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RESEARCH ARTICLE

ESTIMATION OF THE SURVIVAL FUNCTION UNDER THE CONSTANT SHAPE BI-WEIBULL FAILURE TIME DISTRIBUTION BASED ON THREE LOSS FUNCTIONS.

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Abstract

We Consider the Constant Shape Bi-Weibull distribution which has been extensively used in the testing and reliability studies of the strength of materials. Studies have been done vigorously in the literature to determine the best method in estimating its Survival function. In this paper, we examine the performance of Maximum Likelihood Estimator (MLE) and Bayesian Estimator using Extension of Jeffreys' Prior Information with three Loss functions, namely, the Linear Exponential (LINEX) Loss, General Entropy Loss, and Square Error Loss for estimating Survival Function under the Constant Shape Bi-Weibull Failure time distribution. The results show that Bayesian Estimator using Extension of Jeffreys' Prior under Linear Exponential (LINEX) Loss function in most cases gives the smallest Mean Square Error and Absolute Bias of Survival function S(t) for the given values of Extension of Jeffreys' Prior. An illustrative example is also provided to explain the concepts.

Introduction:-

As a result of the adaptability in fitting time-to-failure of a very widespread multiplicity to multifaceted mechanisms, the Weibull distribution has assumed the centre stage especially in the field of life-testing and reliability/survival analysis. It has shown to be very useful for modeling and analyzing life time data in medical, biological and engineering sciences, Lawless [17]. Much of the attractiveness of the Weibull distribution is due to the wide variety of shapes it can assume by altering its parameters. According to [19], "A data sample is said to be censored when, either by accident or design, the value of the variables under investigation is unobserved for some of the items in the sample." Maximum Likelihood Estimator (MLE) is quiet efficient and very popular both in literature and practice. Bayesian approach has been employed for estimating parameters. Some researchers have made comparisons of MLE and that of the Bayesian approach in estimating the survival function and the parameters of the Weibull distribution. According to [20] determined the Bayes estimates of the reliability function and the hazard rate of the Weibull failure time distribution by employing squared error loss function, [1] studied the approximate Bayesian estimates for the Weibull reliability function and hazard rate from censored data by employing a new method that has the potential of reducing the number of terms in Lindley procedure, and [5] conducted a study on Bayesian survival estimator for Weibull distribution with censored data using squared error loss function with Jeffreys prior amongst others. [10] applied Bayesian estimation, for the two-parameter Weibull distribution using extension of Jeffreys' prior information with three loss functions, [21] considered Bayesian estimation and prediction for Weibull model with progressive censoring. Other recent papers employing different

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models can be seen in [6], [22], and [7, 8]. Similar work can be seen in [11], [23], [2], [3], [9], [18], and a work on generalized exponential distribution: Bayesian estimations, [12] which are somehow similar to the Weibull distribution.

In recent, work we developed Functional Relationship between Brier Score and Area Under the Constant Shape Bi-Weibull ROC Curve [16], Confidence Intervals Estimation for ROC Curve, AUC and Brier Score under the Constant Shape Bi-Weibull Distribution [13], Asymmetric and Symmetric Properties of Constant Shape Bi-Weibull ROC Curve Described by Kullback-Leibler Divergences [14], and Bayesian Estimation of Parameters under the Constant Shape Bi-Weibull Distribution Using Extension of Jeffreys' Prior Information with Three Loss Functions[15].

Now the main objective of this paper is to compare the traditional Maximum Likelihood Estimation of the Survival function of the Constant Shape Bi-Weibull distribution with its Bayesian counterpart using Extension of Jeffreys' Prior Information obtained from Lindley's approximation procedure with three Loss Functions.

In this paper, the Bayesian Estimation of Survival function under the Constant Shape Bi-Weibull Distribution is studied by Using Extension of Jeffreys' Prior Information with Three Loss Functions. This paper is organized as follows: In Section 2, estimation of Survival function under MLE, Jeffreys' Prior Information and Extension of Jeffreys' Prior Information with Three Loss functions is discussed. Section 3, provides simulation study for proposed theory. In Section 4, the proposed theory is validated by using real data. Finally conclusions are provided in Section 5.

