the mean discriminant score, to ensure consistency and interpretability across the banks. Nonetheless, real-world loan classification often reflects a more nuanced, multi-criteria framework that balances statistical analysis with managerial discretion and regulatory compliance.

Table 2: - BwLDA commercial loan classification counts against actual bank records.

Source	Classification	IBLL		GTBLL		ABLL		EBLL		UBA	
		High PD	Low PD	High PD	Low PD	High PD	Low PD	High PD	Low PD	High PD	Low PD
BwLDA Model	High Risk	90	0	28	0	263	0	54	0	73	0
	Low Risk	0	98	0	33	0	261	0	52	0	76
Bank Record	High Risk	68	0	27	0	271	0	56	0	70	0
	Low Risk	0	120	0	34	0	253	0	50	0	79

Estimated BwLDA Model Parameters using Personal Loans

Table 3 shows the estimated posterior coefficient θ_j(adjustment parameter) and the posterior variances σ_j^2 for personal loans financial indicators across the five banks under the BwLDA model. The results indicate consistently high θ_j values across all indicators, demonstrating the strong discriminant influence of variables like DTIR, LTIR, and loan tenure. GTBLL and EBLL show particularly high coefficients (for example, θ_j=1.0364 for expected asset value in GTBLL and $\theta_j = 1.0492$ for Ph_{ihk} in EBLL), suggesting that these features play major roles in classifying personal loan risk within those institutions. In contrast, IBLL and UBA exhibit slightly lower but still substantial weights, paired with modest variances, reflecting stable but more evenly distributed feature importance. Relatively low σ_j^2 values across most banks indicate high confidence in the estimates. Overall, the results affirm that the BwLDA model effectively captures the nuanced contribution of financial indicators in personal loan classification, with flexibility to adjust across different banking profiles.

Table 3: - Estimated BwLDA model parameters across banks using personal loans.

Personal Loans Indicator	IBLL		GTBLL		ABLL		UBA		EBLL	
	θ_j	σ_j^2	θ_j	σ_j^2	θ_j	σ_j^2	θ_j	σ_j^2	θ_j	σ_j^2
$\widehat{DTIR}_{ihk}$	0.7975	0.347	0.8348	0.3685	0.8498	0.3961	0.7535	0.3355	0.8242	0.3259
$\widehat{\mathrm{LTIR}}_{\mathrm{ihk}}$	0.7871	0.3728	0.8622	0.3425	0.7977	0.374	0.7849	0.3633	0.8937	0.3826

Ph_{ihk}	0.7752	0.3766	0.8575	0.3838	0.7815	0.3516	0.7869	0.3967	1.0492	0.3727
DD_{ihk}	0.7422	0.3778	0.9493	0.3663	0.784	0.4219	0.8178	0.3657	0.9055	0.3284
$G(\widehat{A^t_{ihk}})$	0.7833	0.367	1.0364	0.327	0.8172	0.3651	0.8019	0.3422	0.9201	0.3632
r_{ihk}	0.8383	0.3258	1.0642	0.3571	0.7879	0.3502	0.7717	0.3699	1.0105	0.3848
t_{ihk}	0.799	0.3632	1.0809	0.3788	0.8191	0.3806	0.798	0.3708	1.0388	0.3647

Estimated Classification Counts for Personal Loans Under BwLDA Model

Table 4 presents the Bayesian weighted linear discriminant analysis (BwLDA) model, which predicted the classification of personal loans, and compares it with the actual bank record counts across five banks: IBLL, GTBLL, ABLL, EBLL, and UBA. The table includes the counts of loans categorized as high-risk or low-risk by the BwLDA model, along with their corresponding high PD or low PD outcomes and actual bank record count outcomes. The table reveals that BwLDA achieves perfect precision across all banks, whereas every loan predicted as high risk by the BwLDA model corresponds to a loan with high PD. This resulted in zero false positives and false negatives, reflecting a higher accuracy in risk identification.

However, the comparison between the BwLDA model's classifications and the actual bank record counts for personal loans reveals a consistent overestimation across all five banks. For instance, ABLL shows the most substantial overestimation, with the model predicting 285 high-risk loans compared to a bank record count of 200. This suggests that the BwLDA model obtained an excess of 42.5%, which significantly indicates that ABLL employs a more conservative internal risk classification system or that its portfolio includes a broader set of compensating factors (e.g., strong collateral or long-term client relationships) that are not captured by the model's statistical inputs. For GTBLL and UBA, the BwLDA model classified 31 and 90 personal loans as high-risk, respectively, compared to the actual bank record counts of 29 and 85, respectively, suggesting a notable overestimation of 6.9% and 5.88%, respectively. In the case of EBLL and IBLL, the BwLDA model estimated 45 and 65 in classifying high-risk loans, respectively, compared with the actual bank record counts of 44 and 64. The discrepancies were 2.27% and 1.56%, respectively.

