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RESEARCH ARTICLE

TRACE DISTANCE: A MEASURE OF QUANTUMNESS.

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Abstract

We use trace distance as a measure of quantum character in qubits and have calculated the success probability. When a qubit is subjected to environmental noise particularly amplitude damping noise it is expected that its quantum character is diminishes. In this work we also have used weak measurement and quantum measurement reversal operations for the protection of qubits from decoherence. From this study we found that trace distance is actually a good measure of closeness of two states.

Introduction:-

Quantum computing and open quantum systems are two of the most emerging fields of quantum information science [1]. Basic phenomenon of quantum physics have been used to proceed this science for decades like no cloning theorem[1], superposition theorem, uncertainty principle etc. [2]. For quantum information science to work properly there are many other things such as qubits, entanglement, discord etc. A qubit as we know that is a basic unit for the communication through any quantum channel. It is a superposition of two states [1]. In information theory its preservation is quite essential. The entanglement between two qubits is one of the special feature of quantum information science. It is frequently used in making many protocols, such as teleportation of unknown states [3].Quantum key distribution [4]quantum cryptography [5, 6] and quantum computation [7, 8]. The study of open quantum system is also important because environmental noises called, decoherence badly affects the quantum states. Among various kind of noises which can produce decorhence amplitude damping, depolarising, phase damping are so popular [1]. There were a lot of work has been done to suppress these kind of decoherences. Feedback control [9], Quantum Zeno effect [10], weak measurement [11]etc are some methods which has been used to protect qubits (single as well as entangled) against decoherence. The measures of entanglement used in these methods are negativity, concurrence, discord etc. [1]. Recently we have described a measure of quantumness called trace distance [1] [12]. We have shown that trace distance can be used as a suitable measure for describing quantum character between two qubit states which we have taken before decoherence and after applying weak measurement.

Weak measurement and its reversal also plays an important role for the suppression of decohrence [13].

Now it's important to protect a qubit as well as entangles qubits from decoherence. In this letter we try to show that how trace distance between two qubit states changes with success probability, which is defined as probability of occurrence of decoherence when the qubit is subjected to amplitude damping noise and after that weak measurements . The paper is organized in following manner:

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Section 1 contains introduction. Section 2 comprises materials and methods. The results will be shown in section 3 and finally the conclusion will be in section 4.

Materials and Methods:-

Let us suppose we have a qubit which in the pure state is given by:

$$(|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle) \quad (1)$$

The density matrix of equation (3.1) can be written as

$$\rho(0) = \begin{pmatrix} \alpha^2 & \alpha\beta^* \\ \alpha^*\beta & \beta^2 \end{pmatrix} \quad (2)$$

Where,

$|\alpha^2| + |\beta^2| = 1$ and the environment E is initially in the state $|0\rangle_E$.

After interacting with environment the qubit subjected to amplitude damping noise.

The state after amplitude damping noise is given as

$$\rho_{AD} = E_0\,\rho(0)E_0^\dagger + E_1\,\rho(0)E_1^\dagger \quad (3)$$

Where E_0 and E_1 are [1],

$$E_0 = \begin{pmatrix} 1 & 0 \\ 1 & \sqrt{1-D} \end{pmatrix}, E_1 = \begin{pmatrix} 0 & \sqrt{D} \\ 0 & 0 \end{pmatrix} \quad (4)$$

or

$$\rho_{AD} = \begin{pmatrix} \alpha^2 + \beta^2 D & \alpha\beta^*\sqrt{1-D} \\ \alpha^*\beta\sqrt{1-D} & \beta^2(1-D) \end{pmatrix} \quad (5)$$

From equation (5) we see that when D tends to 1 that is when decoherence is maximum the coherence between two states vanishes. It can be seen from the vanishing off diagonal elements with D tends to 1 which are responsible for coherence. The pure state is converted in to mixed state due to amplitude damping noise.

Applying weak measurement given by $M = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-M} \end{pmatrix}$ and reverse measurement $M_{rev} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-M_r} \end{pmatrix}$, we get the state:

$$\rho_{MDM_{2rev}} = \frac{1}{P^2P_1}\begin{pmatrix} \alpha^2\sqrt{(1-M_r)} + D\beta^2\sqrt{1-M}\sqrt{1-M_r} & \alpha\beta^*\sqrt{1-D}\sqrt{1-M_r} \\ \alpha^*\beta\sqrt{1-D}\sqrt{1-M} & \beta^2(1-D)\sqrt{1-M} \end{pmatrix} \quad (6)$$

Where

$$P_1 = \alpha^2\sqrt{(1-M_r)} + D\beta^2\sqrt{1-M}\sqrt{1-M_r} + \beta^2(1-D)\sqrt{1-M} \quad (7)$$

P^2P_1 is defined as the transmittance**[15]**

The first measure we will consider is the trace distance. Let ρ and σ be two density matrices.the trace distance $\delta(\rho,\sigma)$is defined to be

$$\delta(\rho,\sigma) = \frac{1}{2}Tr|\rho - \sigma| \quad (8)$$

The trace is symmetric $\delta(\rho,\sigma) = \delta(\sigma,\rho)$ and nonnegative $\delta(\rho,\sigma) \geq 0$

