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RESEARCH ARTICLE

Three Results for Non-Associative Semi Prime Ring with Unity.

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Abstract

In this paper we have mainly obtained some theorems related to Non-associative ring with unity.

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Introduction:-

Quadri Ashraf (3) generalized Some Results for Associative Rings. They Proved that if R is Non Associative Ring. In which $(xy)^2-yx^2y$ is Centre, Then R is Commutative. In this Paper. We show that a non -Associative Ring with unity such that for $(xy)^2=(xy^2)x$, $xy^2=y^2x$ and $(x, x^3y^3+x^2y^2)=0$ .Throughout the paper Z(R) denotes the centre of non Associative ring R and (x,y)=xy-yx for all x,y in R

Main Results:-

We prove the following theorems

Theorem 1: If R be a Non-associative ring with unity 1 such that for $(xy)^2 = (xy^2)x$ all x,y in R, then R is commutative.

Proof: Given identity is $(xy)^2 = (x y^2)x$

Replacing x by x+1 in the given condition,

$$[(x+1)y]^2 = [(x+1)y^2](x+1)$$

$$(xy+y)^2=[(xy^2+y^2)(x+1)]$$

$$(xy+y)(xy+y)=[(xy^2)x+xy^2+y^2x+y^2]$$

$$(xy)^2+(xy)y+y(xy)+y^2=(xy^2)x+(xy^2)y+y^2x+y^2 \quad \text{(by the condition and cancellation law)}$$

$$(xy)y+y(xy)= (xy^2)+y^2x$$

Replacing y by y+ 1, $[x(y+1)](y+1)+(y+1)[x(y+1)] =x(y^2+2y+1)+((y^2+2y+1)x$

We get xy=yx. $\forall$ x,y $\in$ R(by the condition and cancellation law)

Hence R is commutative ring.

Theorem 2: Let R be a non- associate ring with unity 1 such that $xy^2=y^2x$ for all x, y in R, then R is commutative.

Proof: Given identity $xy^2=y^2x$,

Replacing x by x+1 , $(x+1)^2y^3 = y^3(x+1)^2$

$$(x^2+2x+1)y^3 = y^3(x^2+2x+1)$$

$$x^2y^3+2xy^3+y^3 = y^3x^2+2y^3x+y^3$$

$$x y^3 = y^3x \qquad \ldots(1)$$

Replacing y by y+1 in the above result, $x(y+1)^3 = (y+1)^3x$

$$x(y+1)(y+1)(y+1)=(y+1)(y+1)(y+1)x$$

$$(xy+x)(y^2+2y+1)=(y^2+2y+1)(xy+x)$$

$$xy^3+3xy^2+3xy= y^2xy+y^2x+2yxy+2yx+xy$$,(by cancellation law) since, $xy^2 =yxy$,

$$xy^2=y^2x,$$

$$xy^3+ 2xy = y^2xy+2yx \qquad \ldots(2)$$

again replacing y by y+1 in the above result,

$$x(y+1)^3+2x(y+1)=(y+1)^2x(y+1)+2(y+1)x$$

$$x(y+1)(y+1)(y+1)+2x(y+1)=(y+1)(y+1)x(y+1)+2(y+1)x$$

$$(xy+x)(y^2+2y+1)+2(xy+x) = (y^2+2y+1)(xy+x) +(2yx+x)$$

$$xy^3+3xy^2+4xy= y^2xy+4yx + y^2x+2yxy,$$ by cancellation law

$$2xy^2 +xy^2+2xy=2yx + y^2x+2yxy$$ (from result (2) and given condition)

$$2xy^2+2xy = 2yxy+2yx$$,(by the theorem $x y^2 = yxy$)

$$2xy=2yx$$

R is a commutative ring for all x,y.

Theorem 3: Let R be a 2- divisible associate with unity 1 such that $(x_, x^3y^3+ x^2y^2) =0$ x, y in R. Then R is commutative.

Proof : let x,y be in R, then $(x_, x^3y^3+x^2y^2)=0$

That is $(x^3y^3+x^2y^2)x = x(x^3y^3+x^2y^2)$

Replacing y by y+1 in the above condition

$$[x^3(y+1)^3+x^2(y+1)^2)x = x[x^3(y+1)^3+x^2(y+1)^2]$$

$$[x^3(y+1)(y+1)^2+x^2(y^2+2y+1)]x=x[x^3(y+1)(y+1)(y+1)+ x^2(y+1)(y+1)]$$

$$5 x^3yx+ x^3y^2x+ 2 x^2y x = 5x^4y+ x^4y^2+ 2x^3y$$ (By the theorem $x^n y= x^{n1}y$)

we get, $x^3y^2x=x^4y$

Replacing x by x+1 , $(x+1)(x+1)(x+1)y^2(x+1)=(x+1)(x+1)(x+1)(x+1)y^2$

$$(x^2+2x+1)(x+1)(y^2x+y^2)=(x^2+2x+1)(x^2+2x+1)y^2$$

$$3x^2y^2x+3xy^2x =3x^3y^2+3x^2y^2$$

$$x^2y^2x+xy^2x =x^3y^2+x^2y^2,$$ since , $xy^2=y^2x$

Replacing x by x+1 , ,

$$(x^2+2x+1)(y^2x+y^2)+(xy^2+y^2)(x+1)=(x^2+2x+1)(xy^2+y^2)+(x^2+2x+1)y^2$$

$$2xy^2x+2y^2x=2x^2y^2+2xy^2$$ since, $(xy^2=y^2 x)$

$$xy^2x= x^2y^2$$

again replacing x by (x+1), $(x+1)y^2 (x+1)=(x+1)^2y^2$

$$(xy^2+y^2) (x+1)= (x^2+2x+1)y^2$$

$$xy^2x+xy^2+y^2x+y^2= x^2y^2+2xy^2+y^2$$

$$y^2x =xy^2$$

Replacing y by y+1, $y^2x+2yx+x= xy^2+2xy+x$

$xy=yx \quad \forall x,y \in R$(by the condition and cancellation law)

Hence R is a commutative ring for all x, y.

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