Materials and Methods:-

Let $t_1, t_2 \dots t_n$ be a random sample of size $\boldsymbol{n}$ with respect to the Constant Shape Bi-Weibull distribution, with $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ as the parameters, where $\boldsymbol{\sigma}$ is the scale parameter and $\boldsymbol{\beta}$ is the shape parameter. The probability density function (pdf), cumulative distribution function (cdf) and survival function are given, respectively, as

$$f(t;\sigma,\beta) = \frac{\beta}{\sigma} t^{\beta-1} e^{-\left[\frac{t^{\beta}}{\sigma}\right]}. \tag{1}$$

The Cumulative distribution function (CDF) is

$$F(t;\sigma,\beta) = 1 - e^{-\left[\frac{t^{\beta}}{\sigma}\right]} . \tag{2}$$

The Survival function is

$$S(t;\sigma,\beta) = e^{-\left[\frac{t^{\beta}}{\sigma}\right]} . \tag{3}$$

2.1 Maximum Likelihood Estimation of Constant Shape Bi-Weibull Distribution:-

Since ($t_1, t_2 \dots t_n$) is the set of $\boldsymbol{n}$ random lifetimes from the Constant Shape Bi-Weibull distribution, with $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ as the parameters, where $\boldsymbol{\sigma}$ is the scale parameter and $\boldsymbol{\beta}$ is the shape parameter.

The likelihood function of the pdf is

$$L(t_i,\sigma,\beta) = \prod_{i=1}^{n} \frac{\beta}{\sigma} {t_i}^{\beta-1} e^{-\left[\frac{{t_i}^{\beta}}{\sigma}\right]}. \tag{4}$$

The log-likelihood function is

$$lnL = nln\beta + (\beta-1)\left[\sum_{i=1}^{m} lnt_i\right] - nln\sigma - \frac{1}{\sigma}\sum_{i=1}^{n} {t_i}^{\beta}. \tag{5}$$

By differentiating the equation (5) with respect to $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ and equating to zero, we get

$$\frac{\partial lnL}{\partial \sigma} = -\frac{n}{\sigma} + \frac{\sum_{i=1}^{n} t_i{}^{\beta}}{\sigma^2} = 0\,. \tag{6}$$

$$\frac{\partial lnL}{\partial \beta} = \frac{n}{\beta} + \left[\sum_{i=1}^{n} lnt_i\right] - \frac{1}{\sigma}\sum_{i=1}^{n} t_i{}^{\beta}\, lnt_i \;\; = 0\,. \tag{7}$$

From equation (6), we get

$$\hat{\sigma} = \frac{1}{n}\,\sum_{i=1}^{n} t_i{}^{\beta} \quad . \tag{8}$$

First we shall find $\widehat{\boldsymbol{\beta}}$ and so that $\widehat{\boldsymbol{\sigma}}$ can be determined. So that we propose to find $\widehat{\boldsymbol{\beta}}$ by using Newton-Raphson method as given below. Let $f(\boldsymbol{\beta})$ be the same as equation (6) and taking the first differential of $f(\boldsymbol{\beta})$, we have

$$f'(\beta) = -\left(\frac{n}{\beta^2}\right) - \frac{1}{\sigma}\sum_{i=1}^{n} t_i{}^{\beta}\,(lnt_i\,)^2 \quad . \tag{9}$$

By substituting equation (8) into equation (7), we call $f(\boldsymbol{\beta})$ as

$$f(\beta) = \frac{n}{\beta} + \left[\sum_{i=1}^{n} lnt_i\right] - \frac{\sum_{i=1}^{n} t_i{}^{\beta}\, lnt_i}{\frac{1}{n}\;\sum_{i=1}^{n} t_i{}^{\beta}} \quad . \tag{10}$$

Substituting equation (8) into equation (9), we obtain

$$f'(\beta) = -\left\{\frac{n}{\beta^2} + \frac{\sum_{i=1}^{n} t_i{}^{\beta}\,(lnt_i\,)^2}{\frac{1}{n}\;\sum_{i=1}^{n} t_i{}^{\beta}}\right\} \quad . \tag{11}$$

