ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

Radiative Transfer Problem in a Two-Region Inhomogeneous Slab With an Anisotropic Scattering

A. Elghazaly [1], S. El-Konsol [1], A.S. Sabbah [2] and M. Hosni [1]

1. Reactor &Neutron Physics Department, Nuclear Research Center, Atomic Energy Authority, Cairo, Egypt.

2. Mathematical Department, Faculty of Science, Zagazig University, Zagazig, Egypt.

Manuscript Info

Manuscript History:

Received: 15 August 2014
Final Accepted: 20 September 2014
Published Online: October 2014

Key words:

Radiative Transfer - Two-region inhomogeneous slab- Galerkin technique- Anisotropic scattering - Diffusely and specularly reflecting boundaries.

***Corresponding Author**

A. Elghazaly

Abstract

The problem of radiative transfer has been solved for anisotropic scattering in a two-region inhomogeneous slab of equal or unequal thickness. Diffusely and specularly reflecting boundary, interface transmission and diffuse surface source are considered as the boundary conditions. An exact derivation of the integral equations that describe the irradiance and the net radiative fluxes for each medium is obtained. A Galerkin method is used to solve these integral equations for two-regions and so to calculate the reflection and the transmission coefficients. The calculations are carried out, for anisotropic scattering and exponential single-scattering albedo. Numerical results are obtained for the reflection and the transmission coefficients for isotropic, forward and backward linear anisotropic scattering in the inhomogeneous media with diffusely and specularly reflecting boundaries condition. Results obtained for isotropic scattering in inhomogeneous media are compared well with the published calculations.

Introduction

Radiative transfer problem through scattering media with diffusely and specularly reflecting boundary conditions has attracted considerable interest, particularly for applications through turbid media. [1- 10]. Problems of radiative transfer in composite medium have evoked the wide interest of many researchers and have been studied using several techniques [6- 11]. An analytical approach for obtaining Green's function of the radiative transfer equation(RTE) in two dimensional layered media is derived[7] by considering the exact boundary conditions(BC). The analytical discreate-ordinate method [8-9] is applied RTE in multi-layer medium subject to Fresnel boundary/interface condition. Anodal method [10] based on the Synthetic Kernel approximation is developed for solving RTE in one, and two dimensional medium.

In the previous work, The Galerkin [11, 12] technique is used for solving the problem of particle transfer in the two-regions homogeneous and an inhomogeneous media, but only considered isotopic scattering. But in the present work, the same method is used to solve radiative transfer equation for an absorbing, emitting, inhomogeneous, with isotropic and an anisotropic scattering that contains an internal source. An exact derivation of the integral equations that describe the irradiance and the net radiative fluxes for each an inhomogeneous medium is obtained. The coupled integral equations for the two regions inhomogeneous slab with diffusely and specularly reflecting boundaries are solved. Calculations are carried out for equal or unequal thickness and exponential single scattering albedo for isotropic, forward and backward linear anisotropic scattering.

1. BASIC EQUATIONS

Radiative transfer equation for an absorbing, emitting, inhomogeneous, with isotropic and an anisotropic scattering that contains an internal energy source, are described for two region as

$$(\mu \frac{\partial}{\partial x}+1)\, I_1(x,\mu)=\frac{\omega_1(x)}{2}\int_{-1}^{1} I_1(x,\mu')\, P(\mu,\mu')\,\mathrm{d}\mu' + Q_1(x), \tag{1}$$

$$-1 \le \mu \le 1, \quad \text{and} \quad 0 \le x \le a$$

$$(\mu \frac{\partial}{\partial x}+1)\, I_2(x,\mu)=\frac{\omega_2(x)}{2}\int_{-1}^{1} I_2(x,\mu')\, P(\mu,\mu')\, d\mu' + Q_2(x), \tag{2}$$

$$-1 \le \mu \le 1, \quad \text{and} \quad a \le x \le b$$

Where x is the optical distance, μ is the cosine of the direction of propagation of the radiation intensity $I(x,\mu)$ of the particle, $Q(x)$ is the internal energy source, ω is the single scattering albedo, and the subscripts 1 and 2 refer to the region 1 and 2, respectively and $P(\mu,\mu')$ is the scattering phase function. The linear anisotropic scattering function is defined as [13]

$$P(\mu,\mu')=1+\bar{a}\,\mu\mu' \tag{3a}$$

Where $\bar{a}$, the linear anisotropic coefficients, is given in terms of the Legendre - polynomial coefficients a_m by the formula [13]

$$\bar{a}=\sum_{m=0}^{\infty}\left[(-1)^{m} a_{2m+1}(2m)!\right] / \left[2^{2m} m!(m+1)!\right] \tag{3b}$$

The boundary conditions for specular and diffuse reflection, interface transmission and diffuse surface source are taken as

$$I_1(0,\mu)= f_1 \qquad \mu \ge 0 \quad , \tag{4a}$$

$$I_1(a,-\mu) = \gamma_{21} I_2(a,-\mu)+\rho_{2s}\, I_1(a,\mu)+2\rho_{2d}\, J_1^{+}(a), \qquad \mu \ge 0 \quad , \tag{4b}$$

$$I_2(a,\mu) = \gamma_{12} I_1(a,\mu)+\rho_{3s}\, I_2(a,-\mu)+2\rho_{3d}\, J_2^{-}(a), \qquad \mu \ge 0 \quad , \tag{4c}$$

$$I_2(b,-\mu)= \rho_{4s}\, I_2(b,\mu)+2\rho_{4d}\, J_2^{+}(b), \qquad \mu \ge 0 \quad , \tag{4d}$$

Where f_1 represent diffusely emitting sources, ρ_s and ρ_d are the specular and diffuse reflectivities of the boundaries. γ_{ij} the interface transmissivity when radiation enters the j layer from i layer and equal to 1- $\rho_s+\rho_d$. $J_s^{\pm}$ represent the partial heat fluxes and are defined by

$$J_s^{\pm}(x)=\int_0^1 I_s(x,\pm\mu)\,\mu\,\mathrm{d}\mu, \qquad \mu \ge 0 \tag{5}$$

For the first medium, Eq (1) is transformed by using the boundary conditions (4a) and (4b) to the integral equations

$$I_1(x,\mu)=I_1(0,\mu)\,\mathrm{e}^{-x/\mu}+\int_0^x \frac{\mathrm{e}^{-(x-x')/\mu}}{\mu}(G_{01}^{*}(x')+\bar{a}\mu\, G_1^{*}(x'))\,dx', \qquad \mu \ge 0 \tag{6}$$

$$I_1(x,-\mu)=I_1(a,-\mu)\,\mathrm{e}^{-(a-x)/\mu}+\int_x^a \frac{\mathrm{e}^{-(x'-x)/\mu}}{\mu}(G_{01}^{*}(x')-\bar{a}\mu G_1^{*}(x'))\,dx', \qquad \mu \ge 0 \tag{7}$$

For the second medium, Eq (2) is transformed by using the boundary conditions (4c) and (4d) to the integral equations

$$I_2(x,\mu) = I_2(a,\mu)\, \mathrm{e}^{-(x-a)/\mu} + \int_a^x \frac{\mathrm{e}^{-(x-x')/\mu}}{\mu} (G_{02}^*(x') + \bar{a}\mu G_2^*(x'))\, dx', \qquad \mu \ge 0 \tag{8}$$

$$I_2(x,-\mu) = I_2(b,-\mu)\, \mathrm{e}^{-(b-x)/\mu} + \int_x^b \frac{\mathrm{e}^{-(x'-x)/\mu}}{\mu} (G_{02}^*(x') - \bar{a}\mu G_2^*(x'))\, dx', \qquad \mu \ge 0 \tag{9}$$

Where

$$G_{0s}^*(x) = \frac{\omega_s(x)}{2} G_{0s}(x) + Q_s(x), \qquad \mathrm{s} = 1,2 \tag{10}$$

$$G_s^*(x) = \frac{\omega_s(x)}{2} G_s(x) \qquad \mathrm{s} = 1,2 \tag{11}$$

And the irradiance $G_{0s}(x)$ and the net flux $G_s(x)$ defined as

$$G_{0s}(x) = \int_{-1}^{+1} I_{\mathrm{s}}(x,\mu) d\mu \tag{12}$$

$$G_s(x) = \int_{-1}^{+1} \mu I_{\mathrm{s}}(x,\mu) d\mu \tag{13}$$

We rewrite Eqs. (6-9) at $x = a, x = 0, x = b$ and $x = a$ respectively, and by solving the system of the four resulting equations, we obtain

$$I_1(0,-\mu) = \left[(Y_1\rho_{2s}\, \mathrm{e}^{-a/\mu} + Y_2) + \gamma_{21}\, \mathrm{e}^{-a/\mu}(Y_1\rho_{4s}\gamma_{12}\, \mathrm{e}^{-2(\mathrm{b}-a)/\mu} + Y_{34})\right] \tag{14}$$

$$I_1(a,\mu) = Y_1 \tag{15}$$

$$I_2(\mathrm{b},\mu) = \Delta^{-1}\left[(Y_4\rho_{3s}\, \mathrm{e}^{-(\mathrm{b}-a)/\mu} + Y_3) + \gamma_{12}\, Y_1 \mathrm{e}^{-(b-a)/\mu})\right] \tag{16}$$

$$I_2(a,-\mu) = \Delta^{-1}\left[(Y_3\rho_{4s}\, \mathrm{e}^{-(b-a)/\mu} + Y_4) + \gamma_{12}\rho_{4s}\, \mathrm{e}^{-2(b-a)/\mu} Y_1\right] \tag{17}$$