These findings indicate that although the BwLDA model maintains a consistent internal performance, it may not fully reflect the qualitative judgments, contextual nuances, or adaptive thresholds used by banks in Liberia for personal loan risk classification. Such divergences highlight the need to integrate banking institutional insights or adjust model thresholds to better align with the operational definitions of risk at the banking institution level.

Table 4: - BwLDA personal loan classification counts against actual bank recor

Source	Classification	IBLL		GTBLL		ABLL		EBLL		UBA	
		High PD	Low PD	High PD	Low PD	High PD	Low PD	High PD	Low PD	High PD	Low PD
BwLDA Model	High Risk	65	0	31	0	285	0	45	0	90	0
	Low Risk	0	64	0	31	0	269	0	48	0	80
Bank Record	High Risk	64	0	29	0	200	0	44	0	85	0
	Low Risk	0	65	0	33	0	354	0	49	0	85

Comparing Inclusion and Exclusion Distance-to-Default Feature into BwLDA Model

Tables 5 and 6 display results from BwLDA model including and excluding distance-to-default as risk-sensitive, respectively. The inclusion of the distance-to-default as in the BwLDA model led to mixed performance outcomes across the five banks. For instance, IBLL, the distance-to-default added significantly improved the model predictive performance, with accuracy rising from 0.68 to 0.78, and F1 score from 0.33 to 0.49, highlighting distance-to-default's value in capturing risk signals that were otherwise underrepresented. Conversely, GTBLL experienced a decline in most metrics when distance-to-default was included, withaccuracy falling from 0.81 to 0,64, suggesting possible model overfitting or feature redundancy.

Table 5: - BwLDA model with distance-to-default

Bank	Accuracy	AUC	Precision	Recall	F1 score
IBLL	0.78	0.70	0.59	0.48	0.49
GTBLL	0.64	0.66	0.62	0.63	0.61
ABLL	0.74	0.72	0.72	0.95	0.81
EBLL	0.55	0.53	0.59	0.61	0.59
UBA	0.77	0.73	0.77	0.90	0.82

Table 6: - BwLDA model without distance-to-default

Bank	Accuracy	AUC	Precision	Recall	F1 score
IBLL	0.68	0.60	0.39	0.32	0.33
GTBLL	0.81	0.71	0.75	0.89	0.80
ABLL	0.76	0.72	0.74	0.95	0.82
EBLL	0.57	0.53	0.58	0.70	0.63
UBA	0.77	0.70	0.77	0.90	0.82

For ABLL and UBA, performance remained relatively stable, with minimal changes observed across accuracy, AUC and F1 scores, implying that the model was already well-calibrated, and distance-to-default added marginal incremental values. Slight improvements were shown in EBLL with recall maintained low AUC and F1 scores in both models, indicating additional enhancements or features may be needed for this institution regardless of distance-to-default inclusion. In conclusion, the analysis demonstrates that the distance-to-default variable enhances classification performance in some contexts, particularly for banks with weaker initial separation between classes (for example, IBLL). However, its effectiveness is not uniform, emphasizing the importance of context-specific variable selection in credit risk modeling.

On the other hand, Figure 1 provides a visual comparison of the BwLDA model's performance with and without the distance-to-default feature across five banks. It illustrates how key metrics, such as accuracy, AUC, precision, recall, and F1 score vary depending on the inclusion of distance-to-default, helping to assess its impact on classification effectiveness.