Therefore, $\widehat{\boldsymbol{\beta}}$ is obtained from the equation below by carefully choosing an initial value $\boldsymbol{\beta}$ as $\boldsymbol{\beta_i}$ and iterating the process till it converges:

$$\beta_{i+1} = \beta_i - \frac{\frac{n}{\beta} + \left[\sum_{i=1}^{n} lnt_i\right] - \frac{\sum_{i=1}^{n} t_i{}^{\beta}\, lnt_i}{\frac{1}{n}\;\sum_{i=1}^{n} t_i{}^{\beta}}}{-\left\{\frac{n}{\beta^2} + \frac{\sum_{i=1}^{n} t_i{}^{\beta}\,(lnt_i\,)^2}{\frac{1}{n}\;\sum_{i=1}^{n} t_i{}^{\beta}}\right\}}\,. \tag{12}$$

The estimate of the survival function of the Constant Shape Bi-Weibull distribution under MLE is

$$\hat{S}(t) = e^{-\left[\frac{t^{\hat{\beta}}}{\hat{\sigma}}\right]}\,. \tag{13}$$

2.2 Bayesian Estimation of Survival Function for Constant Shape Bi-Weibull Distribution:-

Bayesian Estimation approach has received a lot of attention in recent times for analyzing Failure Time data, which has mostly been proposed as an alternative to that of the traditional methods. Bayesian Estimation approach makes use of once prior knowledge about the parameters as well as the available data. When once prior knowledge about the parameter is not available, it is possible to make use of the noninformative prior in Bayesian analysis. Since we have no knowledge on the parameters, we seek to use the Extension of Jeffreys' Prior Information, where Jeffreys' Prior is the square root of the determinant of the Fisher information. According to [4], the Extension of Jeffreys' prior is obtained by taking $u(\theta) \propto [I(\theta)]^c$, $c \epsilon R^+$, so that

$$u(\theta) \propto \left[\frac{1}{\theta}\right]^{2c}.$$

Thus,

$$u(\sigma,\beta) \propto \left[\frac{1}{\sigma\beta}\right]^{2c}.$$

Given a sample t= ($t_1, t_2, \ldots, t_n$) from the likelihood function of the pdf (1) is

$$L(t_i \mid \sigma,\beta) = \prod_{i=1}^{n} \frac{\beta}{\sigma} {t_i}^{\beta-1} e^{-\left[\frac{{t_i}^{\beta}}{\sigma}\right]}$$

With Bayes theorem, the joint posterior distribution of the parameters $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ is

$$\pi^*(\sigma,\beta|t) \propto L(t|\,\sigma,\beta)u(\sigma,\beta)$$

$$L(t_i \mid \sigma,\beta) = \frac{k}{(\sigma\beta)^{2c}} \prod_{i=1}^{n} \frac{\beta}{\sigma} {t_i}^{\beta-1} e^{-\left[\frac{{t_i}^{\beta}}{\sigma}\right]},$$

where $\boldsymbol{k}$ is the normalizing constant that makes $\boldsymbol{\pi}^*$ a proper pdf.

2.2.1 Asymmetric Loss Functions:-

Here we consider two Asymmetric Loss Functions namely Linear Exponential Loss Function (LINEX) and General Entropy Loss Function.

2.2.1(a) Linear Exponential Loss Function (LINEX):-

The LINEX Loss Function is under the assumption that the minimal loss occurs at $\hat{\theta} = \theta$ and is expressed as

$$L(\hat{\theta} - \theta) \propto \exp\left(a(\hat{\theta} - \theta)\right) - a(\hat{\theta} - \theta) - 1$$

where $\hat{\boldsymbol{\theta}}$ is an estimation of $\boldsymbol{\theta}$ and $a \neq 0$. The sign and magnitude of the shape parameter '**a**' represents the direction and degree of symmetry, respectively. There is overestimation if $a > 0$ and underestimation if $a < 0$ but when $\mathbf{a} \cong 0$, the LINEX Loss Function is approximately the Squared Error Loss Function. The posterior expectation of the LINEX Loss Function, according to [10], is

$$E_\theta L(\hat{\theta} - \theta) \propto exp(a\hat{\theta})E_\theta(\exp(-a\theta)) - a\left(\hat{\theta} - E_\theta(\theta)\right) - 1\,. \qquad (14)$$

The Bayes Estimator of $\boldsymbol{\theta}$, represented by $\hat{\boldsymbol{\theta}}_{\boldsymbol{BL}}$ under LINEX Loss Function, is the value of $\hat{\boldsymbol{\theta}}$ which minimizes equation (14) and is given as

$$\hat{\theta}_{BL} = -\frac{1}{a} ln\, E_\theta(exp(-a\theta)).$$

Provided $E_\theta(exp(-a\theta)$ exists and is finite.