If $|\Psi\rangle\langle\Psi|$ is a pure state then $\delta(\rho,\sigma) = \sqrt{1 - \langle\Psi|\sigma|\Psi\rangle}$(3.22)

The success proabability which is defined as the probability of the effect of decherence is given by

$$\text{Success probability} = \frac{1}{2} + \frac{1}{2}\delta(\rho,\sigma) \quad (9)$$

Results and Discussion:-

In the case described above we get the trace distance and then the success probability which are given by

$$\delta(\rho,\sigma) = \sqrt{1 - \frac{(\alpha^4\sqrt{(1-M_r)} + D\beta^2\alpha^2\sqrt{1-M}\sqrt{1-M_r} + \alpha^2\beta^2\sqrt{1-D}\sqrt{1-M_r} + \alpha^2\beta^2\sqrt{1-D}\sqrt{1-M} + \beta^4(1-D)\sqrt{1-M})}{P^2\,P_1}}$$

and $\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{\left(\alpha^4\sqrt{(1-M_r)}+D\beta^2\alpha^2\sqrt{1-M}\sqrt{1-M_r}+\alpha^2\beta^2\sqrt{1-D}\sqrt{1-M_r}+\alpha^2\beta^2\sqrt{1-D}\sqrt{1-M}+\beta^4(1-D)\sqrt{1-M}\right)}{P^2\,P_1}}$

Now the qubit is subjected to weak measurement, decoherence, and reverse of weak measurement and we get the trace distance given by equation (3.74). At unknown D we assume $M = M_r$ and when $M \rightarrow 1$ we get the trace distance as a function of α. At D=0.5 and D=0.7 there is a decrease in trace distance with increasing measurement strength with $\alpha > \beta$. Fig1 and Fig.2 shows a variation of trace distance and success probability with measurement strength M and M_r

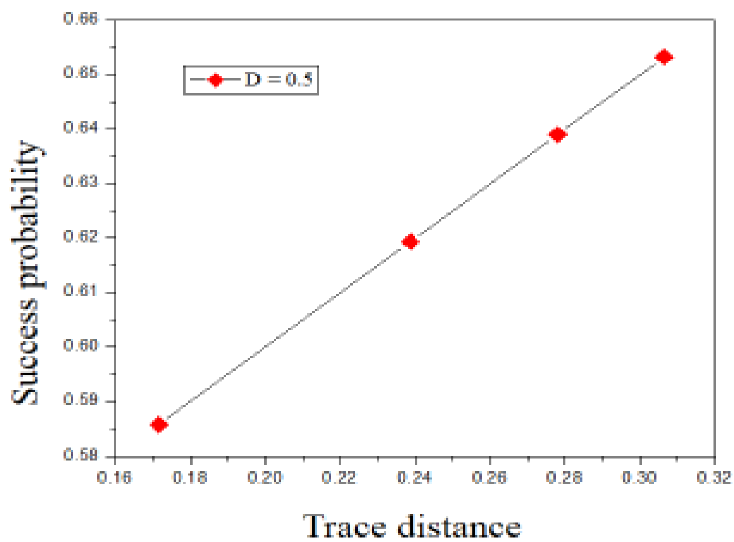

Fig.1:- Increase in success probability with increasing trace distance under amplitude damping noise at D = 0.5, $\alpha > \beta$.

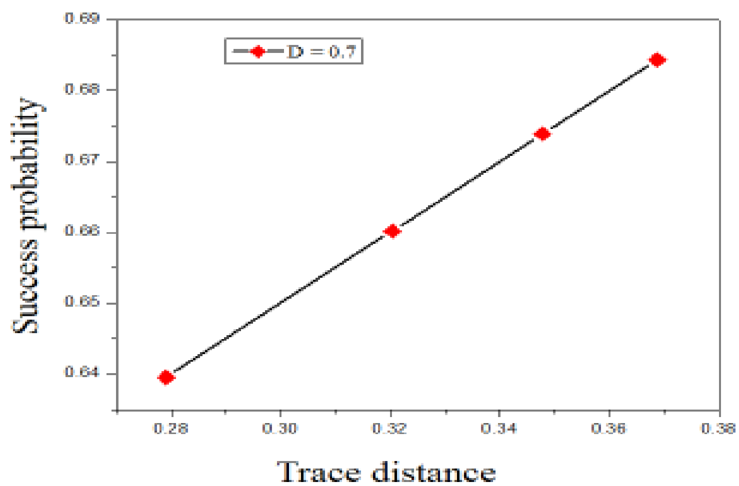

Fig.2:-Increase in success probability with increasing trace distance under amplitude damping noise at D= 0.7,$\alpha > \beta$.

Conclusion:-

From the above study we conclude that success probability is increased as we increase trace distance that is the probability of the effect of decoherence increases. Trace distance is another measure of the closeness of the states. We can apply this method similarly on suppressing phase damping and depolarising noises with the help of the application of weak measurement and its reversal. The importance of this paper is that we can save our qubit from decoherence by using the formula of trace distance. It will help us to make quantum computers with strong qubit state.

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