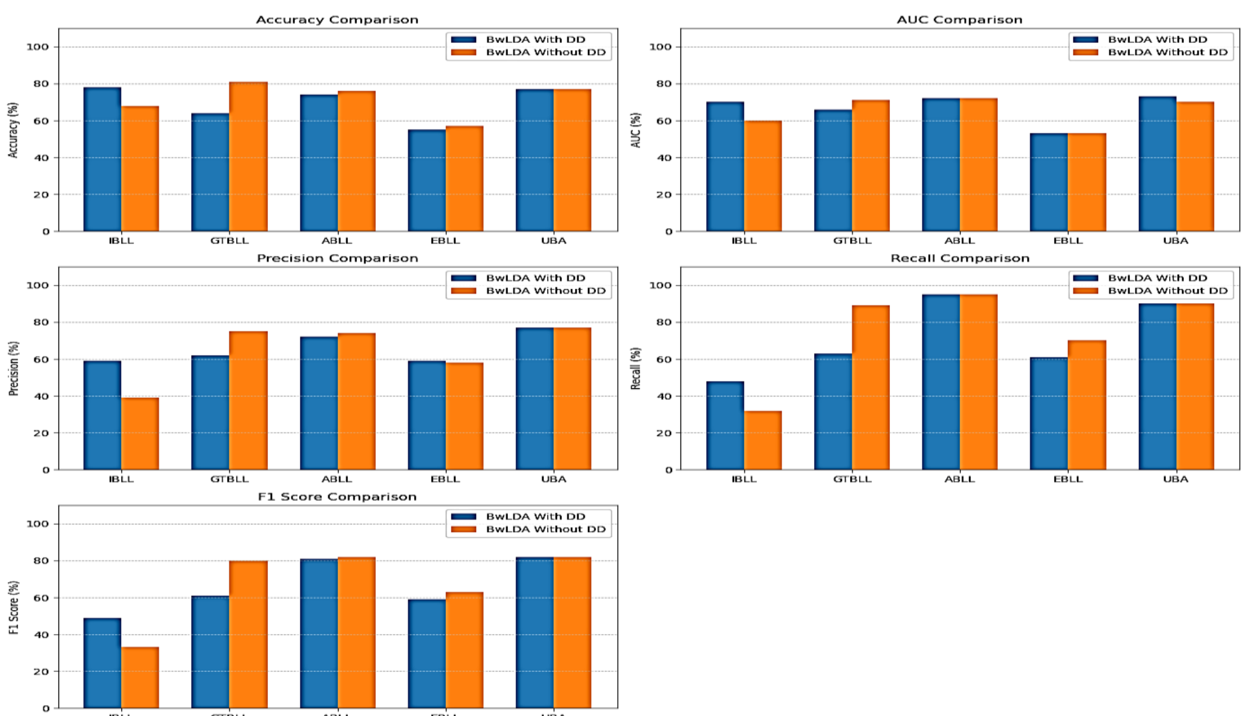

Figure 1: - Performance metrics of BwLDA model with and without distance-to-default.

Implications of the Finding for Credit Risk Management in Liberia's Banking Sector

The empirical findings highlight the critical need for advanced credit risk assessment frameworks, particularly the BwLDA model, to enhance loan classification accuracy and strengthen alignment between high-risk loans and probabilities of default (PD). Liberian banking institutions are recommended to adopt the BwLDA model to significantly reduce misclassification errors and improve risk differentiation, particularly in banks managing complex or high-risk portfolios, such as ABLL and EBLL. The Central Bank of Liberia (CBL) could play a pivotal role by mandating the adoption of BwLDA or similar methodologies across the banking sector to ensure consistency and reliability in credit risk assessments.

Summary and Conclusion:-

The proposed models achieved the goals of this study by demonstrating superior performance in classification precision and risk differentiation. The BwLDA model provided better alignment between risk classifications and default probabilities. Including distance-to-default as a risk-sensitive variable within the BwLDA model was evaluated effectively across five banks. The results showed mixed outcomes. At IBLL, including $\mathrm{DD_{ihk}}$ significantly improved predictive performance with accuracy rising from 0.68 without $\mathrm{DD_{ihk}}$ to 0.78 with $\mathrm{DD_{ihk}}$, and F1 score from 0.33 to 0.49, demonstrating $\mathrm{DD_{ihk}}'\mathrm{s}$ importance in capturing underrepresented risk signals. In contrast, GTBLL declined in performance, with accuracy dropping from 0.81 without $\mathrm{DD_{ihk}}$ to 0.64 with $\mathrm{DD_{ihk}}$, indicating potential overfitting or feature redundancy. For ABLL and UBA, model performance remained largely stable, suggesting that model with $\mathrm{DD_{ihk}}$ contributed minimal incremental value due to prior model calibration. EBLL showed only slight improvement, with maintained but low AUC and F1 scores across both models, highlighting the need for further feature refinement. Overall, the findings suggest that while distance-to-default can enhance credit risk prediction, its effectiveness is context-dependent and varies across institutions.

These findings underscore the importance of tailoring credit risk models to align with the specific characteristics of each bank's portfolio. The BwLDA model excels in addressing complexity and variability. The analysis emphasizes the necessity of adopting a nuanced, institution-specific approach to credit risk assessment and management. This adaptability ensures that each bank can optimize its credit risk strategies based on its unique portfolio dynamics.

Recommendations:-

The empirical application revealed key trends across the five banks, such as the consistently higher risk associated with commercial loans compared to personal loans. The BwLDA model showed superior performance in ensuring classification accuracy and alignment, particularly for banks with complex or high-risk portfolios like ABLL and EBLL. These findings underscore the potential of advanced statistical techniques in addressing challenges in credit risk modeling, especially in emerging markets like Liberia. Furthermore, the paper recommends that the results be used to come up with a roadmap for policymakers and financial institutions to enhance risk management practices and decision-making processes.

Lastly, further study could explore the application of the BwLDA model within the non-banking financial institutions, including microfinance entities or community-based savings groups like Susu clubs and include macroeconomic indicators and industry-specific variables.

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