The posterior density function of the survival function under LINEX loss is given as

$$\hat{S}(t)_{BL} = E\left\{exp\left(-ae^{-\left[\frac{{t_i}^{\beta}}{\sigma}\right]}\right)|t_i\right\} = \frac{\iint exp\left(-ae^{-\left[\frac{{t_i}^{\beta}}{\sigma}\right]}\right)\pi^*(\sigma,\beta)d\sigma d\beta}{\iint \pi^*(\sigma,\beta)d\sigma d\beta}\,. \qquad (15)$$

From (15), it can be observed that ratio of integrals which cannot be solved analytically and for that we employ Lindley's approximation procedure to estimate the parameters. Lindley considered an approximation for the ratio of integrals for evaluating the posterior expectation of an arbitrary function $\hat{\boldsymbol{u}}(\boldsymbol{\theta})$ as

$$E[u(\theta)|x] = \frac{\int u(\theta)v(\theta)[L(\theta)]d\theta}{\int v(\theta)[L(\theta)]d\theta}$$

According to [20], Lindley's expansion can be approximated asymptotically by

$$\hat{\theta} = u + \frac{1}{2}[u_{11}\delta_{11} + u_{22}\delta_{22}] + u_1\rho_1\delta_{11} + u_2\rho_2\delta_{22} + \frac{1}{2}\left[L_{30}u_1{\delta^2}_{11} + L_{03}u_2{\delta^2}_{22}\right], \quad (16)$$

where L is the log-likelihood function in equation (5),

$$u = exp\left(-ae^{-\left[\frac{t_i{}^{\beta}}{\sigma}\right]}\right),$$

$$q = e^{-\left[\frac{t_i{}^{\beta}}{\sigma}\right]},$$

$$u_1 = \frac{\partial u}{\partial \sigma} = auq\left(\frac{t_i{}^{\beta}}{\sigma^2}\right),$$

$$u_{11} = \frac{\partial^2 u}{\partial^2 \sigma} = uq\left(\frac{\left(at_i{}^{\beta}\right)^2 q - a\left(t_i{}^{\beta}\right)^2}{\sigma^4} + \frac{2at_i{}^{\beta}}{\sigma^3}\right),$$

$$u_2 = \frac{\partial u}{\partial \beta} = \frac{auqt_i{}^{\beta}\, lnt_i}{\sigma},$$

$$u_{22} = \frac{\partial^2 u}{\partial^2 \beta} = uq\left[\frac{(lnt_i)^2 at_i{}^{\beta}}{\sigma} + \left(\frac{alnt_i t_i{}^{\beta}}{\sigma}\right)^2\right],$$

$$\rho(\sigma,\beta) = -ln(\sigma^{2c}) - ln(\beta^{2c}),$$

$$\rho_1 = \frac{\partial \rho}{\partial \sigma} = -\frac{1}{\sigma^{2c}}, \qquad \rho_2 = \frac{\partial \rho}{\partial \beta} = -\frac{1}{\beta^{2c}},$$

$$\delta_{11} = (-L_{20})^{-1}, \delta_{22} = (-L_{02})^{-1},$$

$$L_{02} = -\left(\frac{n}{\beta^2}\right) - \frac{1}{\sigma}\sum_{i=1}^{n} t_i{}^{\beta}\,(lnt_i\,)^2,$$

$$L_{03} = 2\left(\frac{n}{\beta^3}\right) - \frac{1}{\sigma}\sum_{i=1}^{n} t_i{}^{\beta}\,(lnt_i\,)^3,$$

$$L_{20} = \frac{n}{\sigma^2} - 2\frac{\sum_{i=1}^{n} t_i{}^{\beta}}{\sigma^3}, and\ L_{30} = -2\frac{n}{\sigma^3} + 6\frac{\sum_{i=1}^{n} t_i{}^{\beta}}{\sigma^4}.$$