Where

$$\Delta = 1 - \rho_{3s}\rho_{4s}\, \mathrm{e}^{-2(\mathrm{b}-a)/\mu} \tag{18}$$

$$Y_1 = f_1\, \mathrm{e}^{-a/\mu} + \int_0^a \frac{\mathrm{e}^{-(a-x')/\mu}}{\mu} (G_{01}^*(x') + Q_1(x') + \bar{a}\mu G_1^*(x'))\, dx', \tag{19}$$

$$Y_2 = 2\rho_{2\mathrm{d}} J_1^+(a)\, \mathrm{e}^{-a/\mu} + \int_0^a \frac{\mathrm{e}^{-x'/\mu}}{\mu} (G_{01}^*(x') + Q(x') - \bar{a}\mu G_1^*(x'))\, dx', \tag{20}$$

$$Y_3 = 2\rho_{3\mathrm{d}} J_2^-(a)\, \mathrm{e}^{-(b-a)/\mu} + \int_a^b \frac{\mathrm{e}^{-(b-x')/\mu}}{\mu} (G_{02}^*(x') + Q_2(x') + \bar{a}\mu G_2^*(x'))\, dx', \tag{21}$$

$$Y_4 = 2\rho_{4\mathrm{d}} J_2^+(b)\, \mathrm{e}^{-(b-a)/\mu} + \int_a^b \frac{\mathrm{e}^{-(x'-a)/\mu}}{\mu} (G_{02}^*(x') + Q_2(x') - \bar{a}\mu G_2^*(x'))\, dx' \tag{22}$$

and

$$Y_{34} = Y_4 + \rho_{4s} Y_3\, \mathrm{e}^{-(b-a)/\mu} \tag{23}$$

The integral forms of the irradiance $G_{0s}(x)$ Eqs. (12) and the net fluxes $G_s(x)$ Eqs (13) with the transparent boundaries $\rho_{\mathrm{is}}\rho_{\mathrm{id}} = 0$ reduce to

$$G_{01}(x) = \int_0^1 I_1(0,\mu)\, \mathrm{e}^{-x/\mu}\, \mathrm{d}\mu + \int_0^1 \mathrm{I}_1(a,-\mu)\, \mathrm{e}^{-(a-x/\mu}\, \mathrm{d}\mu$$
$$+ \frac{\omega_1(x)}{2}\int_0^a E_1(|x-x'|)[G_{01}(x') + Q_1(x')]\, dx' + \frac{\bar{a}\,\omega_1(x)}{2}\int_0^a \mathrm{sign}\ E_2(|x-x'|) G_1(x')\, dx'$$

$$G_{01}(x) = f_1\, E_2(x) + \rho_{2s} f_1\, E_2(2a-x) + \rho_{4s} f_1\, E_2(2b-x) + 2\rho_{2\,\mathrm{d}} J_1^+(a)\, E_2(a-x)$$
$$+ 2\gamma_{21}\rho_{4\,\mathrm{d}} J_2^+(b)\, E_2(b-x) + \frac{1}{2}\int_0^a \omega_1(x) M_0^1(x,x') G_{01}(x')\, dx'$$
$$+ \frac{\bar{a}}{2}\int_0^a \omega_1(x) M_0^2(x,x') G_1(x')\, dx' + \int_0^a Q_1(x) M_0^1(x,x')\, dx' \qquad (24)$$
$$+ \frac{1}{2}\int_a^b \omega_2(x) K_0^1(x,x') G_{02}(x')\, dx') + \bar{a}\int_a^b \omega_2(x) K_0^2(x,x') G_2(x')\, dx')$$
$$+ \int_a^b Q_2(x) K_0^1(x,x')\, dx'$$

$$G_1(x) = \int_0^1 \mu\, I_1(0,\mu)\, \mathrm{e}^{-x/\mu}\, d\mu - \int_0^1 \mu\, I_1(a,-\mu)\, \mathrm{e}^{-(a-x/\mu}\, d\mu$$
$$+ \frac{\omega_1(x)}{2}\int_0^a sign(x-x')\, E_2(|x-x'|)[G_{01}(x') + Q_1(x')]\, dx' + \frac{\bar{a}\,\omega_1(x)}{2}\int_0^a E_3(|x-x'|) G_1(x')\, dx'$$

$$G_1(x) = f_1\, E_3(x) - \rho_{2s} f_1\, E_3(2a-x) - \rho_{4s} f_1\, E_3(2b-x) + 2\rho_{2\,\mathrm{d}} J_1^+(a)\, E_3(a-x)$$
$$- 2\gamma_{21}\rho_{4\,\mathrm{d}} J_2^+(b)\, E_3(b-x) + \frac{1}{2}\int_0^a \omega_1(x) M_1^1(x,x') G_{01}(x')\, dx'$$
$$+ \frac{\bar{a}}{2}\int_0^a \omega_1(x) M_1^2(x,x') G_1(x')\, dx' + \int_0^a Q_1(x) M_1^1(x,x')\, dx' \qquad (25)$$
$$+ \frac{1}{2}\int_a^b \omega_2(x) K_1^1(x,x') G_{02}(x')\, dx') + \bar{a}\int_a^b \omega_2(x) K_1^2(x,x') G_2(x')\, dx'$$
$$+ \int_a^b Q_2(x) K_1^1(x,x')\, dx'$$

Similarly, the integral form for the second region is

$$G_{02}(x) = f_1\, \gamma_{21} E_2(x) + \gamma_{12}\rho_{4s} f_1\, E_2(2b-x) + 2\rho_{3\,\mathrm{d}} J_2^-(a)\, E_2(x-a) + 2\rho_{4\,\mathrm{d}} J_2^+(b)\, E_2(b-x)$$
$$+ \frac{1}{2}\int_0^a \omega_1(x) M_2^1(x,x') G_{01}(x')\, dx' + \frac{\bar{a}}{2}\int_0^a \omega_1(x) M_2^2(x,x') G_1(x')\, dx'$$
$$+ \int_0^a Q_1(x) M_2^1(x,x')\, dx' + \frac{1}{2}\int_a^b \omega_2(x) K_2^1(x,x') G_{02}(x')\, dx' \qquad (26)$$
$$+ \bar{a}\int_a^b \omega_2(x) K_2^2(x,x') G_2(x')\, dx' + \int_a^b Q_2(x) K_2^1(x,x')\, dx'$$

and

$$G_2(x) = f_1\, \gamma_{12} E_3(x) - \gamma_{12}\rho_{4s} f_1\, E_3(2b-x) + 2\rho_{3\,\mathrm{d}} J_2^-(a)\, E_3(x-a) - 2\rho_{4\,\mathrm{d}} J_2^+(b)\, E_3(b-x)$$
$$+ \frac{1}{2}\int_0^a \omega_1(x) M_3^1(x,x') G_{01}(x')\, dx' + \frac{\bar{a}}{2}\int_0^a \omega_1(x) M_3^2(x,x') G_1(x')\, dx'$$
$$+ \int_0^a Q_1(x) M_3^1(x,x')\, dx' + \frac{1}{2}\int_a^b \omega_2(x) K_3^1(x,x') G_{02}(x')\, dx' \qquad (27)$$
$$+ \bar{a}\int_a^b \omega_2(x) K_3^2(x,x') G_2(x')\, dx' + \int_a^b Q_2(x) K_3^1(x,x')\, dx'$$

Where

$$M_0^i(x,x') = (sign(x-x'))^{i-1}\mathrm{E}_i(|x-x'|) + \gamma_{12}\gamma_{21}\rho_{4s}\mathrm{E}_i(2b-x-x') + \rho_{2s}\mathrm{E}_i(2a-x-x'), \qquad i=1,2 \tag{28}$$

$$K_0^i(x,x') = (-1)^{i-1}\mathrm{E}_i(x'-x) + \gamma_{21}\rho_{4s}\mathrm{E}_i(2b-x-x') \tag{29}$$

$$M_1^1(x,x') = sign(x-x')\mathrm{E}_2(|x-x'|) - \rho_{2s}\mathrm{E}_2(2a-x-x') - \gamma_{12}\gamma_{21}\rho_{4s}\mathrm{E}_2(2b-x-x'), \tag{30}$$

$$M_1^2(x,x') = \mathrm{E}_3(|x-x'|) - \rho_{2s}\mathrm{E}_3(2a-x-x') - \gamma_{12}\gamma_{21}\rho_{4s}\mathrm{E}_3(2b-x-x'), \tag{31}$$

$$K_1^i(x,x') = (-1)^i\mathrm{E}_k(x'-x) - \gamma_{21}\rho_{4s}\mathrm{E}_k(2b-x-x'), \qquad k=2,3 \tag{32}$$

$$M_2^i(x,x') = \mathrm{E}_i(x-x') + \gamma_{12}\rho_{4s}\mathrm{E}_i(2b-x-x') \tag{33}$$

$$K_2^1(x,x') = \mathrm{E}_1(|x-x'|) + \rho_{3s}\mathrm{E}_1(x+x'-2a) + \rho_{4s}\mathrm{E}_1(2b-x-x') \tag{34a}$$

$$K_2^2(x,x') = sign(x-x')\mathrm{E}_2(|x-x'|) - \rho_{3s}\mathrm{E}_2(x+x'-2a) + \rho_{4s}\mathrm{E}_2(2b-x-x') \tag{34b}$$

$$M_3^i(x,x') = \gamma_{12}E_k(x-x') - \gamma_{12}\rho_{4s}E_k(2b-x'-x) \tag{35}$$

$$K_3^1(x,x') = sign(x-x')E_2(|x-x'|) + \rho_{3s}E_2(x+x'-2a) - \rho_{4s}E_2(2b-x-x') \tag{36a}$$

$$K_3^2(x,x') = E_3(|x-x'|) - \rho_{3s}E_3(x+x'-2a) - \rho_{4s}E_3(2b-x-x') \tag{36b}$$

$$J_1^+(a) = f_1\, E_3(a) + \frac{1}{2}\int_0^a \omega_1(x)E_2(a-x)G_{01}(x)dx + \frac{\bar{a}}{2}\int_0^a \omega_1(\mathrm{x})E_3(a-x)G_1(x)dx + \int_0^a Q_1(x)E_2(a-x)dx \tag{37}$$

$$\begin{aligned} J_2^-(a) = {} & 2\rho_{4d}\, J_2^+(b)E_3(b-a) + \frac{1}{2}\int_a^b \omega_2(\mathrm{x})E_2(x-a)G_{02}(x)dx - \frac{\bar{a}}{2}\int_a^b \omega_2(\mathrm{x})E_3(x-a)G_2(x)dx \\ & + \int_a^b Q_2(x)E_2(x-a)dx + \gamma_{12}\rho_{4s}\, f_1E_3(2b-a) + \rho_{4s}\{\frac{1}{2}\int_a^b \omega_2(x)E_2(2b\text{-}a\text{-}x)G_{02}(x)dx \\ & + \int_a^b Q_2(x)E_2(2b\text{-}a\text{-}x)\mathrm{d}x + \frac{1}{2}\bar{a}\int_{\mathrm{a}}^b \omega_2(x)E_3(2b\text{-}a\text{-}x)G_2(x)dx + \frac{\gamma_{12}}{2}\int_0^a \omega_1(\mathrm{x})E_2(2b\text{-}a\text{-}x)G_{01}(x)dx \\ & + \gamma_{12}\int_0^a Q_1(x)E_2(2b\text{-}a\text{-}x)dx + \frac{\bar{a}\gamma_{12}}{2}\int_0^a \omega_1(x)E_3(2b\text{-}a\text{-}x)G_1(x)dx\,\} \end{aligned} \tag{38}$$

and

$$\begin{aligned} J_2^+(b) = {} & 2\rho_{3d}\, J_2^-(a)E_3(b-a) + \gamma_{12}\, f_1E_3(b) + \frac{1}{2}\int_{\mathrm{a}}^b \omega_2(x)E_2(b\text{-}x)G_{02}(x)dx\int_{\mathrm{a}}^b Q_2(x)E_2(b\text{-}x)dx \\ & + \frac{\bar{a}}{2}\int_{\mathrm{a}}^b \omega_2(x)E_3(b\text{-}x)G_2(x)dx + \frac{\gamma_{12}}{2}\int_0^a \omega_1(x)E_2(b\text{-}x)G_{01}(x)dx + \gamma_{12}\int_0^a Q_1(x)E_2(b\text{-}x)dx \\ & + \frac{\bar{a}\gamma_{12}}{2}\int_0^a \omega_1(x)E_3(b\text{-}x)G_1(x)dx + \frac{\rho_{3\mathrm{s}}}{2}\int_{\mathrm{a}}^b \omega_2(x)E_2(b\text{-}2a+x)G_{02}(x)dx \\ & + \rho_{3\mathrm{s}}\int_{\mathrm{a}}^b Q_2(x)E_2(b\text{-}2a+x)dx - \frac{\bar{a}\rho_{3\mathrm{s}}}{2}\int_{\mathrm{a}}^b \omega_2(x)E_3(b\text{-}2a+x)G_2(x)dx \end{aligned} \tag{39}$$