2.2.1(b) General Entropy Loss Function

Another useful Asymmetric Loss Function is the General Entropy (GE) Loss which is a generalization of the Entropy Loss and is given as

$$\mathrm{L}(\hat{\theta} - \theta) \propto \left(\frac{\hat{\theta}}{\theta}\right)^k - k\ ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\ .$$

The Bayes Estimator $\hat{\boldsymbol{\theta}}_{BG}$ of $\boldsymbol{\theta}$ under the General Entropy Loss is

$$\hat{\theta}_{BG} = [E_\theta(\theta^{-k})]^{-\frac{1}{k}} \quad ,$$

provided $E_\theta(\theta^{-k})$ exists and is finite.

The posterior density function of the Survival function under general entropy loss is given as

$$\hat{S}(t)_{BG} = E\left\{\left(exp\left[\frac{-t_i{}^{\beta}}{\sigma}\right]\right)^{-k} |t_i\right\} = \frac{\iint \left(exp\left[\frac{-t_i{}^{\beta}}{\sigma}\right]\right)^{-k} \pi^*(\sigma,\beta)d\sigma d\beta}{\iint \pi^*(\sigma,\beta)d\sigma d\beta}.$$

Applying the same Lindley approach here as in (16) with u_1, u_{11} and u_2, u_{22} are the first and second derivatives for $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$, respectively, and are given as

$$u = \left(exp\left[\frac{-t_i{}^{\beta}}{\sigma}\right]\right)^{-k},$$
$$e = \left[\frac{-t_i{}^{\beta}}{\sigma}\right],$$
$$u_1 = \frac{\partial u}{\partial \sigma} = \frac{k}{\sigma}ue,$$
$$u_{11} = \frac{\partial^2 u}{\partial^2 \sigma} = ue^2\left(\frac{k}{\sigma}\right)^2 - \frac{2kue}{\sigma^2},$$
$$u_2 = -kue(lnt_i) \ and \ u_{22} = \frac{\partial^2 u}{\partial^2 \beta} = -kue(lnt_i)^2 + u(kelnt_i)^2$$

2.2.2 Symmetric Loss Function:-

The Symmetric Loss Function is the Squared Error Loss is given by

$$\mathrm{L}(\hat{\theta} - \theta) \propto (\hat{\theta} - \theta)^2.$$

This Loss Function is symmetric in nature, that is, it gives equal weightage to both over and under estimation. In real life, we encounter many situations where overestimation may be more serious than underestimation or vice versa.

The most common loss function used for Bayesian estimation is the squared error (SE), also called quadratic loss. The square error loss denotes the punishment in using to $\hat{\theta}$ estimate θ and is given as $E_\theta(t|\theta) = \left(\hat{\theta}(t) - \theta\right)^2$, where the expectation is taken over the joint distribution of θ and (t).

The posterior density function of the Survival function under the Symmetric loss function are given as

$$\hat{S}(t)_{BS} = E\left\{exp\left[\frac{-t_i{}^{\beta}}{\sigma}\right]|t_i\right\} = \frac{\iint exp\left[\frac{-t_i{}^{\beta}}{\sigma}\right]\pi^*(\sigma,\beta)d\sigma d\beta}{\iint \pi^*(\sigma,\beta)d\sigma d\beta}.$$

Applying the same Lindley approach here as in (16) with u_1, u_{11} and u_2, u_{22} are the first and second derivatives for $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$, respectively, and are given as

$$u = exp\left[\frac{-t_i{}^{\beta}}{\sigma}\right],$$
$$e = \left[\frac{-t_i{}^{\beta}}{\sigma}\right],$$
$$u_1 = \frac{\partial u}{\partial \sigma} = \frac{-ue}{\sigma},$$

$u_{11} = \frac{\partial^2 u}{\partial^2 \sigma} = u\left(\frac{e}{\sigma}\right)^2 - \frac{2ue}{\sigma^2},$

$u_2 = ue(lnt_i)\ and\ u_{22} = \frac{\partial^2 u}{\partial^2 \beta} = ue\ lnt_i(elnt_i + 1)$

Simulation study:-

Since it is difficult to compare the performance of the estimators theoretically and also to validate the data employed in this paper, we have performed extensive simulations to compare the estimators through Mean Squared Errors and Absolute Biases by employing different sample sizes with different parameter values. The Mean Squared Error and Absolute Bias given as

$$MSE = \frac{\sum_{r=1}^{5000}\left(\hat{\theta}^r - \theta\right)^2}{R-1}, and\ Abs = \frac{\sum_{r=1}^{5000}\left|\hat{\theta}^r - \theta\right|}{R-1}.$$

In our Simulation study, we chose a sample size of $\boldsymbol{n}$= 25, 50, and 100 to represent small, medium, and large dataset. The Survival function is estimated for Constant Shape Bi-Weibull distribution with Maximum Likelihood and Bayesian using Extension of Jeffreys' Prior methods. The values of the parameters chosen are $\boldsymbol{\sigma} = 0.5$ and 1.5, $\boldsymbol{\beta} = 0.8$ and 1.2. The values of Jeffreys Extension are $\boldsymbol{c} = 0.4$ and 1.4. The values for the Loss parameters (a, k) are $a = k = \pm 0.6\ and\ \pm 1.6$. These were iterated (*R)* 5000 times and the Survival function for each method was calculated. The results are presented below for the estimated Survival function and their corresponding Mean Squared Error and Absolute Bias values.

In Table 3.1 we present the Mean Square Error estimated values for the Survival function S(t) for both the MLE and Bayesian Estimation using extension of Jeffrey's prior information with the three loss functions.

Table 3.1:- MSE Estimated Survival function.

n	σ	c	β	$\hat{S}(t)_{ML}$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$
						a = k = 0.6		**a= k = -0.6**		**a =k =1.6**		**a = k = -1.6**	
25	0.5	0.4	0.8	7.7286	4.1372	0.8652	1260.5	2.3701	3.7401	3.0528	9342975	45.756	3.3481
	0.5	0.4	1.2	7.8043	4.6105	0.9427	454.47	3.0413	3.4182	2.9345	1382589	67.156	4.8103
	0.5	1.4	0.8	8.1556	4.9309	1.0151	836.12	3.0310	3.9139	3.3621	6652040	63.221	4.6580
	0.5	1.4	1.2	7.4826	4.8884	1.0372	273.62	3.4794	3.3785	3.1383	4080642	79.522	5.4873
	1.5	0.4	0.8	6.9660	4.0078	0.9184	381.69	2.4404	3.3646	3.4869	4566042	45.978	3.7161
	1.5	0.4	1.2	7.1391	4.7772	1.0426	3004.3	3.0765	3.9921	3.5726	2064227	63.870	4.7267
	1.5	1.4	0.8	7.7948	3.5288	0.9943	6.3846	3.3877	1.3838	2.8735	228.883	76.476	3.7329
	1.5	1.4	1.2	6.4431	3.4645	0.7678	4815.7	2.1600	3.1275	2.7644	6412273	43.963	3.2338
50	0.5	0.4	0.8	15.840	8.2012	1.7322	108068	5.1308	7.1835	5.8232	1.0e+16	106.86	7.6403
	0.5	0.4	1.2	16.070	8.6333	1.7655	670.74	5.6865	6.7338	5.5099	8064524	125.13	8.9937
	0.5	1.4	0.8	16.471	8.8129	1.8469	5447.0	5.5562	7.3594	6.0578	4601141	115.18	8.4626
	0.5	1.4	1.2	16.360	8.6203	1.7912	651.85	5.7124	6.5140	5.5683	1218967	123.16	9.0463
	1.5	0.4	0.8	14.849	9.0016	1.9101	356.78	6.3178	6.6708	6.1060	945835	145.15	10.230
	1.5	0.4	1.2	14.591	7.8900	1.6767	145.39	5.3890	5.6621	5.1631	90103.7	115.30	8.6538
	1.5	1.4	0.8	16.470	7.0496	1.6609	99.766	5.9171	4.4831	4.8834	189445	141.07	9.1766
	1.5	1.4	1.2	15.343	9.7015	2.0816	1524.8	6.2387	8.2141	6.9332	3085917	129.65	9.5216
100	0.5	0.4	0.8	33.173	16.822	3.5247	1759.4	10.791	14.157	11.477	1562816	228.47	16.462
	0.5	0.4	1.2	32.955	14.052	2.8044	2797.3	9.8480	10.354	8.1578	3691605	230.01	16.277
	0.5	1.4	0.8	32.689	17.128	3.5921	5548.6	11.059	14.572	11.703	7984880	236.49	16.955
	0.5	1.4	1.2	33.380	16.310	3.3506	4376.0	11.174	11.793	9.9616	3376230	247.71	18.056
	1.5	0.4	0.8	31.816	18.497	3.8209	2719.5	12.575	14.194	11.909	5498157	282.37	20.242
	1.5	0.4	1.2	33.237	18.655	3.7950	1765.2	13.082	13.561	11.217	2354539	302.19	21.358
	1.5	1.4	0.8	32.257	15.335	3.1350	421.93	10.680	10.933	9.3752	1271190	243.30	17.606
	1.5	1.4	1.2	29.012	16.222	3.4484	1306.1	10.470	13.333	11.263	1631574	218.49	16.133
ML: Maximum Likelihood, BS: Squared Error Loss, BL: LINEX Loss function, BG: General Entropy Loss function.													