The reflection coefficient is defined by

$$R = \frac{\int_0^1 \mu\, \mathrm{I}(0,\text{-}\mu)\mathrm{d}\mu}{\int_0^1 \mu\, \mathrm{I}(0,\mu)\mathrm{d}\mu} = \frac{2}{f_1}J_1^-(0) \tag{40}$$

$$R=\frac{2}{f_1}[\rho_{2s} f_1\, E_3(2a)+\rho_{2s}[\frac{1}{2}\int_0^a \omega_1(x)E_2(2a\text{-}x)G_{01}(x)dx+\frac{\bar{a}}{2}\int_0^a \omega_1(x)E_3(2a\text{-}x)G_{01}(x)dx$$
$$+\int_0^a Q_1(x)E_2(2a\text{-}x)dx]++2\rho_{2d}J_1^+(a)E_3(a)+\frac{\gamma_{21}}{2}\int_0^a \omega_1(x)E_2(x)G_{01}(x)dx$$
$$+\frac{\bar{a}\gamma_{21}}{2}\int_0^a \omega_1(x)E_3(x)G_1(x)dx+\gamma_{21}\int_0^a Q_1(x)E_2(x)dx+\gamma_{21}\rho_{4s}[\int_0^a Q_1(x)E_2(2b\text{-}x)dx \tag{41}$$
$$+\frac{1}{2}\int_0^a \omega_1(x)E_2(2b\text{-}x)G_{01}(x)dx+\frac{\bar{a}}{2}\int_0^a \omega_1(x)E_3(2b\text{-}x)G_1(x)dx]+\frac{\gamma_{21}}{2}\int_a^b \omega_2(x)E_2(x)G_{02}(x)dx$$
$$+\frac{\bar{a}\gamma_{21}}{2}\int_a^b \omega_2(x)E_3(x)G_2(x)dx+\gamma_{21}\int_a^b Q_2(x)E_2(x)dx+\frac{\gamma_{21}\rho_{4s}}{2}(\int_a^b \omega_2(x)E_2(2b\text{-}x)G_{02}(x)dx$$
$$+\int_a^b Q_2(x)E_2(2b\text{-}x)dx+\bar{a}\int_a^b \omega_2(x)E_3(2b\text{-}x)G_2(x)dx+\gamma_{12}f_1\, E_3(2b))+2\gamma_{21}\rho_{4d}\,J_2^+(b)E_3(b)]$$

Also, the transmission coefficient is

$$T=(1-\rho_{4s}-\rho_{4d})\frac{\int_0^1 \mu \mathrm{I}(\mathrm{b},\mu)\mathrm{d}\mu}{\int_0^1 \mu \mathrm{I}(0,\mu)\mathrm{d}\mu}=(1-\rho_{4s}-\rho_{4d})\frac{2}{\mathrm{f}_1}\mathrm{J}_2^+(b) \tag{42}$$

$$T=\frac{2(1-\rho_{4s}-\rho_{4d})}{f_1}[2\rho_{3d}\,J_2^-(a)E_3(b-a)+\gamma_{12}f_1E_3(b)+\frac{1}{2}\int_{\mathrm{a}}^b \omega_2(x)E_2(b\text{-}x)G_{02}(x)dx$$
$$+\frac{1}{2}\int_a^b \omega_2(x)E_3(b\text{-}x)G_2(x)dx+\int_{\mathrm{a}}^b Q_2(x)E_2(b\text{-}x)dx+\frac{\gamma_{12}}{2}\int_0^a \omega_1(\mathrm{x})E_2(b\text{-}x)G_{01}(x)dx \tag{43}$$
$$+\frac{\gamma_{12}}{2}\int_0^a \omega_1(x)E_3(b\text{-}x)G_1(x)dx+\gamma_{12}\int_0^a Q_1(x)E_2(b\text{-}x)dx+\frac{\rho_{3s}}{2}(\int_a^b Q_2(x)E_2(b\text{-}2a+x)dx$$
$$+\int_a^b \omega_2(x)E_2(b\text{-}2a+x)G_2(x)dx-\bar{a}\int_a^b \omega_2(x)E_3(b\text{-}2a+x)G_2(x)dx)]$$

2. SOLUTION METHOD AND CALCULATIONS

In the Galerkin method the irradiance $G_{01}(x), G_{02}(x),$ and the net flux $G_1(x), G_2(x)$ in terms of optical distance are expressed by the expansion forms as

$$G_{01}(x)=\sum_{n=0}^{\mathrm{N}} A_n x^n, \qquad G_1(x)=\sum_{n=0}^{\mathrm{N}} B_n x^n,$$
$$G_{02}(x)=\sum_{n=0}^{\mathrm{N}} C_n x^n \quad \text{and} \quad G_2(x)=\sum_{n=0}^{\mathrm{N}} D_n x^n \tag{45}$$

In this worke, we will take the single scattering albedo $\omega_1(x)$ and $\omega_2(x)$ as. [11, 14]

$$\omega_1(x)=\omega_{01}\mathrm{e}^{-\alpha x} \quad \text{and} \quad \omega(x)=\omega_{02}\mathrm{e}^{-\alpha x}. \tag{46}$$

To find the unknown coefficients $A_n, B_n,$ C_n, and D_n, substitute Eqs. (45) and (46) into Eqs.(24-27), multiplying the resultant equations by x^m, $\mathrm{m}=0,1,...,$ N and integrating over $x\in(0,a)$ for Eqs.(24-25) and $x\in(a,b)$ for Eqs.(26-27) respectively. These lead to

$$\sum_{n=0}^{N} A_n H_{nm}^{11} - \bar{a}\sum_{n=0}^{N} B_n H_{nm}^{12} - \sum_{n=0}^{N} C_n H_{nm}^{13} - \bar{a}\sum_{n=0}^{N} D_n H_{nm}^{14} = \mathrm{Q}_{\mathrm{m}}^{1} \quad (47)$$

$$-\sum_{n=0}^{N} A_n H_{nm}^{21} + \sum_{n=0}^{N} B_n H_{nm}^{22} - \sum_{n=0}^{N} C_n H_{nm}^{23} - \bar{a}\sum_{n=0}^{N} D_n H_{nm}^{24} = \mathrm{Q}_{\mathrm{m}}^{2} \quad (48)$$

$$-\sum_{n=0}^{N} A_n H_{nm}^{31} - \bar{a}\sum_{n=0}^{N} B_n H_{nm}^{32} + \sum_{n=0}^{N} C_n H_{nm}^{33} - \bar{a}\sum_{n=0}^{N} D_n H_{nm}^{34} = \mathrm{Q}_{\mathrm{m}}^{3} \quad (49)$$

and

$$-\sum_{n=0}^{N} A_n H_{nm}^{41} - \bar{a}\sum_{n=0}^{N} B_n H_{nm}^{42} - \sum_{n=0}^{N} C_n H_{nm}^{43} + \sum_{n=0}^{N} D_n H_{nm}^{44} = \mathrm{Q}_{\mathrm{m}}^{4} \quad (50)$$

with

$$\begin{aligned} H_{nm}^{11} = {} & \int_0^a x^{n+m} dx - \frac{\omega_{01}}{2}\int_0^a x^{\mathrm{m}} \int_0^a \mathrm{e}^{-\alpha x} x'^{\mathrm{n}} M_0^1(x,x') dx dx' \\ & - \omega_{01}\rho_{2d}\int_0^a x^m E_2(a-x)\int_0^a \mathrm{e}^{-\alpha x} x'^{\mathrm{n}} E_2(a-x') dx dx' \\ & - \omega_{01}\gamma_{12}\gamma_{21}\rho_{4d}\int_0^a x^m E_2(b-x)\int_0^a \mathrm{e}^{-\alpha x} x'^{n} E_2(b-x') dx dx' \end{aligned} \quad (51)$$

$$\begin{aligned} H_{nm}^{12} = {} & \frac{\omega_{01}}{2}\int_0^a x^m \int_0^a e^{-\alpha x} x'^n M_0^2(x,x') dx dx' \\ & + \omega_{01}\rho_{2d}\int_0^a x^m E_2(a-x)\int_0^a e^{-\alpha x} x'^n E_3(a-x') dx dx' \\ & + \omega_{01}\gamma_{12}\gamma_{21}\rho_{4d}\int_0^a x^m E_2(b-x)\int_0^a e^{-\alpha x} x'^n E_3(b-x') dx dx' \end{aligned} \quad (52)$$

$$\begin{aligned} H_{nm}^{13} = {} & \frac{\omega_{02}}{2}\int_0^a x^m \int_a^b e^{-\alpha x'} x'^n K_0^1(x,x') dx dx' \\ & + \gamma_{21}\omega_{02}\rho_{4d}\int_0^a x^m E_2(b-x)\int_a^b e^{-\alpha x'} x'^n E_2(b-x') dx dx' \end{aligned} \quad (53)$$

$$\begin{aligned} H_{nm}^{14} = {} & \frac{\omega_{02}}{2}\int_0^a x^m \int_a^b e^{-\alpha x'} x'^n K_0^2(x,x') dx dx' \\ & + \gamma_{21}\omega_{02}\rho_{4d}\int_0^a x^m E_2(b-x)\int_a^b e^{-\alpha x'} x'^n E_3(b-x') dx dx' \end{aligned} \quad (54)$$