From Table 3.1 it is observed that Bayes estimation with LINEX loss function provides the smallest MSE values in most cases especially the loss parameter value is 0.6. Also when sample size increases MLE and Bayes estimation under all loss functions have increases in MSE values.

In Table 3.2 we present the Absolute Bias estimated values for the Survival function S(t) for both the Maximum Likelihood Estimation and Bayesian Estimation using extension of Jeffrey's prior information with the three loss functions.

Table 3.2:- Absolute Bias Estimated Survival function.

n	σ	c	β	$\hat{S}(t)_{ML}$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$
						a = k = 0.6		**a= k = -0.6**		**a =k =1.6**		**a = k = -1.6**	
25	0.5	0.4	0.8	11.728	8.1340	3.7390	100.42	6.0524	7.8513	7.0736	15739.2	25.842	7.0204
	0.5	0.4	1.2	12.002	8.6631	3.9072	50.117	7.0931	7.3018	6.7416	5134.64	33.119	8.9042
	0.5	1.4	0.8	12.222	8.9252	4.0344	69.207	7.0255	7.8055	7.2348	11514.6	32.027	8.6982
	0.5	1.4	1.2	11.506	9.0313	4.1551	40.931	7.5496	7.4276	7.1743	2944.71	35.354	9.4509
	1.5	0.4	0.8	11.015	8.0853	3.8906	58.898	6.2195	7.4513	7.5695	3708.94	26.308	7.5583
	1.5	0.4	1.2	11.262	8.8719	4.1262	109.02	7.1112	7.9811	7.4981	54317.2	31.922	8.7544
	1.5	1.4	0.8	12.020	7.9611	4.1551	9.5026	7.7009	4.8241	6.8889	50.2055	35.838	8.0285
	1.5	1.4	1.2	10.745	7.4052	3.5269	135.24	5.6727	7.1197	6.7392	95734.9	24.005	6.6825
50	0.5	0.4	0.8	24.449	16.066	7.4706	1515.9	12.384	15.381	13.862	1174836	53.733	14.539
	0.5	0.4	1.2	24.560	16.978	7.6637	99.217	13.705	14.814	13.342	5797.83	62.965	17.057
	0.5	1.4	0.8	24.843	17.089	7.7968	198.65	13.539	15.382	13.919	103093	60.826	16.590
	0.5	1.4	1.2	24.584	16.920	7.6999	91.062	13.711	14.545	13.421	6220.23	62.624	17.130
	1.5	0.4	0.8	23.031	17.272	7.9167	77.734	14.476	14.609	13.762	2701.55	68.393	18.323
	1.5	0.4	1.2	22.610	16.135	7.4197	56.454	13.330	13.543	12.825	1001.77	61.055	16.800
	1.5	1.4	0.8	24.631	15.054	7.2968	38.028	14.035	11.724	12.021	806.880	68.410	17.448
	1.5	1.4	1.2	23.343	17.868	8.2410	154.18	14.323	16.239	14.791	14965.1	64.490	17.544
100	0.5	0.4	0.8	49.655	33.306	15.297	247.69	26.364	30.564	27.451	14373.8	117.59	31.850
	0.5	0.4	1.2	50.390	30.411	13.517	185.12	25.429	25.409	22.453	34752.9	120.92	32.502
	0.5	1.4	0.8	49.580	33.751	15.453	340.34	26.890	30.830	27.554	69092.5	120.99	32.760
	0.5	1.4	1.2	49.750	32.855	14.835	188.29	27.222	27.554	25.146	81042.8	126.93	34.451
	1.5	0.4	0.8	48.045	35.047	15.862	217.18	28.856	30.284	27.434	35799.8	134.84	36.308
	1.5	0.4	1.2	49.311	34.992	15.669	168.31	29.433	29.303	26.282	22217.4	140.80	37.493
	1.5	1.4	0.8	48.631	31.905	14.353	115.36	26.620	26.551	24.244	3063.02	125.47	33.988
	1.5	1.4	1.2	45.594	32.716	15.134	210.24	26.004	29.683	27.189	11143	115.24	31.631
ML: Maximum Likelihood, BS: Squared Error Loss, BL: LINEX Loss function, BG: General Entropy Loss function.													

From Table 3.2 it is observed that Bayes estimation with LINEX loss function provides the smallest Absolute Bias values in most cases. As the sample size increases Absolute values of the MLE and Bayes estimation under all loss functions increases.

Illustration:-

The real data set is about a clinical Trial in the Treatment of Carcinoma of the Oropharynx (PHARYNX) Data extracted from [24]. The data file gives the data for a part of a large clinical trial carried out by the Radiation Therapy Oncology Group in the United States. This data consists of a total of 195 respondents of which 53 are alive and 142 are dead. Here we considered Survival time in days from day of diagnosis is the most factor. Table 4.1 depicts the Standard Error values for Estimated Survival function S(t) using PHARYNX Data.

Table 4.1:- Standard Error values for Estimated Survival function S(t) using PHARYNX Data

Estimates	$\hat{S}(t)_{ML}$	$\hat{S}(t)_{BS}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$	$\hat{S}(t)_{BL}$	$\hat{S}(t)_{BG}$
			a = k = 0.6		**a= k = -0.6**		**a=k=1.6**		**a=k=-1.6**	
c=0.4	0.0036	2.3e-89	0.000	3.1e+54	0.000	2.6e-54	0.000	1.9e+147	0.000	5.5e-142
c=1.4	0.0036	2.6e-87	0.000	1.4e+57	0.000	2.3e-52	0.000	9.2e+149	0.000	7.1e-140

From Table 4.1, we observe that, Bayesian estimator under LINEX loss function has the smallest values for Survival function S(t). So that the Bayes estimators of Survival function S(t) under LINEX loss function is best estimation method for Constant Shape Bi-Weibull Distribution using PHARYNX Data.

Conclusion:-

In this paper, we have addressed the problem of Bayesian estimation of Survival function for the Constant Shape Bi-Weibull distribution, under Asymmetric and Symmetric loss functions and that of Maximum Likelihood Estimation. Bayes estimators were obtained using Lindley approximation while MLE were obtained using Newton-Raphson method. A Simulation study was conducted to examine and compare the performance of the estimates for different sample sizes with different values for the extension of Jeffreys' prior and the loss functions. From the results, we observe that in most cases, Bayesian estimator under LINEX loss function has the smallest Mean Squared Error values and minimum Bias for Survival function S(t) in most cases especially compared when the loss parameter values are 0.6 and 1.6, for both values of the extension of Jeffreys' prior information. As the sample size increases the Mean Squared Error and the Absolute Bias for Maximum Likelihood Estimator and Bayes estimator under all the loss functions increases correspondingly.

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