$$\begin{aligned} H_{nm}^{21} = {} & \frac{\omega_{01}}{2}\int_0^a x^m \int_0^a e^{-\alpha x} x'^n M_1^1(x,x') dx dx' \\ & - \omega_{01}\rho_{2d}\int_0^a x^m E_3(a-x)\int_0^a e^{-\alpha x''} x'^n E_2(a-x') dx dx' \\ & - \omega_{01}\gamma_{21}\rho_{4d}\int_0^a x^m E_3(b-x)\int_0^a e^{-\alpha x'} x'^n E_2(b-x') dx dx' \end{aligned} \quad (55)$$

$$\begin{aligned} H_{nm}^{22} = {} & \int_0^a x^{n+m} dx - \frac{\bar{a}\omega_{01}}{2}\int_0^a x^m \int_0^a e^{-\alpha x'} x'^n M_1^2(x,x') dx dx' \\ & + \bar{a}\omega_{01}\rho_{2d}\int_0^a x^m E_3(a-x)\int_0^a e^{-\alpha x'} x'^n E_3(a-x') dx dx' \\ & + \bar{a}\omega_{01}\gamma_{12}\gamma_{21}\rho_{4d}\int_0^a x^m E_3(b-x)\int_0^a e^{-\alpha x'} x'^n E_3(b-x') dx dx' \end{aligned} \quad (56)$$

$$H_{nm}^{23} = \frac{\omega_{02}}{2}\int_0^a x^m \int_a^b e^{-\alpha x'} x'^n K_1^1(x,x')dxdx' - \gamma_{21}\omega_{02}\rho_{4d}\int_0^a x^m E_3(b-x)\int_a^b e^{-\alpha x'} x'^n E_2(b\text{-}x')dxdx' \tag{57}$$

$$H_{nm}^{24} = \frac{\omega_{02}}{2}\int_0^a x^m \int_a^b e^{-\alpha x'} x'^n K_1^2(x,x')dxdx' - \gamma_{21}\omega_{02}\rho_{4d}\int_0^a x^m E_3(b-x)\int_a^b e^{-\alpha x'} x'^n E_3(b\text{-}x')dxdx' \tag{58}$$

$$H_{nm}^{31} = \frac{\omega_{01}}{2}\int_a^b x^m \int_0^a e^{-\alpha x'} x'^n M_2^1(x,x')dxdx' + \omega_{01}\gamma_{12}\gamma_{21}\rho_{4d}\int_a^b x^m E_2(b-x)\int_0^a e^{-\alpha x'} x'^n E_2(b\text{-}x')dxdx' \tag{59}$$

$$H_{nm}^{32} = \frac{\omega_{01}}{2}\int_a^b x^m \int_0^a e^{-\alpha x'} x'^n M_2^2(x,x')dxdx' + \omega_{01}\gamma_{12}\gamma_{21}\rho_{4d}\int_a^b x^m E_2(b-x)\int_0^a e^{-\alpha x'} x'^n E_3(b\text{-}x')dxdx' \tag{60}$$

$$H_{nm}^{33} = \int_a^b x^{n+m}dx - \frac{\omega_{02}}{2}\int_a^b x^m \int_a^b e^{-\alpha x'} x'^n K_2^1(x,x')dxdx' - \omega_{02}\rho_{3d}\int_a^b x^m E_2(x-a)\int_a^b e^{-\alpha x'} x'^n E_2(x'\text{-}a)dxdx' - \omega_{02}\rho_{4d}\int_a^b x^m E_2(b-x)\int_a^b e^{-\alpha x'} x'^n E_2(b\text{-}x')dxdx' \tag{61}$$

$$H_{nm}^{34} = \frac{\omega_{02}}{2}\int_a^b x^m \int_a^b e^{-\alpha x'} x'^n K_2^2(x,x')dxdx' - \omega_{02}\rho_{3d}\int_a^b x^m E_2(x-a)\int_a^b e^{-\alpha x'} x'^n E_3(x'\text{-}a)dxdx' + \omega_{02}\rho_{4d}\int_a^b x^m E_2(b-x)\int_a^b e^{-\alpha x'} x'^n E_3(b\text{-}x')dxdx' \tag{62}$$

$$H_{nm}^{41} = \frac{\omega_{01}}{2}\int_a^b x^m \int_0^a e^{-\alpha x'} x'^n M_3^1(x,x')dxdx' - \omega_{01}\rho_{4d}\int_a^b x^m E_3(b-x)\int_0^a e^{-\alpha x'} x'^n E_2(b\text{-}x')dxdx' \tag{63}$$

$$H_{nm}^{42} = \frac{\omega_{01}}{2}\int_a^b x^m \int_0^a e^{-\alpha x'} x'^n M_3^2(x,x')dxdx' - \omega_{01}\rho_{4d}\int_a^b x^m E_3(b-x)\int_0^a e^{-\alpha x'} x'^n E_3(b\text{-}x')dxdx' \tag{64}$$

$$H_{nm}^{43} = \frac{\omega_{02}}{2}\int_a^b x^m \int_a^b e^{-\alpha x'} x'^n K_3^1(x,x')dxdx' + \omega_{02}\rho_{3d}\int_a^b x^m E_3(x-a)\int_a^b e^{-\alpha x'} x'^n E_2(x'\text{-}a)dxdx' - \omega_{02}\rho_{4d}\int_a^b x^m E_3(b-x)\int_a^b e^{-\alpha x'} x'^n E_2(b\text{-}x')dxdx' \tag{65}$$

$$H_{nm}^{44} = \int_a^b x^{n+m} dx - \frac{\bar{a}\omega_{02}}{2} \int_a^b x^m \int_a^b e^{-\alpha x'} x'^n K_3^2(x,x') dx dx'$$
$$+ \bar{a}\omega_{02}\rho_{3d} \int_a^b x^m E_3(x-a) \int_a^b e^{-\alpha x'} x'^n E_3(x'-a) dx dx'$$
$$+ \bar{a}\omega_{02}\rho_{4d} \int_a^b x^m E_3(b-x) \int_a^b e^{-\alpha x'} x'^n E_3(b-x') dx dx' \quad (66)$$

$$Q_m^l = \int_0^a x^m H_l(x) dx, \qquad l = 0,1 \quad (67a)$$

and

$$Q_m^k = \int_a^b x^m H_k(x) dx, \qquad k = 2,3 \quad (67b)$$

where

$$H_0(x) = f_1 E_2(x) + \gamma_{12}\gamma_{21}\rho_{4s} f_1 E_2(2b-x) + \rho_{2s} f_1 E_2(2a-x) + 2\rho_{2d} f_1 E_3(a) E_2(a-x)$$
$$+ 2\gamma_{12}\gamma_{21}\rho_{4d} f_1 E_3(b) E_2(b-x) + \int_0^a Q_1(x') M_0^1(x,x') dx' + \int_a^b Q_2(x') K_0^1(x,x') dx'$$
$$+ 2\gamma_{21}\rho_{4d} E_2(b-x)\{\gamma_{12}\int_0^a Q_1(x') E_2(b-x) dx' + \int_a^b Q_2(x') E_2(b-x) dx'\}$$
$$+ 2\rho_{2d} E_2(a-x) \int_0^a Q_1(x') E_2(a-x) dx' \quad (68)$$

$$H_1(x) = f_1 E_3(x) - \rho_{4s} f_1 E_3(2b-x) - \rho_{2s} f_1 E_3(2a-x) + 2\rho_{2d} f_1 E_3(a) E_3(a-x)$$
$$- 2\rho_{4d} f_1 E_3(b) E_3(b-x) - \int_0^a Q_1(x') M_1^1(x,x') dx' - \int_a^b Q_2(x') K_1^1(x,x') dx'$$
$$- 2\gamma_{21}\rho_{4d} E_3(b-x)\{\gamma_{12}\int_0^a Q_1(x') E_2(b-x') dx' + \int_a^b Q_2(x') E_2(b-x') dx'\}$$
$$+ 2\rho_{2d} E_3(a-x) \int_0^a Q_1(x') E_2(a-x') dx' \quad (69)$$

$$H_2(x) = \gamma_{12} f_1 E_2(x) + \gamma_{12}\rho_{4s} f_1 E_2(2b-x) + 2\rho_{4d} f_1 E_3(b) E_2(b-x) + \int_0^a Q_1(x') M_2^1(x,x') dx'$$
$$+ \int_a^b Q_2(x') K_2^1(x,x') dx' + 2\rho_{3d} E_2(x-a) \int_a^b Q_2(x') E_2(x'-a) dx'$$
$$+ 2\rho_{4d} E_2(b-x)\{\gamma_{12}\int_0^a Q_1(x') E_2(b-x') dx' + \int_a^b Q_2(x') E_2(b-x') dx'\} \quad (70)$$

$$H_3(x) = \gamma_{12} f_1 E_3(x) - \gamma_{12}\rho_{4s} f_1 E_3(2b-x) - 2\rho_{4d} f_1 E_3(b) E_3(b-x) + \int_0^a Q_1(x') M_2^1(x,x') dx'$$
$$+ \int_a^b Q_2(x') K_2^1(x,x') dx' + 2\rho_{3d} E_3(x-a) \int_a^b Q_2(x') E_2(x'-a) dx'$$
$$- 2\rho_{4d} E_3(b-x)\{\gamma_{12}\int_0^a Q_1(x') E_2(b-x') dx' + \int_a^b Q_2(x') E_2(b-x') dx'\} \quad (71)$$

The simultaneous solution of the set of Eqs. (47-50) and (40) will give the unknown coefficients $A_n, B_n,$ C_n, and D_n which are used to calculate the physical quantities reflection and transmission.

3. RESULTS AND DISCUSSION

The coefficients of reflection R and transmission T with different optical thickness a, b and different values of single scattering albedo ω_{01} and ω_{02} are calculated. The calculations are carried out for two-region inhomogeneous media with anisotropic scattering by using five terms of expansions (45).

We present in Table 1 the effects of interchange of the values of the single scattering albedo ω_{01} and ω_{02} on the R and T for inhomogeneous slab of thickness $a = 2, b - a = 2$ with linear anisotropic scattering. The reflection and transmission are higher with single scattering albedo w_{01} than w_{02}.

Table 2 shows the effect of interchange of thickness with interchange of ω_{01} and ω_{02} on R and T for inhomogeneous slab with thickness $a = 1, b - a = 2$ and $a = 2, b - a = 1$ respectively. The R, and T is influenced not only by the magnitude of ω_{01} and ω_{02} but also by the optical thicknesses. The calculations in table 1 and 2 are carried out for interface transmission with reflecting coefficients $\rho_{is} = \rho_{id} = 0$, i=2, 3, 4.

Tables 3, 4, 5 and 6 show the effects of interchange of specular and diffuse reflecting boundaries on reflection and transmission with thickness $a = 1, b - a = 1$ and $a = 1, b - a = 2$ respectively. The calculations are carried out for different values of interface transmission γ_{ij}, diffuse and specular ρ_{id}, ρ_{is} reflecting boundaries and for different values of single scattering albedo ω_{01} and ω_{02}.

The numerical results of the reflectivity of an inhomogeneous slab of isotropic scattering is slightly higher with the specularly reflecting boundary than with the diffusely reflecting boundary at $x = b$ with interchange single scattering albedo ω_{0s} and reflecting boundaries ρ_i. Physically, these results can be explained by noting that, the radiant energy undergoes more multiple reflections in a system with specularly reflecting than in a system with diffusely reflecting. While the transmissivity for the same case is kindly higher with the diffusely reflecting boundary than with the specularly reflecting boundary with interchange ω_{0s} and different values of ρ_4. But the numerical results of the transmissivity are more higher with the specularly reflecting boundary than with the diffusely reflecting boundary at $x = b$ with interchange single scattering albedo ω_{0s} and reflecting boundaries ρ_2, ρ_3.

All the calculations of Tables 1, 2, 3, 4, 5 and 6 are carried out for diffuse surface sources f_1 =1and has no internal energy source and are given for isotropic, forward and backward anisotropic scattering. The medium of forward anisotropic scattering has reflecting index n=1.2and spherical particles of size parameter x=2 while the medium backward anisotropic scattering has reflecting index n=∞ and spherical particles of size parameter x=1 [13]. The Legendre expansion coefficients [13] of the above two media are taken in Eq. (3b) to give $\bar{a}$ =1.81517 and $\bar{a}$ =-0.58659 for the forward and backward linear anisotropic scattering respectively. The results are compared in case isotropic scattering with those of Ref. [11], which show good agreement

In Table 7, we calculate R., T of the same as of Tables 1,2 but with internal energy source $Q_2 = 1$. Also, Tables 8, 9 show the effect of interchange of specular and diffuse reflection on transmissivity and reflectivity of an inhomogeneous slab for the internal source $Q_2 = 1$ and ω_{01}=2 with interchange of ω_{02} for the optical thickness $a = 1, b - a = 1$ and $a = 1, b - a = 2$ respectively. The calculations of Tables 7, 8 and 9 are given for isotropic scattering medium.

Table 1: The reflectivity and transmissivity for inhomogeneous slabs with $a = 2, b - a = 2$ and $\rho_{is} = \rho_{id} = 0$.

ω_{01}	ω_{02}	α	Reflectivity				Transmissivity			
			Isotropic scattering	Ref[11]	Forward scattering	Backward scattering	Isotropic scattering	Ref[11]	Forward scattering	Backward scattering
0.2	0.2	0.1	0.0441088	0.0441088	0.0723886	0.0356493	0.0069919	0.0069919	0.0088472	0.0065288
		0.01	0.0459902	0.0459904	0.0752329	0.0372339	0.0073375	0.0073375	0.0962223	0.0067506
		0.001	0.0461372	0.0461372	0.0754726	0.0373566	0.0073813	0.0073813	0.0972781	0.0067809
0.2	0.5	0.1	0.0443930	0.0443930	0.0724082	0.0359940	0.0085662	0.0085662	0.0125978	0.0076469
		0.01	0.0464001	0.0464001	0.0752940	0.0377234	0.0099205	0.0099205	0.0158167	0.0864481
		0.001	0.0465643	0.0465643	0.0755401	0.0378651	0.0101122	0.0101122	0.0162747	0.0087900
0.2	0.8	0.1	0.0448329	0.0448329	0.0725382	0.0364918	0.0113798	0.0113798	0.0191279	0.0097981
		0.01	0.0471896	0.0471896	0.0756487	0.0385824	0.0162460	0.0162460	0.0296624	0.0137189
		0.001	0.0474137	0.0474137	0.0759381	0.0364701	0.0170995	0.0170995	0.0313998	0.0144271
0.5	0.2	0.1	0.1372299	0.1372299	0.2135822	0.1167163	0.0091935	0.0091935	0.0139361	0.0081416
		0.01	0.1445848	0.1445848	0.2236782	0.1235230	0.0101225	0.0101225	0.0160256	0.0087234
		0.001	0.1452935	0.1452935	0.2246652	0.1241814	0.0101343	0.0101343	0.0215629	0.0087983
	0.5	0.1	0.1378199	0.1378199	0.2136753	0.1174063	0.0114195	0.0114195	0.0201460	0.0096684
		0.01	0.1455211	0.1455211	0.2238937	0.1245908	0.0138374	0.0138374	0.0268884	0.0114199
		0.001	0.1462806	0.1462806	0.2249010	0.1253035	0.0141834	0.0141834	0.0278525	0.0116738
	0.8	0.1	0.1387402	0.1387402	0.2139968	0.1184159	0.0154441	0.0154441	0.0310593	0.0126291
		0.01	0.1473506	0.1473506	0.2247805	0.1265215	0.0234286	0.0234286	0.0516124	0. 187930
		0.001	0.1482739	0.1482739	0.2259000	0.1273980	0.0248585	0.0248585	0.0550677	0.0199251
0.8	0.2	0.1	0.2984574	0.2984574	0.4243809	0.2682727	0.0140795	0.0140795	0.0245079	0.0120399
		0.01	0.3259324	0.3259324	0.4540355	0.2957942	0.0167979	0.0167979	0.0306912	0.0141903
		0.001	0.3291230	0.3291230	0.4573594	0.2990141	0.0171598	0.0171598	0.0315110	0.0144788
	0.5	0.1	0.3001582	0.3001582	0.4248199	0.2701754	0.017820	0.017820	0.0359805	0.0145938
		0.01	0.3293239	0.3293239	0.4551705	0.2994975	0.0239957	0.0239957	0.0527110	0.0192713
		0.001	0.3327935	0.3327935	0.4586186	0.3030131	0.0249218	0.0249218	0.0551888	0.0199786
	0.8	0.1	0.3028552	0.3028552	0.4259656	0.2730320	0.0247219	0.0247219	0.0563931	0.0196160
		0.01	0.3362004	0.3362004	0.4588914	0.3065780	0.0429079	0.0429079	0.1042726	0.0337201
		0.001	0.3405153	0.3405153	0.4628911	0.3109461	0.0463607	0.0463607	0.1127050	0.0364644

Table 2: Effect of interchange of the thickness with interchange of ω_{01} and ω_{02} on transmissivity and reflectivity of the inhomogeneous slab with $\rho_{is} = \rho_{id} = 0$.

a	$b-a$	ω_{01}	ω_{02}	α	Reflectivity				Transmissivity			
					Isotropic scattering	Ref[11]	Forward scattering	Backward scattering	Isotropic scattering	Ref[11]	Forward scattering	Backward scattering
1.0	2.0	0.2	0.5	0.1	0.0486445	0.0486445	0.07098615	0.04181997	0.02812497	0.02822497	0.0404655	0.0253667
				0.01	0.0511159	0.0511159	0.07271137	0.0443006	0.03144749	0.03148748	0.0475282	0.0278447
				0.001	0.0513799	0.0513799	0.07295170	0.0445634	0.03186119	0.03190119	0.0484310	0.0281686
			0.8	0.1	0.0553512	0.0553512	0. 07374854	0.04930179	0.04047170	0.04057170	0.0668020	0.0352645
				0.01	0.0608348	0.0608348	0. 07761407	0.0548455	0.05346264	0.05350263	0.0919878	0.0459723
				0.001	0.0615281	0.0615281	0. 07818209	0.0555428	0.05539043	0.05543043	0.0955778	0.0476175
		0.8	0.2	0.1	0.2703228	0.2703228	0.3849063	0.2420696	0.03247788	0.03257788	0.04735871	0.0292104
				0.01	0.2855958	0.2855958	0.4015226	0.2570117	0.03464696	0.03468696	0.0517634	0.0308355
				0.001	0.2872243	0.2872243	0.4033309	0.2586330	0.03489655	0.03493655	0.0523038	0.0310305
			0.5	0.1	0.2822193	0.2822193	0.3884490	0.2559142	0.04294720	0.04304720	0.0737979	0.0370308
				0.01	0.3016663	0.3016663	0.4068647	0.2754418	0.05009108	0.05013108	0.0902037	0.0423641
				0.001	0.3038155	0.3038155	0.4089073	0.2776355	0.05099201	0.05103201	0.0923315	0.0430679
2.0	1.0	0.2	0.5	0.1	0.0443438	0.0443438	0.0723899	0.0359294	0.02478067	0.02482067	0.0327809	0.0228581
				0.01	0.04632765	0.0463277	0.0752805	0.0376423	0.02681945	0.02685945	0.0368908	0.0243601
				0.001	0.0464870	0.0464870	0.0755245	0.0377792	0.02706952	0.02710952	0.0374167	0.0245543
			0.8	0.1	0.0447025	0.0447025	0.0724864	0.0363537	0.02913069	0.02917069	0.415601	0.0263360
				0.01	0.0469121	0.0469121	0.0754945	0.0383001	0.03414981	0.03418981	0.0509332	0.0303997
				0.001	0.0471043	0.0471043	0.0757581	0.0384716	0.03484450	0.03488450	0.0522173	0.0309848
		0.8	0.2	0.1	0.2983745	0.2983745	0.4243776	0.2682093	0.04595306	0.04599306	0.0768841	0.0398961
				0.01	0.3258377	0.3258377	0.4540374	0.2956835	0.05441349	0.05445348	0.0936412	0.0468019
				0.001	0.3290209	0.3290209	0.7573607	0.2988952	0.05549198	0.05553198	0.0956527	0.0477063
			0.5	0.1	0.2998651	0.2998651	0.4247864	0.2699565	0.05317630	0.05321629	0.0966016	0.0452779
				0.01	0.3288455	0.3288455	0.4550341	0.2989989	0.06717705	0.06721705	0.1265945	0.0563518
				0.001	0.3322639	0.3322639	0.4584590	0.3024633	0.06906196	0.06910196	0.1306033	0.0578971

Table 3: Effect of interchange of specular and diffuse reflection on reflectivity of the inhomogeneous slab for the optical thickness $a = 1, b - a = 1$ and ω_{01}=2 with interchange of ω_{02}.

ω_{02}	ρ_2	ρ_3	ρ_4	α	Reflectivity with specular reflecting boundary				Reflectivity with Diffusely reflecting boundary			
					Isotropic scattering	Ref[11]	Forward scattering	Backward scattering	Isotropic scattering	Ref[11]	Forward scattering	Backward scattering
0.5	0.0	0.0	0.5	0.1	0.0522280	0.0522282	0.0755729	0.0453752	0.0514552	0.0514552	0.07614293	0.04454865
				0.01	0.0553335	0.0553335	0.0791606	0.0480240	0.0543812	0.0543812	0.07905689	0.04696793
				0.001	0.0556106	0.055611	0.0794708	0.0482907	0.0546614	0.0546614	0.08822548	0.04723612
0.8	0.0	0.0	0.5	0.1	0.0598259	0.0598264	0.0821204	0.0543752	0.0590341	0.0590341	0.08214601	0.05262439
				0.01	0.0658444	0.0658444	0.0880812	0.0588902	0.0650627	0.0650627	0.08822548	0.05798600
				0.001	0.0665075	0.0665075	0.0887992	0.0595353	0.0657369	0.0657369	0.08897122	0.05863907
0.5	0.0	0.0	0.9	0.1	0.0558491	0.0558490	0.0766829	0.0463177	0.0546842	0.0546842	0.08018746	0.04756157
				0.01	0.0597137	0.0597137	0.0845741	0.0521251	0.0580823	0.0580823	0.0837355	0.05038808
				0.001	0.0600432	0.0600432	0.0849607	0.0524382	0.0584198	0.0584198	0.0841425	0.05070730
0.8	0.0	0.0	0.9	0.1	0.0648612	0.0648616	0.0887320	0.0575446	0.0647698	0.0647698	0.0898637	0.05787215
				0.01	0.0741481	0.0741481	0.0989461	0.0665419	0.0729819	0.0729819	0.0987923	0.06522991
				0.001	0.0751034	0.0751034	0.1000301	0.0674597	0.0739682	0.0739682	0.0999330	0.06617357
0.5	0.4	0.4	0.0	0.1	0.0645763	0.0645763	0.09314611	0.06577411	0.0689690	0.0689690	0.09500105	0.06146949
				0.01	0.0757323	0.0757323	0.1008339	0.0680287	0.0710226	0.0710226	0.09729368	0.06313859
				0.001	0.0759205	0.0759205	0.1010589	0.06820565	0.0712138	0.0712138	0.09753302	0.06331672
0.8	0.4	0.4	0.0	0.1	0.0665679	0.0665679	0.0940798	0.0684048	0.0712416	0.0712416	0.09582004	0.06420389
				0.01	0.0789474	0.0789474	0.1021222	0.07171796	0.0742302	0.0742302	0.09856347	0.06681493
				0.001	0.0792461	0.0792461	0.1024078	0.07201641	0.0745315	0.0745315	0.09886300	0.06711392
0.5	0.6	0.8	0.0	0.1	0.0776263	0.0776263	0.1002345	0.0774739	0.0799454	0.0799454	0.1101607	0.07107103
				0.01	0.0887349	0.0887349	0.1150180	0.0806689	0.0817282	0.0817282	0.1129576	0.07243818
				0.001	0.0889025	0.0898155	0.1152444	0.08081967	0.0819012	0.0819012	0.1132530	0.07258057
0.8	0.6	0.8	0.0	0.1	0.0781251	0.0781251	0.10018004	0.0774993	0.0805335	0.0805335	0.1126590	0.07144927
				0.01	0.0896135	0.0896135	0.11534018	0.0817072	0.0825992	0.0825992	0.1179205	0.07295192
				0.001	0.0898155	0.0898155	0.1155825	0.0818976	0.0828056	0.0828056	0.1186760	0.07311160
0.5	0.8	0.6	0.0	0.1	0.0918038	0.0918038	0.1171804	0.0925469	0.0926310	0.0926310	0.1222951	0.08388089
				0.01	0.1038623	0.1038623	0.1295483	0.09589534	0.0946111	0.0946111	0.1252150	0.08545380
				0.001	0.1040478	0.1040478	0.1297877	0.09606322	0.0948041	0.0948041	0.1255224	0.08561780
0.8	0.8	0.6	0.0	0.1	0.0922972	0.0922972	0.1173894	0.09290705	0.0931822	0.0931822	0.1243645	0.08424758
				0.01	0.1046718	0.1046718	0.1298237	0.0966631	0.0954189	0.0954189	0.1289196	0.08594670
				0.001	0.1048869	0.1048869	0.1300758	0.09685467	0.0956414	0.0956414	0.1295017	0.08612666

Table 4: Effect of interchange of specular and diffuse reflection on transmissivity of the inhomogeneous slab for the optical thickness $a = 1, b - a = 1$ and ω_{01} =2 with interchange of ω_{02}.

ω_{02}	ρ_2	ρ_3	ρ_4	α	transmissivity with specular reflecting boundary				Transmissivity with Diffusely reflecting boundary			
					Isotropic scattering	Ref[11]	Forward scattering	Backward scattering	Isotropic scattering	Ref[11]	Forward scattering	Backward scattering
0.5	0.0	0.0	0.5	0.1	0.0419687	0.0419689	0.04549981	0.0397273	0.0436629	0.0436629	0.0538074	0.0412656
				0.01	0.0463662	0.0463662	0.0574102	0.0434346	0.0466886	0.0466886	0.0580805	0.0436534
				0.001	0.0466862	0.0466862	0.0579062	0.0437127	0.0470158	0.0470158	0.0585931	0.0439372
0.8	0.0	0.0	0.5	0.1	0.0551149	0.0551156	0.06899632	0.0499325	0.0567334	0.0567334	0.0734440	0.0528139
				0.01	0.0650330	0.0650330	0.0841178	0.0602174	0.0656486	0.0656486	0.0854659	0.0606970
				0.001	0.0661348	0.0661348	0.0855882	0.0612311	0.0667653	0.0667653	0.0869748	0.0617242
0.5	0.0	0.0	0.9	0.1	0.0086570	0.0086570	0.00854750	0.0082712	0.0091535	0.0091535	0.0109605	0.0087279
				0.01	0.0097891	0.0097891	0.00976750	0.0092699	0.0099089	0.0099089	0.0118702	0.0093573
				0.001	0.0098699	0.0098699	0.00978853	0.0093477	0.0099924	0.0099924	0.0119830	0.0094334
0.8	0.0	0.0	0.9	0.1	0.0116589	0.0116589	0.01171714	0.0110090	0.0125461	0.0125461	0.0153496	0.0118716
				0.01	0.0148508	0.0148508	0.01447829	0.0140426	0.0150789	0.0150789	0.0181971	0.0142326
				0.001	0.0151779	0.0151779	0.01486357	0.0143591	0.0154108	0.0154108	0.0185664	0.0145541
0.5	0.4	0.4	0.0	0.1	0.0481370	0.0481370	0.0646591	0.0478005	0.0517957	0.0517957	0.0640005	0.0490096
				0.01	0.0549028	0.0549028	0.0682967	0.0513666	0.0551483	0.0568006	0.0690296	0.0515183
				0.001	0.0552454	0.0552454	0.0687842	0.0516574	0.0554945	0.0572676	0.0696083	0.0518112
0.8	0.4	0.4	0.0	0.1	0.0608080	0.0608080	0.0910124	0.0606401	0.0660887	0.0668055	0.0863478	0.0614733
				0.01	0.0749258	0.0749258	0.0982459	0.0691168	0.0753493	0.0762719	0.0995372	0.0694028
				0.001	0.0760372	0.0760372	0.09983239	0.0701222	0.0764682	0.0774150	1.011519	0.0704126
0.5	0.6	0.8	0.0	0.1	0.0167373	0.0167373	0.0185996	0.0165718	0.0180005	0.0184767	0.0263076	0.0163174
				0.01	0.0191969	0.0191969	0.0229769	0.0181523	0.0193544	0.0191641	0.0301346	0.0171002
				0.001	0.0193336	0.0193336	0.0231748	0.0182735	0.0194934	0.0200930	0.0306057	0.0171895
0.8	0.6	0.8	0.0	0.1	0.0218601	0.0218601	0.0270818	0.0225443	0.0240611	0.0239282	0.0535645	0.0200194
				0.01	0.0279286	0.0279286	0.0340936	0.0262627	0.0281995	0.0280231	0.0862848	0.0222518
				0.001	0.0284351	0.0284351	0.0347031	0.0267416	0.0287101	0.0285284	0.0922048	0.0225284
0.5	0.8	0.6	0.0	0.1	0.0332149	0.0332149	0.0352775	0.0327292	0.0355518	0.0378954	0.0504738	0.0324595
				0.01	0.0378177	0.0378177	0.0456619	0.0356450	0.0381001	0.0409877	0.0571779	0.0356550
				0.001	0.0380725	0.0380725	0.0461499	0.0358746	0.0383592	0.0413010	0.0579939	0.0357735
0.8	0.8	0.6	0.0	0.1	0.0428698	0.0428698	0.4491636	0.0425517	0.0463886	0.0484243	0.0693492	0.0392171
				0.01	0.0531843	0.0531843	0.0668799	0.0496097	0.0536679	0.0563304	0.0781938	0.0432783
				0.001	0.0540573	0.0540573	0.0680168	0.0504179	0.0545498	0.0572895	0.0792646	0.0437776

Table 5: Effect of interchange of specular and diffuse reflection on reflectivity of the inhomogeneous slab for the optical thickness $a=1, b-a=2$ and ω_{01}=2 with interchange of ω_{02}.

ω_{02}	ρ_2	ρ_3	ρ_4	α	Reflectivity with specular reflecting boundary				Reflectivity with Diffusely reflecting boundary			
					Isotropic scattering	Ref[11]	Forward scattering	Backward scattering	Isotropic scattering	Ref[11]	Forward scattering	Backward scattering
0.5	0.0	0.0	0.5	0.1	0.0489195	0.0486607	0.07119292	0.0447619	0.0490690	0.0490662	0.0873891	0.0381514
				0.01	0.0518247	0.0515261	0.0764108	0.0473660	0.0516476	0.0516489	0.0936152	0.0398999
				0.001	0.0521047	0.0518057	0.0766911	0.0476404	0.0519271	0.0519285	0.0943667	0.0400924
0.8	0.0	0.0	0.5	0.1	0.0558121	0.0555562	0.0770589	0.0520769	0.0562706	0.0562725	0.1140787	0.0426815
				0.01	0.0627271	0.0624215	0.0843192	0.058149	0.0625279	0.0625304	0.1411277	0.0463572
				0.001	0.0635611	0.063255	0.0852210	0.0596232	0.0633626	0.0633653	0.1511695	0.0468220
0.5	0.0	0.0	0.9	0.1	0.0492840	0.0488291	0.0740913	0.0475596	0.0494348	0.0495209	0.0879027	0.0384933
				0.01	0.0523663	0.0519050	0.0793877	0.0498706	0.0521342	0.0521368	0.0942122	0.0403428
				0.001	0.0526624	0.0522008	0.0797023	0.0501579	0.0524305	0.0524331	0.1013633	0.0405488
0.8	0.0	0.0	0.9	0.1	0.0566418	0.0561849	0.0737151	0.0564432	0.0571690	0.0572551	0.1156703	0.0434174
				0.01	0.0646405	0.0641713	0.0901130	0.0624333	0.0644109	0.0644163	0.1435530	0.0476949
				0.001	0.0656665	0.0651967	0.0913656	0.0634029	0.0654438	0.0654494	0.1542169	0.0482677
0.5	0.4	0.4	0.0	0.1	0.0645492	0.0645492	0.0889655	0.0635279	0.0693500	0.0693500	0.1024987	0.0597291
				0.01	0.0760535	0.0760535	0.1009098	0.0683920	0.0713464	0.0713464	0.1066059	0.0611884
				0.001	0.0762562	0.0762562	0.1011413	0.0685842	0.0715521	0.0715521	0.1070823	0.0613435
0.8	0.4	0.4	0.0	0.1	0.0665277	0.0665277	0.0901473	0.0650110	0.0722295	0.0722295	0.1160327	0.0614701
				0.01	0.0803883	0.0803883	0.1028851	0.0732272	0.0756841	0.0756841	0.1366147	0.0636428
				0.001	0.0808047	0.0808047	0.1032532	0.0736450	0.0761041	0.0761041	0.1398064	0.0638954
0.5	0.6	0.8	0.0	0.1	0.0775549	0.0775549	0.1087382	0.0728891	0.0800452	0.0800452	0.1103841	0.0710979
				0.01	0.0888141	0.0888141	0.1150356	0.0807604	0.0818090	0.0818090	0.1133634	0.0724798
				0.001	0.0889855	0.0889855	0.1152635	0.0809153	0.0819854	0.0819854	0.1142649	0.0726240
0.8	0.6	0.8	0.0	0.1	0.0780080	0.0780080	0.0996860	0.0736206	0.0808096	0.0808096	0.1154748	0.0715411
				0.01	0.0896135	0.0896135	0.1155295	0.0821579	0.0830191	0.0830191	0.1234895	0.0731226
				0.001	0.0902670	0.0902670	0.1157931	0.0823887	0.0832638	0.0832638	0.1243974	0.0732947
0.5	0.8	0.6	0.0	0.1	0.0917435	0.0917435	0.1173677	0.0871400	0.0927339	0.0927339	0.1224994	0.0839087
				0.01	0.1039389	0.1039389	0.1295722	0.0960317	0.0946888	0.0946888	0.1255762	0.0854960
				0.001	0.1041279	0.1041279	0.1298133	0.0962051	0.0948853	0.0948853	0.1259088	0.0856618
0.8	0.8	0.6	0.0	0.1	0.0921966	0.0921966	0.1190417	0.0878648	0.0934496	0.0934496	0.1263420	0.0843374
				0.01	0.1050406	0.1050406	0.1300414	0.0972848	0.0957931	0.0957931	0.1480209	0.0861080
				0.001	0.1052874	0.1052874	0.1303156	0.0975211	0.0960478	0.0960478	0.1644305	0.0862993

Table 6: Effect of interchange of specular and diffuse reflection on transmissivity of the inhomogeneous slab for the optical thickness

$a = 1, b - a = 2$ and ω_{01}=2 with interchange of ω_{02}.

ω_{02}	ρ_2	ρ_3	ρ_4	α	transmissivity with specular reflecting boundary				Transmissivity with Diffusely reflecting boundary			
					Isotropic scattering	Ref[11]	Forward scattering	Backward scattering	Isotropic scattering	Ref[11]	Forward scattering	Backward scattering
0.5	0.0	0.0	0.5	0.1	0.0124870	0.0124985	0.0188565	0.0113778	0.0148047	0.0148257	0.0235576	0.0130099
				0.01	0.0167921	0.0168126	0.0242476	0.0150523	0.0169064	0.0169279	0.0302476	0.0143058
				0.001	0.0170552	0.0170757	0.0247373	0.0152696	0.0171738	0.0171954	0.0312135	0.0144772
0.8	0.0	0.0	0.5	0.1	0.0179319	0.0179491	0.0252713	0.0132269	0.0225007	0.0225229	0.0415739	0.0177234
				0.01	0.0314210	0.0314411	0.0396434	0.0275546	0.0316695	0.0316932	0.0522727	0.0226275
				0.001	0.0328590	0.0328791	0.0418941	0.0288238	0.0331196	0.0331435	0.0538596	0.0233693
0.5	0.0	0.0	0.9	0.1	0.0024392	0.0024436	0.0038777	0.0020395	0.0309147	0.0310713	0.0479059	0.0273658
				0.01	0.0035550	0.0035591	0.0049349	0.0032225	0.0359776	0.0360234	0.0619190	0.0306911
				0.001	0.0036190	0.0036681	0.0050399	0.0032776	0.0366349	0.0366809	0.0667991	0.0311394
0.8	0.0	0.0	0.9	0.1	0.0037387	0.0037458	0.0096868	0.0032575	0. 0494269	0. 0496577	0.0963744	0.0393305
				0.01	0.0073374	0.0073412	0.0106455	0.0065760	0. 0743230	0. 0743786	0.1299474	0.0534656
				0.001	0.0077553	0.0077592	0.0111949	0.00696011	0. 0785462	0. 0786030	0.1344678	0.0557437
0.5	0.4	0.4	0.0	0.1	0.0156145	0.0156386	0.0262592	0.0147218	0.0201037	0.0202769	0.0328105	0.0165822
				0.01	0.0198463	0.0198703	0.0287751	0.0177609	0.0226727	0.0231084	0.0428267	0.0177309
				0.001	0.0201269	0.0201509	0.0293335	0.0179868	0.0229939	0.0234560	0.0442528	0.0178778
0.8	0.4	0.4	0.0	0.1	0.0223511	0.0223752	0.0377014	0.0204766	0.0302720	0.0298622	0.0507415	0.0218265
				0.01	0.0360034	0.0360274	0.0487734	0.03144748	0.0411751	0.0406157	0.0626402	0.0268105
				0.001	0.0374519	0.0374759	0.0496884	0.0327097	0.0428305	0.0422305	0.0646255	0.0275248
0.5	0.6	0.8	0.0	0.1	0.0053048	0.0053128	0.00702558	0.00489928	0.0074632	0.00746182	0.0130313	0.0058396
				0.01	0.0069532	0.0069122	0.00968713	0.00629016	0.0086489	0.00862013	0.0175664	0.0063726
				0.001	0.0070592	0.0070672	0.00987948	0.00637807	0.0087630	0.00876253	0.0192431	0.0064499
0.8	0.6	0.8	0.0	0.1	0.0076831	0.0076911	0.01029947	0.0069694	0.0113991	0.0114583	0.0528040	0.0083709
				0.01	0.0137172	0.0137252	0.02026523	0.01222319	0.0161136	0.0162032	0.0764670	0.0105313
				0.001	0.0143485	0.0143565	0.02118143	0.01279029	0.0168535	0.0169380	0.0797924	0.0108430
0.5	0.8	0.6	0.0	0.1	0.0105961	0.0106121	0.01477982	0.0100398	0.0158490	0.0158906	0.0238999	0.0125936
				0.01	0.0136749	0.0136909	0.01929182	0.01233001	0.0183309	0.0183774	0.0316756	0.0135129
				0.001	0.0138766	0.0138926	0.01967097	0.01249538	0.0186576	0.0186794	0.0327929	0.0136305
0.8	0.8	0.6	0.0	0.1	0.0152539	0.0152699	0.02120616	0.01321328	0.0237276	0.0237380	0.0693474	0.0161211
				0.01	0.0258004	0.0258164	0.0315080	0.02278954	0.0328812	0.0329055	0.0966410	0.0196372
				0.001	0.0269077	0.0269237	0.03322523	0.02376962	0.0342914	0.0342890	0.0987746	0.0201899

Table 7: Effect of interchange of the optical thickness with interchange of ω_{01} and ω_{02} on transmissivity and reflectivity of the inhomogeneous slab with $a = 1$, and $\rho_{is} = \rho_{id} = 0$.

ω_{01}	ω_{02}	α	$a=1,\ b-a=2$		$a=2,\ b-a=1$		$a=2,\ b-a=2$	
			R	T	R	T	R	T
0.2	0.5	0.1	0. 3814835	1. 361524	0.113397	1.053299	0.137394	1.289984
		0.01	0. 4145237	1.513266	0.122478	1.148988	0.152344	0.148347
		0.001	0. 4184291	1. 532435	0.123536	1.160980	0154311	1.510178
		0.0	0. 4188720	1.534621	0.123655	1.162346	01545364	1.513258
	0.8	0.1	0. 5204531	1.845730	0.129398	1.293150	0.168193	1.665240
		0.01	0. 3643330	2.364343	0.149192	1.545168	0.212837	2.293429
		0.001	0. 6501982	2.440855	0.151779	1.580205	0.220101	2.399846
		0.0	0. 6520351	2.449762	0.152077	1.584249	0.220958	2.412490
0.5	0.2	0.1	0. 4557831	1.096580	0.224911	0.915371	0.244440	1.066463
		0.01	0. 4756244	1.133781	0.240757	0.949374	0.262205	1.115876
		0.001	0. 4777374	1.138119	0.242540	0.953378	0.264294	1.122020
		0.0	0. 4779744	1.138609	0.242741	0.953831	0,264530	1.122719
	0.8	0.1	0.7355903	1.876124	0.264621	1.340474	0.316901	1.686137
		0.01	0.9004105	2.416842	0.308855	1.626356	0.400044	2.342524
		0.001	0.9231965	2.497045	0.314624	1.666655	0.413561	2.454906
		0.0	0.9258348	2.506388	0.315288	1.671316	0.415158	2.468281
0.8	0.2	0.1	0.7064057	1.112348	0.452372	0.961484	0.478551	1.080431
		0.01	0.7527069	1.152303	0.514472	1.015771	0.546109	1.136449
		0.001	0.7577783	1.156978	0.522182	1.022507	0.554644	1.143519
		0.0	0.7583480	1.157506	0.523060	1.023274	0.555617	1.144324
	0.5	0.1	0.8481859	1.402647	0.481612	1.147350	0.526053	1.323054
		0.01	0.9472394	1.567813	0.566423	1.294563	0.632254	1.540340
		0.001	0.9589230	1.588800	0.577395	1.313903	0.646852	1.570904
		0.0	0.9602472	1.591194	0.578653	1.316120	0.648535	1.574436

1. CONCULUSION

The integral form of the radiative transfer problem in two-region inhomogeneous slab of diffuse and specular reflecting boundaries for anisotropic scattering is considered. The integral form of this problem is solved using the Galerkin method. The reflection and the transmission coefficients for diffusely and specularly reflecting boundaries are calculated respectively. The calculations are given for isotropic, forward and backward anisotropic scattering. The calculations are carried out for two cases. The first is for no internal energy source and external incident of unit radiation. The second is for internal energy source $Q_2 = 1$.0. The results for two inhomogeneous medium with equal and unequal optical distance with isotropic scattering are compared with published calculation [11] and show good agreement.

Table 8: Effect of interchange of specular and diffuse reflection on transmissivity and reflectivity of the inhomogeneous slab for the optical thickness $a=1, b-a=1$, $Q_2=1$ and ω_{01}=2 with interchange of ω_{02}.

ω_{02}	ρ_2	ρ_3	ρ_4	α	Transmissivity and Reflectivity of slab with			
					Specular reflecting boundary		Diffusely reflecting boundary	
					Reflectivity	Transmissivity	Reflectivity	Transmissivity
0.5	0.0	0.0	0.5	0.1 0.01 0.001 0.0	0. 3546917 0. 3651958 0. 3677977 0. 3680912	0. 6466569 0. 6509193 0. 6560981 0. 6566834	0. 3738568 0. 3715201 0. 3743314 0.3746478	0. 6527141 0. 8334378 0. 8419726 0.8429334
0.8	0.0	0.0	0.5	0.1 0.01 0.001 0.0	0. 4787479 0. 5148840 0. 5232911 0. 5242505	0. 9196985 0. 9535426 0. 9713025 0. 9733327	0. 4823624 0. 5302991 0. 5382742 0.5391791	0. 8807284 1.189732 1.209082 1.211275
0.5	0.0	0.0	0.9	0.1 0.01 0.001 0.0	0. 3864378 0. 4123319 0. 4157300 0. 4161139	0.1313027 0.1382965 0.1396012 0.1397489	0. 4514177 0. 4540694 0. 4585897 0.4590992	0.1447056 0.2081196 0.2108055 0.2111081
0.8	0.0	0.0	0.9	0.1 0.01 0.001 0.0	0.5232358 0.6233333 0.6363379 0.6378307	0.1873337 0.2199712 0.2252285 0.2258329	0. 6057888 0. 7081355 0. 7219549 0.7235289	0.2066181 0.3174298 0.3237925 0.3245160
0.5	0.4	0.4	0.0	0.1 0.01 0.001 0.0	0.2354860 0.2445398 0.2459452 0.2461035	1.286189 1.308093 1.317207 1.318236	0. 2179859 0. 2311302 0. 2324162 0.2325610	1.209795 1.282340 1.290807 1.291762
0.8	0.4	0.4	0.0	0.1 0.01 0.001 0.0	0.3252573 0.3207769 0.3249056 0.3253756	1.786892 1.851074 1.880692 1.884067	0. 2661656 0. 2996752 0. 3034295 0.3038566	1.567088 1.782993 1.810329 1.813445
0.5	0.6	0.8	0.0	0.1 0.01 0.001 0.0	0.2054246 0.2086519 0.2098057 0.2099357	1.373371 1.419884 1.430887 1.432130	0. 1812061 0. 1911994 0. 1921834 0.1922942	1.277909 1.358492 1.367943 1.369011
0.8	0.6	0.8	0.0	0.1 0.01 0.001 0.0	0.2635124 0.2742287 0.2780915 0.2784322	1.958354 2.138828 2.179671 2.184342	0. 2170451 0. 2441528 0. 2472743 0.2476306	1.698711 1.965225 1.999735 2.003681
0.5	0.8	0.6	0.0	0.1 0.01 0.001 0.0	0.1610162 0.1627094 0.1633526 0.1634249	1.227846 1.360093 1.370045 1.371169	0. 1433568 0. 1494019 0. 1499897 0.1500558	1.242152 1.318452 1.327353 1.328358
0.8	0.8	0.6	0.0	0.1 0.01 0.001 0.0	0.1892425 0.1917698 0.1934985 0.1936962	1.669666 1.980003 2.014366 2.018289	0. 1603586 0. 1742037 0. 1757357 0.1759100	1.627874 1.865999 1.896395 1.899864

Table 9: Effect of interchange of specular and diffuse reflection on transmissivity and reflectivity of the inhomogeneous slab for the optical thickness $a = 1, b - a = 2$, $Q_2 = 1$ and ω_{01}=2 with interchange of ω_{02}.

ω_{02}	ρ_2	ρ_3	ρ_4	α	Transmissivity and Reflectivity of slab with			
					Specular reflecting boundary		Diffusely reflecting boundary	
					Reflectivity	Transmissivity	Reflectivity	Transmissivity
0.5	0.0	0.0	0.5	0.1	0. 3767966	0.7713258	0. 4771107	0. 9885572
				0.01	0. 4390382	0. 8137857	0. 3855164	2.368677
				0.001	0. 4436574	0. 8262807	0. 3940336	2.425375
				0.0	0.4441824	0.8277095	0.3949913	2.431775
0.8	0.0	0.0	0.5	0.1	0. 5381053	1.308850	0. 6576882	1.445733
				0.01	0. 7077279	1.401495	0. 8580848	3.392830
				0.001	0. 7294837	1.460554	0. 8847248	3.459412
				0.0	0.7320253	1.467496	0. 8876850	3.466709
0.5	0.0	0.0	0.9	0.1	0.3704121	0.1248865	0. 5685857	0.2518574
				0.01	0.4613299	0.1732800	0. 4470273	0.7688254
				0.001	0.4667129	0.1763733	0. 4632526	0.7903335
				0.0	0.4673256	0.1767279	0.4650834	0.7927656
0.8	0.0	0.0	0.9	0.1	0.6467082	0.2563708	0. 8062372	0.3913420
				0.01	0.7888413	0.3295830	1.274966	1.170322
				0.001	0.8188987	0.3471799	1.326885	1.199923
				0.0	0.8224497	0.3492719	1.332689	1.203212
0.5	0.4	0.4	0.0	0.1	0.2856890	1.435037	0. 2739157	1.387920
				0.01	0.3092294	1.562846	0. 2961077	1.544750
				0.001	0.3120145	1.583372	0. 2987373	1.564637
				0.0	0.3123303	1.583372	0.2990356	1.566905
0.8	0.4	0.4	0.0	0.1	0.4301608	2.078108	0. 3680471	1.909618
				0.01	0.4747610	2.535650	0. 4503211	2.478608
				0.001	0.4872117	2.624683	0. 4620986	2.564158
				0.0	0.4872117	2.635077	0.4634671	2.574144
0.5	0.6	0.8	0.0	0.1	0.2272421	1.471443	0. 2195906	1.416635
				0.01	0.2546923	1.620716	0. 2361632	1.579939
				0.001	0.2568992	1.642965	0. 2381340	1.600743
				0.0	0.2571496	1.645505	0.2383576	1.603117
0.8	0.6	0.8	0.0	0.1	0.3283002	2.156215	0. 2913330	1.988321
				0.01	0.3969400	2.771350	0. 3592155	2.632467
				0.001	0.4082949	2.880089	0. 3693120	2.732113
				0.0	0.4096203	2.892838	0.3704913	2.743792
0.5	0.8	0.6	0.0	0.1	0.1655614	1.451559	0.1625287	1.401569
				0.01	0.1851336	1.589979	0. 1716894	1.561281
				0.001	0. 1862739	1.611290	0. 1727534	1.581577
				0.0	0.1864031	1.613721	0.1728739	1.583893
0.8	0.8	0.6	0.0	0.1	0.2147393	2.113678	0. 1963356	1.945667
				0.01	0.2480688	2.640754	0. 2282887	2.547071
				0.001	0.2530503	2.738195	0. 2328842	2.638620
				0.0	0.2536296	2.749591	0.2334188	2.649325

2. REFERENCES

[1] Shouman SM, Ozisik MN. Radiative transfer in an isotropically scattering two region slab with reflecting boundaries. J. Quant. Spec. Radiat. Transfer; 26, 1 (1981):26: 1-9.

DOI: 10.1016/0022-4073(81)90014-5

[2] Siewert C H, Anote on radiative transfer in a finite layer. J. physics A 2007; 40 : 1785-1789

DOI: 10.1088/1751-8113/40/8/007

[3] Elghazaly A and Soliman N F. Solution of transport equation for two medium slab lattice with anisotropic scattering. . Arab J. of Nucl Scie and Applications 2012; 45(3): 136-144.

[4] Maruyama S. Radiative transfer in anisotropic scattering media with specular boundary subjected to collimated irradiation. Int J Heat Mass Transfer 1998; 41: 2847-2856.

DOI: 10.1016/S0017-9310(98)00055-6

[5] Elghazaly A, Particle Transfer Problem for Inhomogeneous Finite Medium With Generalized Boundary Condition. Arab J. of Nucl Scie and Applications 2004; 37(1): 187.

[6] Zhou H C, The influence of Anisotropic scattering on the radiative intensity in a gray plane – parallel medium calculated by the DRESOR method, J. Quant. Spec. Radiat. Transfer. 2007; 104: 99-115.

DOI: 10.1016/j.jqsrt.2006.08.015

[7] Andre` Liemert and Alwin Kienle. Analytical approach for solving radiative transfer equation in two-dimensional layered media. J. Quant. Spec. Radiat. Transfer 2012; 113: 559-564.

DOI:10.1016/j.jqsrt.2012.01.013

[8] Liou B-T, Wu C-Y. Composite discrete-ordinate solutions for radiative transfer in a two-layer medium with Fresnel interfaces. Numer Heat Transfer Part A 1996; 30: 739-751.

DOI: 10.1080/10407789608913868

[9] Garacia RDM. Radiative transfer with polarization in a multi-layer medium subject to Fresnel boundary and interface conditions J. Quant. Spec. Radiat. Transfer 2013; 115: 28-45.

DO:10.1016/j.jqsrt.2012.09.004

[10] Zekeriya Altac and Messut Tekkalmaz. Nodal synthetic kernel (N-SK_N) method for solving radiative heat transfer problem in one and two dimensional participating medium with isotropic scattering. J. Quant. Spec. Radiat. Transfer 2013; 129: 214-235.

DOI: 10.1016/j.jqsrt.2013.06.017

[11] Elghazaly A. Particle transfer problem in a two-region inhomogeneous slab with generalized boundary condition. J. Quant. Spec. Radiat. Transfer 2006; 97: 99-111.

DOI: 10.1016/j.jqsrt.2004.12.023

[12] Awatif A. Hendi, Elghazaly A. The Solution of the Neutron Transport Equation In Slab with Anisotropic Scattering. J. Quant. Spec. Radiat. Transfer 2004; 84(3): 339.

DOI: 10.1016/S0022-4073(03)00188-2

[13] Ozisik MN. Radiative transfer and Inetaction with conduction and convection, Wiley, New York, NY (1973).

[14] Attia MT, Madkour MA, Abulwafa EM, Abd-Elnaby MM. Anisotropic radiation transfer in a plane medium with specularly-reflecting boundary conditions. J. Quant. Spec. Radiat. Transfer. 1992; 47: 221-227.

DOI: 10.1016/0022-4073(92)90030-8